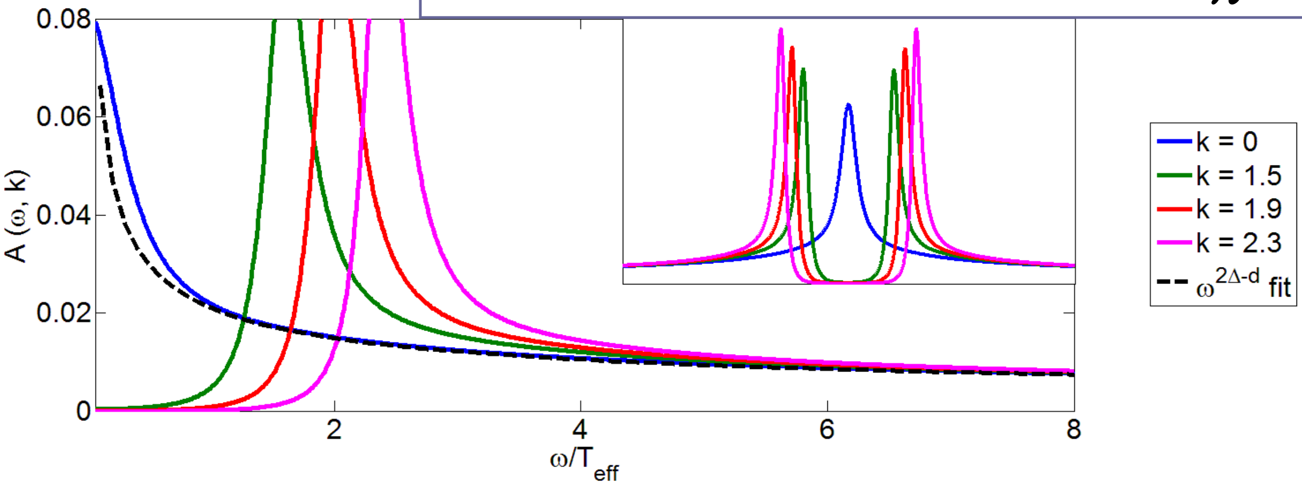
Part III

Fermi liquid phase of the boundary theory

Spectral functions and quasiparticle peak

$$\Delta = 1.25$$

$$\text{Spectral function: } A(\omega, k) = -\frac{1}{\pi}\,\mathrm{ImTr}\left(i\gamma_0 G_R(\omega, k)\right)$$



$$\frac{\mu_0}{T} \to 0:$$

$$G = \left(-\omega^2 + k^2\right)^{\frac{2\Delta - d}{2}}$$

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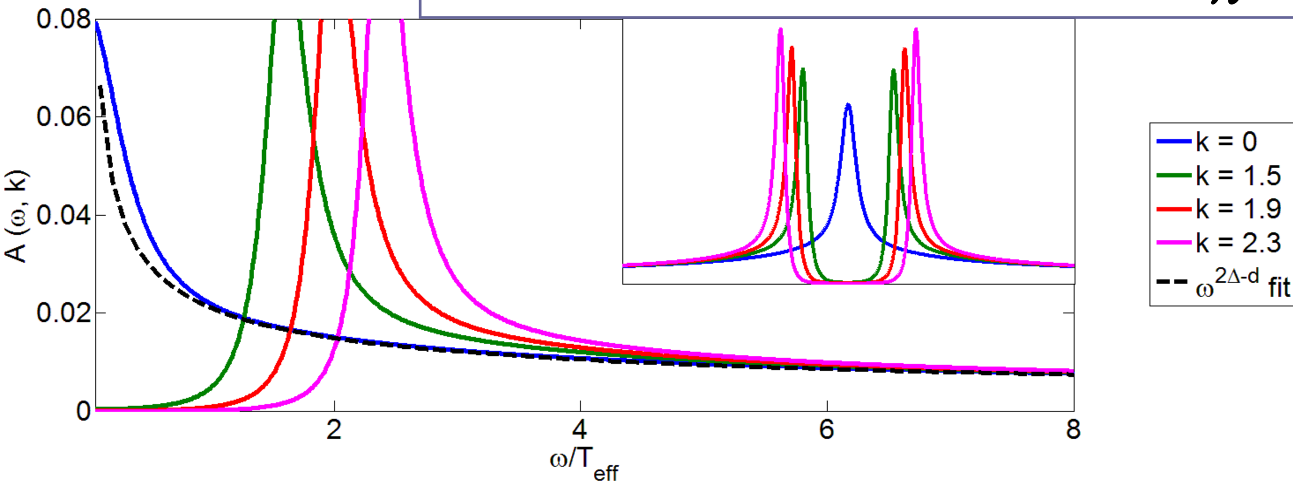

