

Non-linear supersymmetry and MSSM

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- ① Non-linear SUSY and goldstino
- ② Description of low energy couplings

I.A.-Tuckmantel '04; Komargodski-Seiberg '09

- ③ Goldstino couplings to MSSM
- ④ Phenomenology: Higgs potential and invisible decays

I.A.-Dudas-Ghilencea-Tziveloglou '10

Non-linear supersymmetry \Rightarrow goldstino mode χ

Volkov-Akulov '73

Why study goldstino interactions:

- Effective field theory of SUSY breaking at low energies $m_\chi \ll m_{susy}$
e.g. gauge mediation dominant vs gravity mediation

$$\chi: \text{longitudinal gravitino with } m_\chi \simeq \frac{m_{susy}^2}{M_{Planck}} \lesssim m_{soft} \ll m_{susy}$$

$M_{Planck} \rightarrow \infty$: SUGRA decoupled

massless χ coupled to matter $\sim 1/m_{susy}$

- Brane dynamics: half SUSY of the bulk broken but NL realized
 \Rightarrow strongly constrain coupling of brane to bulk fields

Non-linear SUSY transformations: [5]

$$\delta\chi_\alpha = \frac{\xi_\alpha}{\kappa} + \color{red}\kappa\Lambda_\xi^\mu\partial_\mu\chi_\alpha\color{black} \quad \Lambda_\xi^\mu = -i(\chi\sigma^\mu\xi - \xi\sigma^\mu\bar{\chi})$$

κ : goldstino decay constant (SUSY breaking scale) $\kappa = 1/(\sqrt{2}m_{susy}^2)$

Goldstino interactions: 3 formulations

- Standard realization

Volkov-Akulov '73, Clark-Love '96, Clark-Lee-Love-Wu '98

- Superfield formalism

Ivanov-Kapustnikov '78, Samuel-Wess '83

Brignole-Feruglio-Zwirner '97, Luty-Ponton '98, I.A.-Tuckmantel '04

- Constrained superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Standard realization

Define the ‘vierbein’: $E_\mu^a = \delta_\mu^a + \kappa^2 t_\mu^a$ $t_\mu^a = i\chi \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\chi}$

$\delta(\det E) = \kappa \partial_\mu (\Lambda_\xi^\mu \det E)$ \Rightarrow invariant action:

$$S_{VA} = -\frac{1}{2\kappa^2} \int d^4x \det E = -\frac{1}{2\kappa^2} - \frac{i}{2} \chi \sigma^\mu \overset{\leftrightarrow}{\partial}_\mu \bar{\chi} + \dots$$

Generalization to matter and gauge fields:

$$S_{\text{eff}} = \int d^4x \det E \mathcal{L}_{SM}(\phi) \quad \text{invariant if } \delta\phi = \kappa \Lambda_\xi^\mu \partial_\mu \phi \quad \text{and so } \mathcal{L}_{SM}$$

However problem with derivatives \Rightarrow define SUSY covariant ones:

$$\mathcal{D}_a \phi \equiv (E^{-1})_a^\nu D_\nu \phi \quad \mathcal{F}_{ab} \equiv (E^{-1})_a^\lambda (E^{-1})_b^\rho F_{\lambda\rho}$$

$$\mathcal{L}_{\text{eff}} = \det E \mathcal{L}_{SM}(\phi, \mathcal{D}_\mu \phi) = \mathcal{L}_{SM}(\phi, D_\mu \phi) + \kappa^2 t^{\mu\nu} T_{\mu\nu} + \dots$$

universal coupling to stress-tensor but NOT the most general inv action

Superfield formalism

Recipe: $\phi(x) \rightarrow \Phi(x, \theta, \bar{\theta}) \equiv \phi(\tilde{x}) \quad \tilde{x}^\mu = x^\mu + \Lambda_\theta^\mu(\tilde{x})$ [3] [10]

