Conformal defects in gauged WZW models

40th Ahrenshoop Symposium/ Crete - Kolymbari

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based on: CB, S.Monnier, **JHEP 1002:003, 2010**

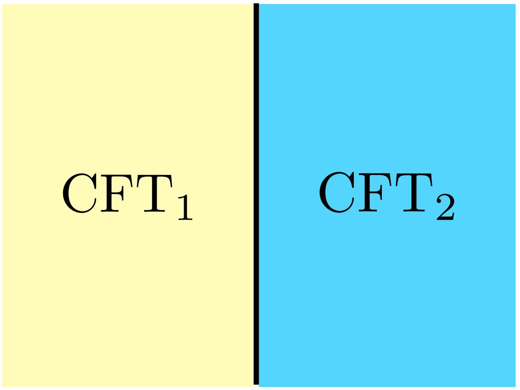
& earlier: CB, M.Gaberdiel, **JHEP 0411:065, 2004**
A.Alekseev, S.Monnier, **JHEP 0708:039, 2007**

Outline

- Conformal Interfaces/Defects: what are they good for?
- Loop operators in WZW models and universal matrix model
- Reduction to GKO models
- Distances between CFTs
- Outlook

CB, Brunner, Douglas, Rastelli, in progress

Conformal Interfaces/Defects



Interface

conformal if $T_{\sigma\tau} = T_{++} - T_{--}$

is continuous (no energy flow to interface)

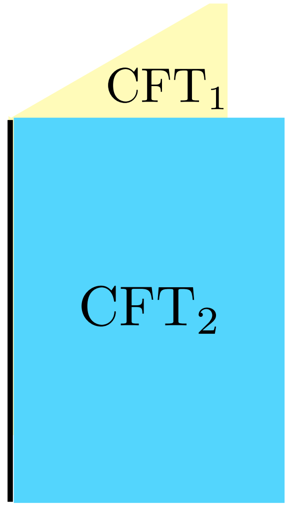
special cases:

<u>boundary</u> $CFT_2 = \emptyset$

<u>defect</u> $CFT_1 = CFT_2$

<u>topological</u> T_{++} , T_{--} separately continuous

Folding:



interface = boundary of special (tensor) CFTs

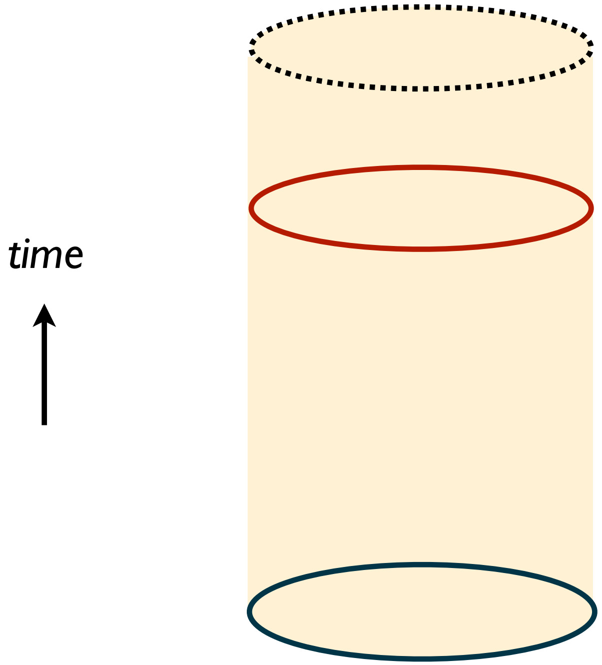
$CFT_1 \otimes CFT_2$

In this talk: focus on conformal defects , described by

* A n-dimensional space of quantum states*

* An interaction Hamiltonian H_{imp} which is an $n \times n$ matrix with entries depending on the local bulk fields*

Exchanging the roles of space and time:



defect

Associate an operator acting on the states of the CFT

$$\mathcal{O} = tr(P e^{-i \oint H_{\text{imp}}})$$

$$[T_{++} - T_{--}, \mathcal{O}] = 0$$

boundary

Associate a state of the CFT on the circle, such that

$$(T_{++} - T_{--})|\mathcal{B}\rangle = 0 \ .$$

Note: Defect operator $\sim$ Wilson loop of gauge theories ("quark defect") e.g. for non-linear σ-model, general scale-invariant defect:

$$\int_C ds\, \mathcal{H}_{\rm imp} = \int_C d\zeta^\alpha \left[\partial_\alpha \quad \cdot \mathbf{B}(\quad) + \epsilon_{\alpha\beta}\partial^\beta \quad \cdot \mathbf{C}(\quad)\right] \quad \equiv \int d\zeta^\alpha\, \mathcal{W}_\alpha$$

pull-back form

matrix-valued vector fields

(doubling degrees of freedom)