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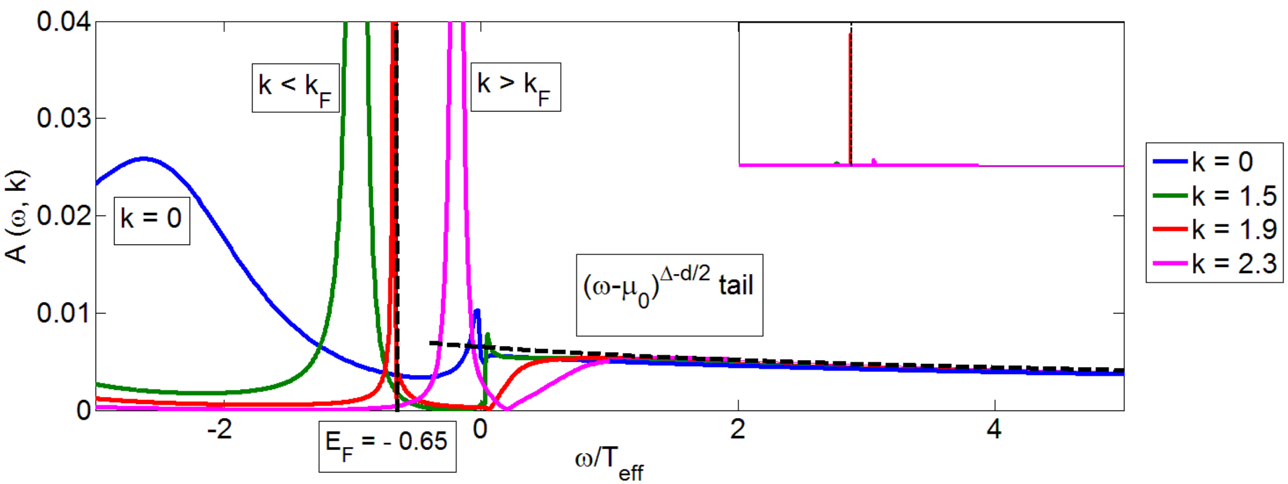
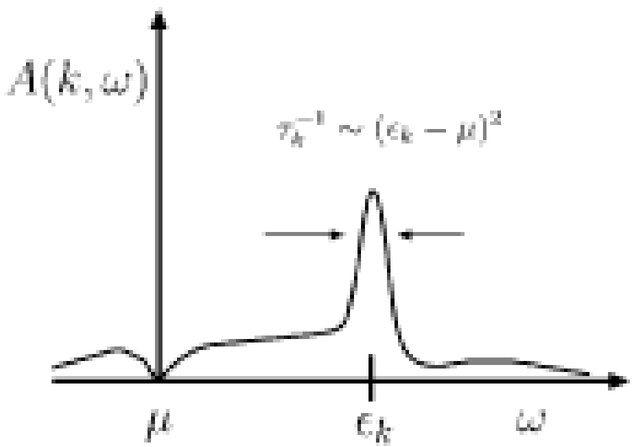


$$\frac{\mu_0}{T} = 30.9 > \left(\frac{\mu_0}{T}\right)_c :$$

QP peaks

Basic Fermi liquid phenomenology

- Single-particle propagators capture all the physics
- Quasiparticle peak (QP) & sharp Fermi surface:



Peak weight $Z \equiv$ field renormalization

- Landau theory

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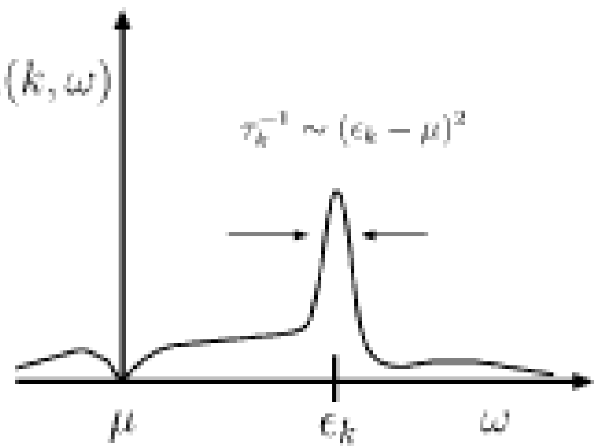


- Landau theory
- Single-particle propagator:

$$G(\omega,k) = \frac{1}{\omega - \mu_0 - k^2/2m - (\Sigma' + i\Sigma'')} = \frac{Z}{(\omega - E_F) - v_R(k - k_F) + \ldots}$$

- Spectral function (energy distribution curve - EDC):

$$A(\omega,k) = \frac{\Sigma''(\omega,k)}{\left|\omega + \mu + (k - k_F)^2/2m + \Sigma'(\omega,k)\right|^2 + \left|\Sigma''(\omega,k)\right|^2}$$