$$= \phi(x) + \kappa \Lambda_\theta^\mu \partial_\mu \phi + \dots \Rightarrow$$

Goldstino (spinor) superfield: $\mathcal{G}_\alpha = \frac{\theta_\alpha}{\kappa} + \chi_\alpha(\tilde{x})$

space-time derivatives: use the 'vierbein' $E(\tilde{x})$

$$\text{e.g. } \mathcal{F}_{ab}(x, \theta, \bar{\theta}) \equiv \left[(E^{-1})_a^\lambda (E^{-1})_b^\rho F_{\lambda\rho} \right] (\tilde{x})$$

List of lowest dim operators

2 operators of dim 6 linear in χ [8]

$$S_1 = C_1 \int d^4x \kappa F_{\mu\nu} \psi \sigma^\mu \partial^\nu \bar{\chi} + h.c. \quad S_2 = C_2 \int d^4x \kappa (\psi \partial_\alpha \chi) D^\alpha \phi + h.c.$$

Quadratic in χ : 1 operator of dim 7

$$S_7 = C_7 \int d^4x \kappa^{3/2} \phi_1 \phi_2 \partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi + h.c. \quad J_{(\frac{1}{2},0)}^{\mu\nu} = \frac{i}{4} \sigma^{[\mu} \bar{\sigma}^{\nu]}$$

+ 5 operators of dim 8

$$S_3 = C_3 \int d^4x \kappa^2 (\psi_1 \partial^\mu \chi) (\bar{\psi}_2 \partial_\mu \bar{\chi}) + h.c. \quad S_4 = C_4 \int d^4x \kappa^2 (\psi_1 \psi_2) (\partial_\mu \chi \partial^\mu \chi) + h.c.$$

$$S_5 = C_5 \int d^4x \kappa^2 \phi_1 \overset{\leftrightarrow}{D}_\mu \phi_2 i \partial_\alpha \chi \sigma^\mu \partial^\alpha \bar{\chi} + h.c.$$

$$S_6 = C_6 \int d^4x \kappa^2 \partial^\alpha \chi \sigma^\mu \partial^\nu \bar{\chi} \partial_\alpha F_{\mu\nu} + h.c.$$

$$S_8 = C_8 \int d^4x \kappa^2 \phi_1 \phi_2 \phi_3 (\partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi) + h.c.$$

D-brane examples

Type II (closed) strings on $4d$ Minkowski $M_4 \times X_6$ internal $6d$ manifold

X_6 flat $\Rightarrow N = 8$ SUSY ; X_6 Calabi-Yau $\Rightarrow N = 2$ SUSY

Single stack of N Dp -branes \Rightarrow half SUSY is spontaneously broken $p \geq 3$

($p - 3$) dims wrapped around cycles in $X_6 \Rightarrow 4d$ effective field theory

- Gauge group: $G = U(N)$ (generically)
- SUSY: half remains unbroken Q_e ; other half NL realized Q_o
broken SUSY commutes with $G \Rightarrow$ goldstino = $U(1)$ gaugino of Q_e

Intersecting branes: all SUSY is generally broken except for special angles

e.g. $\theta_1 + \theta_2 + \theta_3 = 0$ for $X_6 = T^2 \times T^2 \times T^2$

goldstino becomes a combination of the 2 gauginos

1) Goldstino decay constant: sum of brane tensions

$$\frac{1}{2\kappa^2} = T_1 + T_2 \quad T_i = \frac{M_s^4}{4\pi^2 g_i^2} N_i$$

2) Goldstino couplings: only 3 non-vanishing up to order κ^2 [6]

$$C_1 = \sqrt{2} \quad ; \quad C_2 = 2 \quad ; \quad C_3 = 2$$

- universal coefficients independent of brane-angles
- C_3 : fixes the field theory ambiguity of 4-fermion operator