Flat connection $\quad \partial_\alpha \mathcal{W}_\beta - \partial_\beta \mathcal{W}_\alpha + [\mathcal{W}_\alpha, \mathcal{W}_\beta] = 0 \quad \Longrightarrow \quad$ classically topological

Main problems:

* Quantization (RG flows, fixed points)
* Fusion (analog of OPE ?)

Why interesting ?

- Impurities in condensed-matter systems *(quantum dots)*

Fisher, Kane '92
Affleck, Oshikawa '96 . .....

- Natural (non-local) observables of CFT

e.g. Drukker, Gaiotto, Gomis '10

- Spectrum-generating symmetry of (O)SFT ?

Graham, Watts '03
CB '08

NB: perturbative symmetries generated by topological $g=1$ defects, but "algebra" includes non-invertible $g>1$ symmetries

Frohlich, Fuchs, Runkel, Schweigert '04, '06
CB, Brunner '08

more references:

Bazhanov, Lukyanov, Zamolodchikov ’94, ’97, ‘99

Petkova, Zuber ‘00

CB, de Boer, Dijkgraaf, Ooguri ‘01

Quella, Schomerus ‘02

Lindstrom, Zabzine ‘02

Quella, Runkel, Watts ‘06

Kapustin, Witten ‘06

Runkel ‘07

Mikhailov, Schafer-Nameki ‘07

CB, Brunner ‘07

Azeyanagi, Karch, Takayanagi, Thompson ‘07

Brunner, Jockers, Roggenkamp ‘08

Gang, Yamaguchi ‘08

Sakai, Sato ‘08

Brunner, Roggenkamp ’09, ‘10

Sarkissian ‘09

Chiodaroli, Gutperle, Krym ‘10 ....

Gauged WZW models

largest class of exact CFTs

G/H fields: $g \in G$, $A_\pm \in \mathfrak{h} := \mathrm{Lie}(H) \subseteq \mathfrak{g}$

$$I_{\mathrm{GKO}}(g,A) = I_{\mathrm{WZW}}(g) + \frac{k}{2\pi}\int_\Sigma \mathrm{Tr}'\left(A_+ g^{-1}\partial_- g + A_- g \partial_+ g^{-1} + A_+ g^{-1} A_- g - A_+ A_-\right)$$

$$I_{\mathrm{WZW}} = \frac{k}{16\pi}\int_\Sigma \mathrm{Tr}'\left(\partial^\alpha g\, \partial_\alpha g^{-1}\right) - \frac{k}{24\pi}\int_{\mathcal{B}} \mathrm{Tr}'\left(g^{-1}\partial_\alpha g\, g^{-1}\partial_\beta g\, g^{-1}\partial_\gamma g\right)\epsilon^{\alpha\beta\gamma}$$

where $\mathrm{Tr}'(XY) = \mathrm{tr}_R(XY)/x_R$

Gawedzki, Kupiainen ‘88
Karabali, Park, Schnitzer, Yang ‘89

gauge invariance: $g \to hgh^{-1}$ and $A_\alpha \to hA_\alpha h^{-1} + h\partial_\alpha h^{-1}$

the (non-local) field redefinition $A_- := h_1\partial_- h_1^{-1}$ and $A_+ := h_2\partial_+ h_2^{-1}$
gives (Polyakov-Wiegman) :

$$I_{\mathrm{GKO}}(g, A) = I_{\mathrm{WZW}}(h_1^{-1} g h_2) - I_{\mathrm{WZW}}(h_1^{-1} h_2)$$

$\tilde{g}$ $\tilde{h}$

Field equations:

$$D_+(g^{-1}D_-g) = 0$$

$$g^{-1}D_-g\Big|_{\mathfrak{h}} = gD_+g^{-1}\Big|_{\mathfrak{h}} = 0$$

$$F(A) = 0$$

$\Leftrightarrow$

$$\partial_\pm \mathcal{J}^G_\mp = \partial_\pm \mathcal{J}^H_\mp = 0$$

$$\mathcal{J}^G_\pm\Big|_{\mathfrak{h}} = \mathcal{J}^H_\pm \qquad \text{(A)}$$

$$\mathcal{J}^H_\pm = \mp ik\,\Lambda_H \qquad \text{(B)}$$

constant in Cartan

Quantization:

$$J_-^a(\sigma) = \sum_{n\in\mathbf{Z}} J_n^a\, e^{-in\sigma} \quad \text{with} \quad [J_n^a, J_m^b] = i f^{abc} J^c_{n+m} + kn\, \delta^{ab}\delta_{n+m,0}$$

(A) : operator equations

(B) : (weak) conditions on physical states

GKO

$$\hat{\mathfrak{h}} = \hat{\mathfrak{h}}_{(-)} \oplus \hat{\mathfrak{h}}_{(0)} \oplus \hat{\mathfrak{h}}_{(+)} \quad , \quad J_n^a\,|\text{phys}\rangle = 0 \qquad \forall\, J_n^a \in \hat{\mathfrak{h}}_{(+)}$$

state space :

$$L^{\mathfrak{g}}_{(\nu,k)} = \bigoplus_{\gamma} L^{\mathfrak{h}}_{(\gamma,xk)} \otimes L^{\mathfrak{g}/\mathfrak{h}}_{[\nu,\gamma]}$$

quantize separately $\widehat{\mathfrak{g}}_k$ and $\widehat{\mathfrak{h}}_{-xk-2\check{h}_{\mathfrak{h}}}$

impose conditions via BRST cohomology

BRST

Karabali, Schnitzer '90

Hwang, Rhedin '93

Defects in WZW models

currents: $\mathcal{J}_- = ik\, g^{-1}\partial_- g \quad , \quad \mathcal{J}_+ = ik\, g\partial_+ g^{-1}$

generic scale-invariant defect: $(\mathcal{W}_-\,,\,\mathcal{W}_+) = (\ \mathcal{M}^a(g)\mathcal{J}_-^a\ ,\ \bar{\mathcal{M}}^a(g)\mathcal{J}_+^a\ )$

$2\dim(\mathfrak{g}) \times \dim(V) \times \dim(V)$

coupling functions

Can reduce coupling-space by imposing symmetry under

$H \subseteq (G_{\text{left}} \times G_{\text{right}})$

or under its affine extension

classically :

V must carry a representation R of H, and under transformation of bulk fields

$$\mathcal{W}_\alpha \;\rightarrow\; R(\Omega)\,\mathcal{W}_\alpha\,R(\Omega)^{-1} + R(\Omega)\partial_\alpha\,R(\Omega)^{-1}$$

in affine case

To reduce to finite parameter space, need a transitive symmetry

e.g. global left symmetry $\quad g(\zeta^\alpha) \rightarrow \overline{\Omega}\,g(\zeta^\alpha)\;,\;\overline{\Omega} \in G_{\rm left}$

$$\Longrightarrow \quad \mathcal{M}^a(g) = -\frac{i}{k}\,\bar{R}(g)M^a\,\bar{R}(g^{-1}) \quad \text{and} \quad \bar{\mathcal{M}}^a(g) \;=\; -\frac{i}{k}\,[{\rm Adj}(g^{-1})]^{ab}\,\bar{R}(g)\,\bar{M}^b\,\bar{R}(g^{-1})$$

constant matrices

for full affine left symmetry

$$\mathcal{M}^a(g) = -\frac{i}{k}\,\bar{R}(g) M^a\,\bar{R}(g^{-1}) \quad \text{and} \quad \bar{\mathcal{M}}^a(g) = -\frac{i}{k}\,\bar{T}^a$$

generators in $\bar{R}$

Classically topological, but don't know how to quantize in general (?)

except when $\bar{R}$ is the trivial representation, in which case

$$\mathcal{W}^{\text{holo}} = -\frac{i}{k}\,M^a \mathcal{J}^a_-\;d\zeta^-$$

couples only to right currents, not to g

Since form fixed by symmetry, must be preserved by RG flow

$\Longrightarrow$ gradient flow of entropy-function

$$\frac{dM^a}{d\log\epsilon} = -\frac{\partial S_0(M^1,\cdots M^{\dim G})}{\partial M^a}$$

Universal matrix model $S_0(M^a)$ [more generally $S_R(M^a, \bar{M}^a)$?]