Brignole-Feruglio-Zwirner '97, I.A.-Benakli-Laugier '01

Constrained superfields

spontaneous global SUSY: no supercharge but still conserved supercurrent

\Rightarrow superpartners exist in operator space (not as 1-particle states)

\Rightarrow constrained superfields: 'eliminate' superpartners

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$

$$X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F \quad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$
$$= F\Theta^2 \quad \Theta = \theta + \frac{\chi}{\sqrt{2}F}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{VA}$$

$$F = \frac{1}{\sqrt{2}\kappa} + \dots$$

Constrained matter superfields

- Fermions: Q_{NL} satisfying $Q_{NL}X_{NL} = 0$ (eliminate sfermions) \Rightarrow

$$Q_{NL} = \sqrt{2} \left(\psi - \frac{F_Q \chi}{F} \right) \Theta + F_Q \Theta^2$$

- Complex scalars: H_{NL} with $X_{NL}\bar{H}_{NL} = \text{chiral}$ (eliminate 'higgsinos') [5]

$$\Rightarrow H_{NL} = H(\hat{y}) \quad \hat{y} = y^\mu + i\sqrt{2}\theta\sigma^\mu\bar{\chi}(\hat{y})/\bar{F}(\hat{y})$$

- Gauge fields: V_{NL} convenient gauge choice: $X_{NL}V_{NL} = 0$
eliminate gauginos: $X_{NL}W_{NL} = 0$ field strength $W = -\frac{1}{4}\bar{D}^2DV$

$$\Rightarrow V_{NL} = -\Theta \left(\sigma^m V_m + \frac{D}{|F|^2} \chi \bar{\chi} \right) \bar{\Theta} + \frac{1}{2}\Theta^2 \bar{\Theta}^2 D + \text{derivatives}$$

Goldstino couplings to matter supermultiplets

Before: goldstino coupling to non-SUSY matter $E \ll m_{soft}, m_{susy}$
→ constrained matter superfields



However if $m_{soft} \lesssim E \ll m_{susy} \Rightarrow$ linear SUSY in matter sector
→ goldstino coupling to ordinary matter superfields

constrained X_{NL} coupled to MSSM

Komargodski-Seiberg '09

Assumption in the following: gaugino, higgsino, slepton masses $\ll m_{susy}$
results independent on squark masses \Rightarrow can be much higher

Equivalence theorem \Rightarrow leading goldstino couplings:

$$\sim \int \partial_\mu \chi J^\mu = - \int \chi \partial_\mu J^\mu \leftarrow \text{supercurrent}$$

Equations of motion: $\partial_\mu J^\mu \sim \text{soft terms}$

\rightarrow generalization to non-linear terms: superfield formalism

Usually parametrization of soft SUSY terms using auxiliary spurion:

$$S = m_{\text{soft}} \theta^2$$

Non-linear MSSM: replace $S \rightarrow \sqrt{2} \kappa m_{\text{soft}} X_{NL} = \frac{m_{\text{soft}}}{m_{\text{susy}}} X_{NL}$

F -auxiliary in X_{NL} : dynamical field with no derivatives to be solved

$$-\bar{F} = m_{\text{susy}}^2 + \frac{B\mu}{m_{\text{susy}}^2} h_1 h_2 + \frac{A_u}{m_{\text{susy}}^2} \tilde{u}_R \tilde{q} h_2 + \dots$$

\Rightarrow compact form for all goldstino couplings at linear and non-linear level