In perturbation theory:

$$S_0 = \frac{1}{8k}\sum_{a,b}\mathrm{Tr}([M^a, M^b]^2) - \frac{1}{6k}\sum_{a,b,c} i f^{abc}\mathrm{Tr}(M^a[M^b, M^c]) + O(1/k^2)$$

Alekseev, Recknagel, Schomerus '00

Monnier '05

scheme-dependent

Is there a scheme in which it is integrable?

(potential in NADBI action)

* Critical points even at leading order not fully mapped out

further symmetry reductions within the space of M^a:

Global $H \subseteq G_{\text{right}}$ symmetry $\Longrightarrow$

the M^a must form invariant *H*-tensors:

$$M^a \mathcal{J}_-^a = \sum_j \Theta^j \mathcal{J}_-^j + \sum_s \tilde{\Theta}^s \mathcal{J}_-^s$$

invariant tensors in

$R \otimes R^* \otimes \mathfrak{h}$ $\qquad$ $R \otimes R^* \otimes \mathfrak{g}/\mathfrak{h}$

Affine $\hat{H} \subseteq \hat{G}_{\text{right}}$ symmetry $\Longrightarrow$

$$\Theta^j = \Theta^j_R$$

→ H-generators in *R*

Regularization (current-frequency cutoff)

preserves

* global H symmetry $[J_0^j\ ,\ O_{\rm ren}(M)]\ =\ 0$

* cylinder translations $[L_0\ ,\ O_{\rm ren}(M)]\ =\ 0$

breaks manifest

* affine $\hat{H}$ symmetry $[J_n^j\ ,\ O_{\rm ren}(M)]\ =\ 0$?

central in envelopping algebra

adjust couplings order by order:

$$0 = F^j(\Theta, \tilde{\Theta}) = (\Theta^j - \Theta_R^j) + O(1/k)$$

possible ?