Non-linear MSSM

with

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{X_{NL}} + \mathcal{L}_H + \mathcal{L}_m + \mathcal{L}_{AB} + \mathcal{L}_g$$

$$\begin{aligned}\mathcal{L}_H &= \sum_{i=1,2} \frac{m_i^2}{m_{susy}^4} \int d^4\theta X_{NL}^\dagger X_{NL} H_i^\dagger e^{V_i} H_i \\ \mathcal{L}_m &= \sum_{\Phi} \frac{m_\Phi^2}{m_{susy}^4} \int d^4\theta X_{NL}^\dagger X_{NL} \Phi^\dagger e^V \Phi \quad ; \quad \Phi = Q, U^c, D^c, L, E^c \\ \mathcal{L}_{AB} &= \frac{1}{m_{susy}^2} \int d^2\theta X_{nl} (A_u H_2 Q U^c + A_d Q D^c H_1 + A_e L E^c H_1) \\ &\quad + \frac{B\mu}{m_{susy}^2} \int d^2\theta X_{NL} H_1 H_2 + h.c. \\ \mathcal{L}_g &= \sum_{i=1}^3 \frac{1}{8g_i^2} \frac{m_{\lambda_i}}{m_{susy}^2} \int d^2\theta X_{NL} \text{Tr} [W^\alpha W_\alpha]_i + h.c.\end{aligned}$$

Phenomenological analysis

I.A.-Dudas-Ghilencea-Tziveloglou '10

Higgs potential is modified:

$$V = V_{MSSM} + 2\kappa^2 \left| m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + B\mu h_1 h_2 \right|^2 + \mathcal{O}(\kappa^4) \Rightarrow$$

$m_{1,2}, B\mu$: soft mass parameters, μ : higgsino mass

Classical value of light higgs mass can be increased above the LEP bound

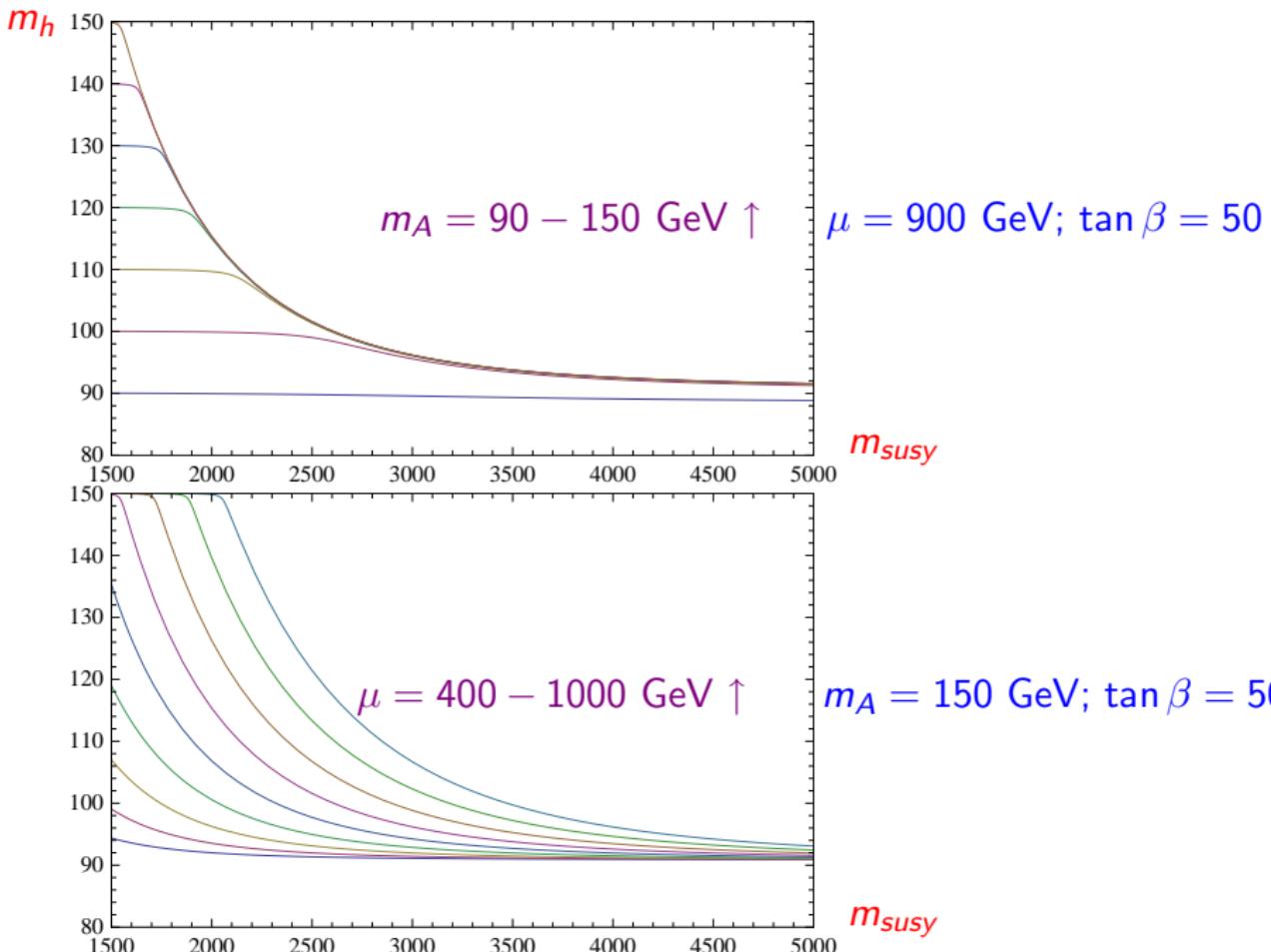
for $m_{susy} \sim$ a few TeV

large $\tan\beta$ limit: $m_h^2 = m_Z^2 + \frac{v^2}{2m_{susy}^2} (2\mu^2 + m_Z^2)^2 + \dots$

$$\rightarrow \text{e.g. } \mu = 900 \text{ GeV, } m_{susy} = 2 \text{ TeV} \Rightarrow m_h = 114.4 \text{ GeV}$$

Quartic higgs coupling increases for large soft masses \Rightarrow [18]

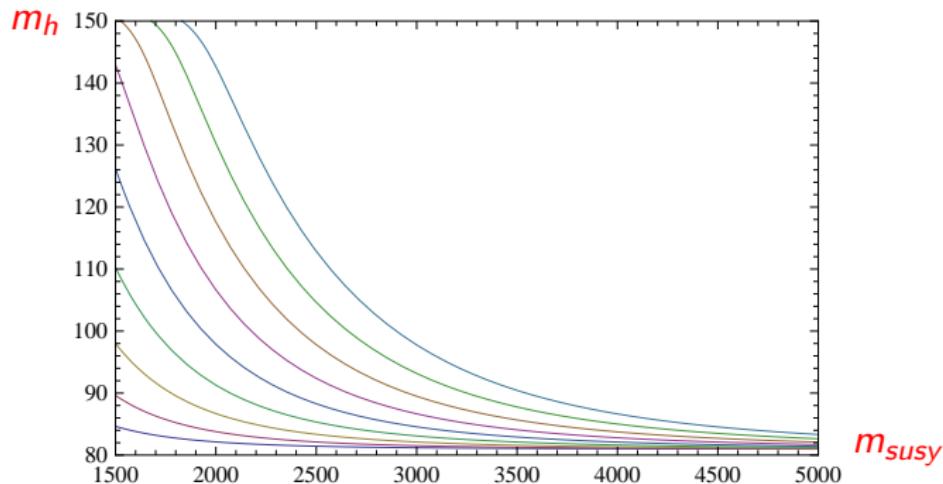
MSSM 'little' fine tuning of the EW scale is alleviated