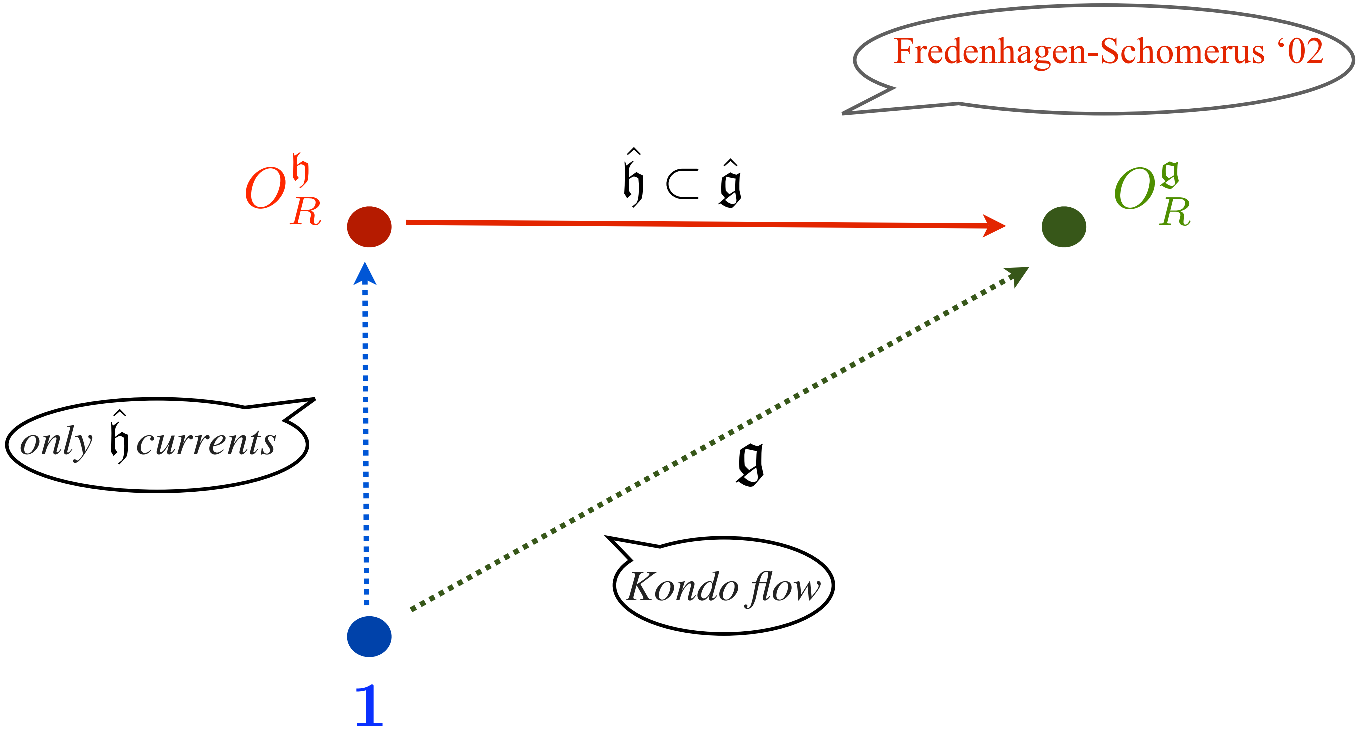
* no anomaly in 0+1 dimension

* explicit proof at RG fixed points

Alekseev, Monnier '07

A generic flow diagram

[*when R=complete G-representation*]



The FS flows take place on a $\widehat{h}_k$ invariant subspace

They descend to flows in the GKO coset models

Kondo flow   for G=SU(2)

Famous problem: screening of magnetic impurity by conduction electrons

Wilson; Nozières; Andrei; Wiegman; Affleck-Ludwig

In appropriate units

forget

$$H = \int_0^\infty dr \left[\frac{\vec{J}\cdot\vec{J}}{2\pi(k+2)} + \frac{\vec{\tilde{J}}\cdot\vec{\tilde{J}}}{2\pi(k+2)} + \lambda \vec{S}_{\rm imp}\cdot\vec{J}\,\delta(r) + \text{charge} + \text{flavor} \right]$$

The **IR** fixed point is then given by **spectral flow** from the **UV** fixed point