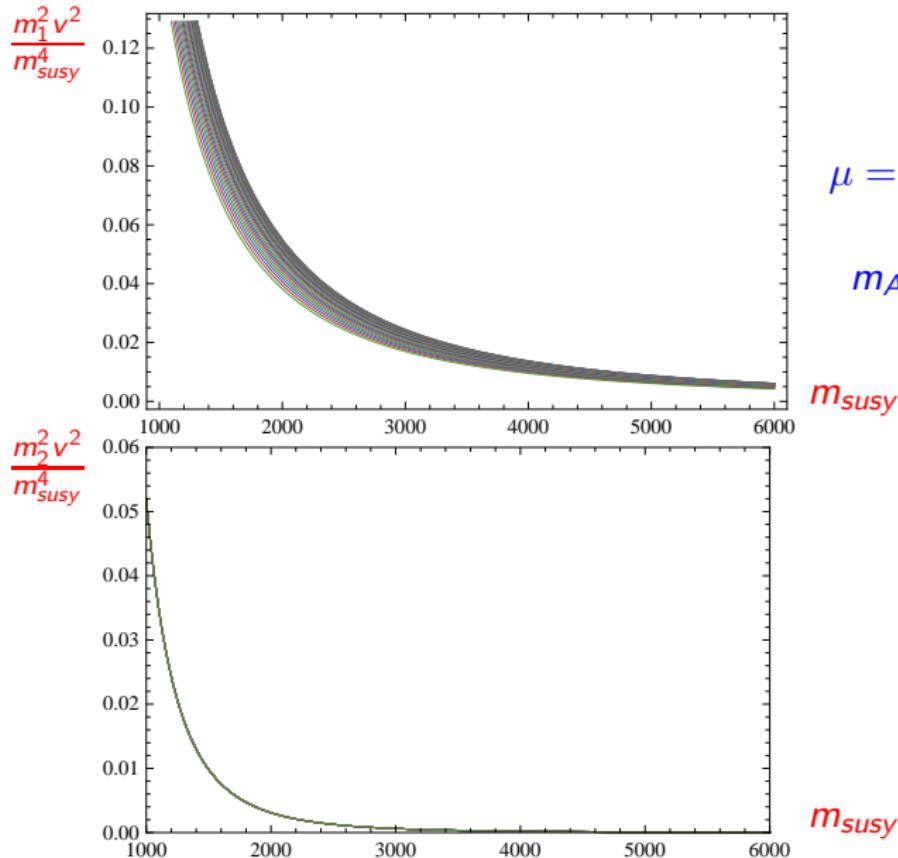
Mild dependence on $\tan \beta$

$\mu = 400 - 1000 \text{ GeV} \uparrow$

$m_A = 150 \text{ GeV}; \tan \beta = 5$



Validity of perturbative expansion: $m_i^2 v^2 / m_{susy}^4 \ll 1$



$\mu = 900 \text{ GeV}; \tan \beta = 50$

$m_A = 90 - 650 \text{ GeV} \uparrow$ [14]

m_{susy}

m_{susy}

Invisible decays of Higgs and Z boson

Other relevant couplings at order $\mathcal{O}(\kappa)$: $\frac{1}{m_{susy}^2} \times$

$$\left(m_1^2 \chi \psi_{h_1^0} h_1^{0*} + m_2^2 \chi \psi_{h_2^0} h_2^{0*} \right) + B\mu \left(\chi \psi_{h_2^0} h_1^0 + \chi \psi_{h_1^0} h_2^0 \right) +$$

$$\sum_{i=1,2} \frac{m_{\lambda_i}}{\sqrt{2}} \tilde{D}_i^a \chi \lambda_i^a + \sum_{i=1}^2 \frac{m_{\lambda_i}}{\sqrt{2}} \chi \sigma^{\mu\nu} \lambda_i^a F_{\mu\nu,i}^a + h.c.$$

\Rightarrow invisible higgs decay $h \rightarrow \chi + \text{NLSP}$ if NLSP is light enough

otherwise inverse decay studied in the past

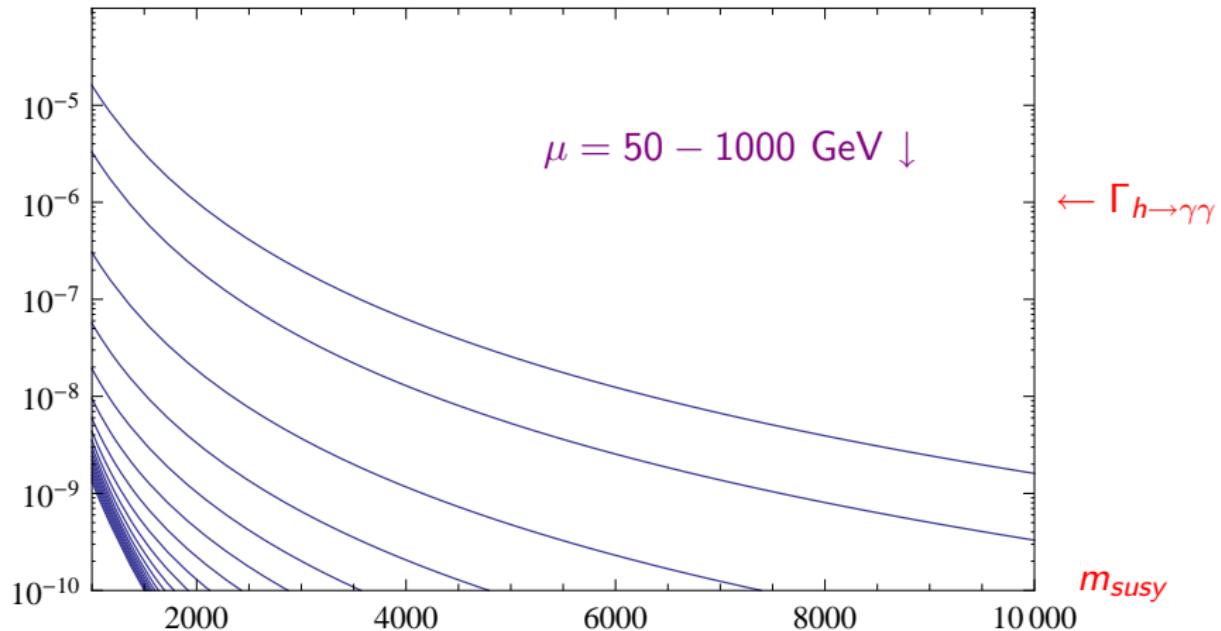
taking also into account the goldstino components of higgsinos/gauginos

from SUSY terms: $\sim h_i^0 \lambda \tilde{h}_i^0$

Similarly $Z \rightarrow \chi + \text{NLSP} \Rightarrow m_{susy} \gtrsim 400\text{-}700 \text{ GeV}$ from Z -width

$\Delta\Gamma_Z \lesssim 2.3 \text{ MeV}$

$$\Gamma_{h \rightarrow \chi + \text{NLSP}} \quad m_A = 150 \text{ GeV}; \tan \beta = 50; \quad (m_{\lambda_1}, m_{\lambda_2}) = (70, 150) \text{ GeV}$$



Conclusions

Non-linear supersymmetry: powerful tool for studying:

- low energy SUSY breaking $E \ll m_{susy} \sim 1/\sqrt{\kappa}$
Volkov-Akulov action and goldstino χ couplings to matter
standard coupling to stress-tensor *not* the most general
- $m_{soft} \lesssim E \ll m_{susy}$: goldstino \equiv spurion coupled to supermultiplets
→ Non-linear MSSM : narrow but interesting region
 - new quartic higgs coupling ⇒ • can increase the higgs mass
 - reduce the MSSM fine tuning of the EW scale
 - new goldstino couplings e.g. to higgs/higgsinos, gauge bosons/gauginos
 - light neutralino ⇒ • invisible higgs decay $h \rightarrow \chi + \text{NLSP} \sim \Gamma_{h \rightarrow \gamma\gamma}$
 - similar Z-decay ⇒ $m_{susy} \gtrsim 400\text{-}700 \text{ GeV}$ from Γ_Z