$\lambda = 0$

J^a_n

$H \propto \vec{J}\cdot\vec{J}$

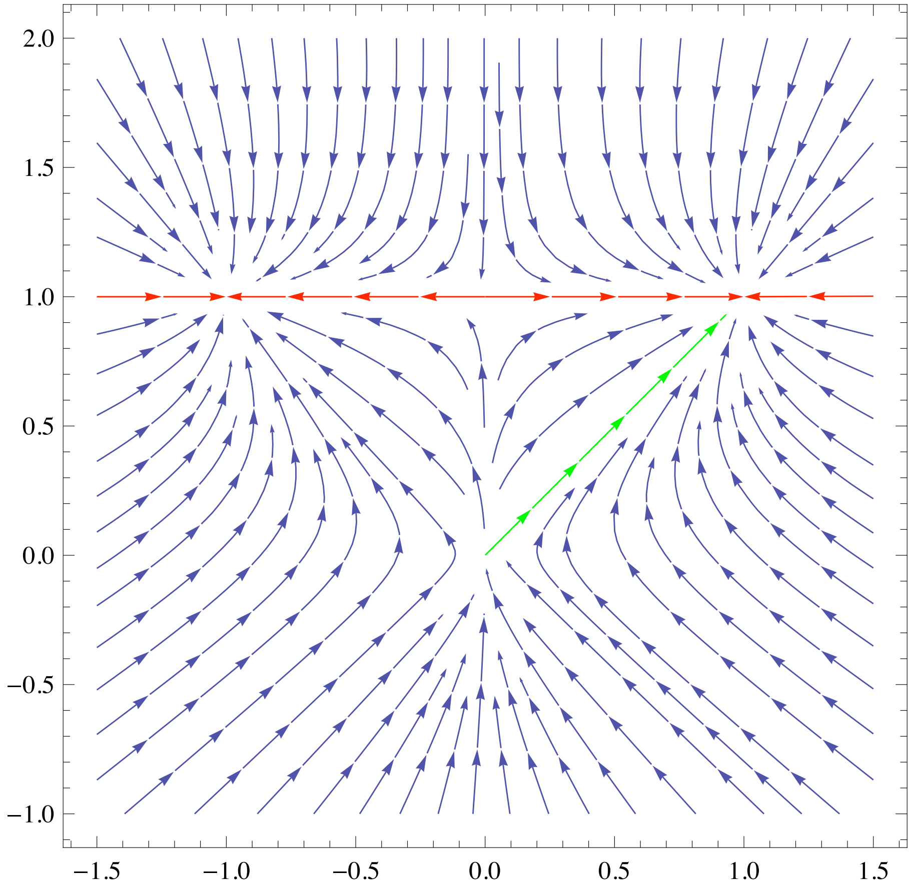
⟷

$\lambda = \frac{1}{k+2}$

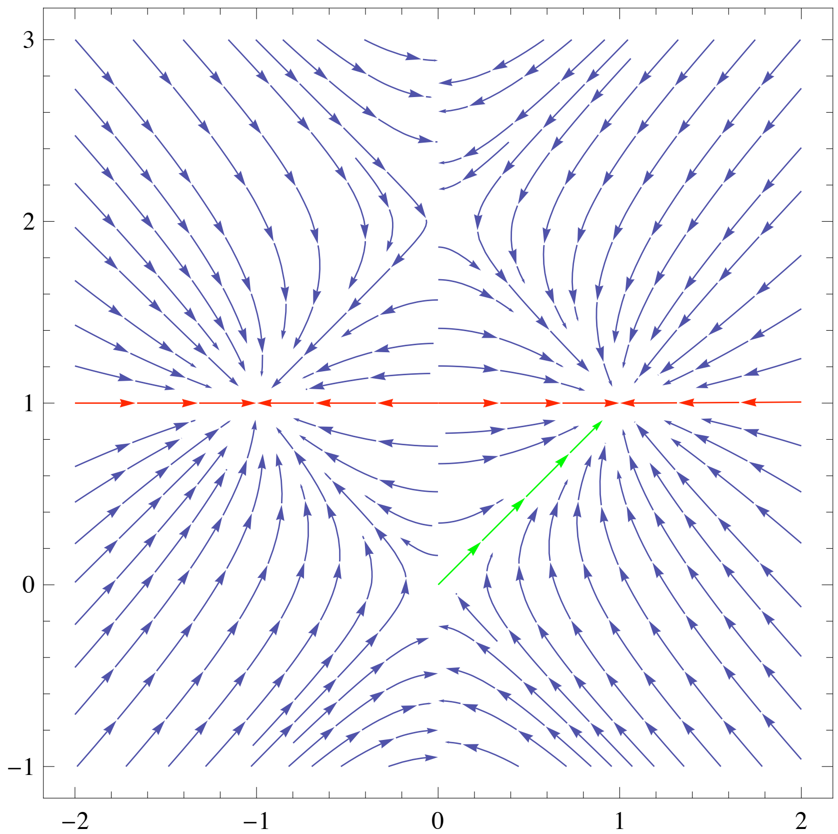
$J^a_n{}' = J^a_n + S^a_{\rm imp}$

$H \propto \vec{J}'\cdot\vec{J}' + \text{constant}$

$$G = SU(2) \times SU(2) \qquad H = SU(2)_{\text{diag}}$$



$$G = SU(2) \qquad H = O(2)$$



Reduction, and q-monodromies

gauge- invariant defects of *G/H* model must obey:

$$\mathcal{W}_\alpha \;\to\; R(h)\,\mathcal{W}_\alpha\,R(h)^{-1} + R(h)\partial_\alpha\; R(h^{-1})$$

arbitrary, not just in loop group

simple choice:

$$\mathcal{W} \;=\; \sum_j \Theta^j_R A^j_\alpha\, d\zeta^\alpha + \sum_a \tilde{\Theta}^a\,(g^{-1}D_- g)^a\, d\zeta^-$$

minimal coupling

transforms homogeneously if $\tilde{\Theta}^a$ = $\mathfrak{h}$-invariant tensor in $\mathfrak{g}\otimes R\otimes R^*$

NB can check that $\mathcal{W}$ is flat $\Longrightarrow$ classically-topological for any $\tilde{\Theta}^a$

In the gauge $h_2 = 1 \Longrightarrow A_+ = 0$ one finds

$$\mathcal{W}_+ = 0 \, , \quad ik\,\mathcal{W}_- = \sum_j \left[\Theta_R^j (\mathcal{J}_-^H)^j + \tilde{\Theta}^j (\mathcal{J}_-^G - \mathcal{J}_-^H)^j\right] + \sum_s \tilde{\Theta}^s (\mathcal{J}_-^G)^s$$

0 in GKO

i.e. precisely the form of the $\hat{\mathfrak{h}}$-invariant WZW defects

Notice that for WZW models, the restriction of RG flow to a finite # of parameters was dictated by symmetry

$\widehat{G}_{\text{left}} \times \widehat{H}_{\text{right}}$

This restriction is non-trivial in the GKO coset models

Loops at special values $\tilde{\Theta}^s = 0$ and $\tilde{\Theta}^s = \Theta^s_R$ measure <u>classical monodromies</u> :

General solution:

$$\tilde{h}(\zeta^+, \zeta^-) = \tilde{h}_+^{-1}(\zeta^+)\,\tilde{h}_-(\zeta^-) \quad \text{and} \quad \tilde{g}(\zeta^+, \zeta^-) = \tilde{g}_+^{-1}(\zeta^+)\,\tilde{g}_-(\zeta^-)$$

$$\text{with} \quad \tilde{h}_\pm(\zeta^\pm \pm 2\pi) = u^H\,\tilde{h}_\pm(\zeta^\pm) \quad \text{and} \quad \tilde{g}_\pm(\zeta^\pm \pm 2\pi) = u^G\,\tilde{g}_\pm(\zeta^\pm)$$

$$\Longrightarrow \quad \mathcal{O}^{\mathfrak{h}}_R = \mathrm{tr}_R(u^H) \;, \quad \mathcal{O}^{\mathfrak{g}}_R = \mathrm{tr}_R(u^G)$$

Quantum operators can be constructed explicitly, and commute with the vertex-operator algebra of the coset model

$$O^{\mathfrak{g}}_\mu = \frac{S^{\mathfrak{g}}_{\mu\nu}}{S^{\mathfrak{g}}_{0\nu}}\mathbf{1} \;, \quad \mathbf{O}^{\mathfrak{h}}_\mu = \sum_\alpha \mathbf{b}_{\mu\alpha}\frac{\mathbf{S}^{\mathfrak{h}}_{\alpha\gamma}}{\mathbf{S}^{\mathfrak{h}}_{\mathbf{0}\gamma}}\mathbf{1} \quad \text{on} \quad \mathbf{L}^{\mathfrak{g}/\mathfrak{h}}_{[\nu,\gamma]}$$

highest weight *branching coefficients*

Fusion in WZW models:

$$O^{\mathfrak{g}}_{\sigma} O^{\mathfrak{g}}_{\mu} = \sum_{\nu} \mathcal{N}_{\sigma\mu}{}^{\nu} O^{\mathfrak{g}}_{\nu} \qquad \text{and} \qquad O^{\mathfrak{g}}_{\nu} B^{\mathfrak{g}}_{0} = B^{\mathfrak{g}}_{\nu}$$

boundary state

Defect flow $\dim(S)\mathbf{1} \rightarrow O^{\mathfrak{g}}_{\sigma}$ imprints universal boundary flows

$$\dim(S) B^{\mathfrak{g}}_{\mu} \rightarrow \sum_{\nu} \mathcal{N}_{\mu\sigma}{}^{\nu} B^{\mathfrak{g}}_{\nu}$$

Affleck-Ludwig "absorption of boundary spin" rule

Similarly coset-defect flows imply the Fredenhagen-Schomerus flows

$$\sum_{\alpha} b_{\mu\alpha} \sum_{J} (N_{[0,\alpha]})_{I}{}^{J} B_{J} \quad \mapsto \quad \sum_{J} (N_{[\mu,0]})_{I}{}^{J} B_{J}$$

generalized Affleck-Ludwig rule

Quantum Symmetries of OSFT

Distance(s) between CFTs

Why? One reason: prove compactness of CY moduli space for fixed volume.

M.Douglas, **arXiv:1005.2779** [hep-th]

A nice candidate:

$$d(T1, T2) = [\min_U \ \log g_U(T1, T2)]^{1/2}$$

conformal interface

Reduces to Zamolodchikov metric for nearby theories:

$$\log g = \frac{\pi^2}{2} \Delta t^j \Delta t^k g^{(Z)}_{jk} + O(\Delta t^3)$$

In c=1 case:

$$d^2(R_1, R_2) = \log \frac{R_1^2 + R_2^2}{2R_1 R_2}$$

obeys triangle inequality.

But for general large-volume CY threefolds:

$$d^2(t_1, t_2) = K(t_1, \bar{t}_1) + K(t_2, \bar{t}_2) - 2\log \left| \int_M \Omega_1 \wedge \bar{\Omega}_2 \right|$$

Calabi diastatic function

fails triangle inequality. By finite amount?

Work in progress ......

Summary+Outlook

Derived largest known class of (FS) defect flows, by reduction to finite-d matrix model

- CM realizations of FS flows ?
- Extension to non-compact CFTs ?
- How do these quantum symmetries of OSFT fit into a larger structure?

Thank you!