List of Fermi liquid tests

- Quasiparticle peaks
- Zero density of states at the Fermi surface: $A\left(E_F, k_F\right) = 0$
- Analytical structure of self-energy
- Quadratic scaling of QP peak widths with temperature
- Linear dispersion of QP excitations

Outline of the calculations

- Retarded propagator $G_R(\omega, k)$
- Spectral function:

$$A(\omega, k) = -\frac{1}{\pi}\,\mathrm{Tr}\,\mathrm{Im}\left(i\gamma_0 G_R(\omega, k)\right)$$

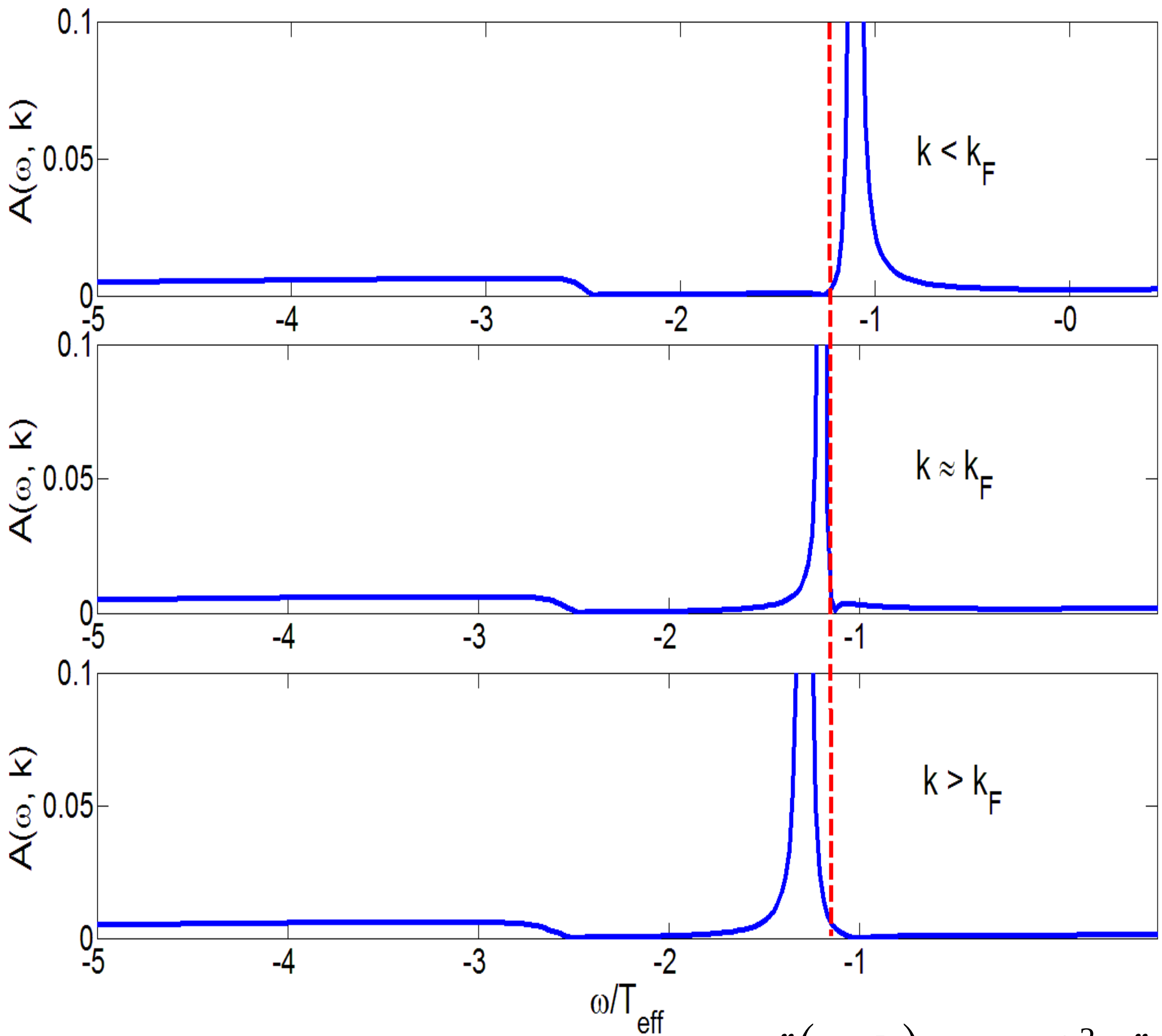
- Computational units:

$$A(\omega, k) = T^{2m} f\left(\omega/T_{eff}\,, k/T_{eff}\,; \Delta, \mu_0/T\right)$$

$$T_{eff} = T\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\mu_0^2}{T^2}}\right)$$

- “AdS/ARPES“ correspondence

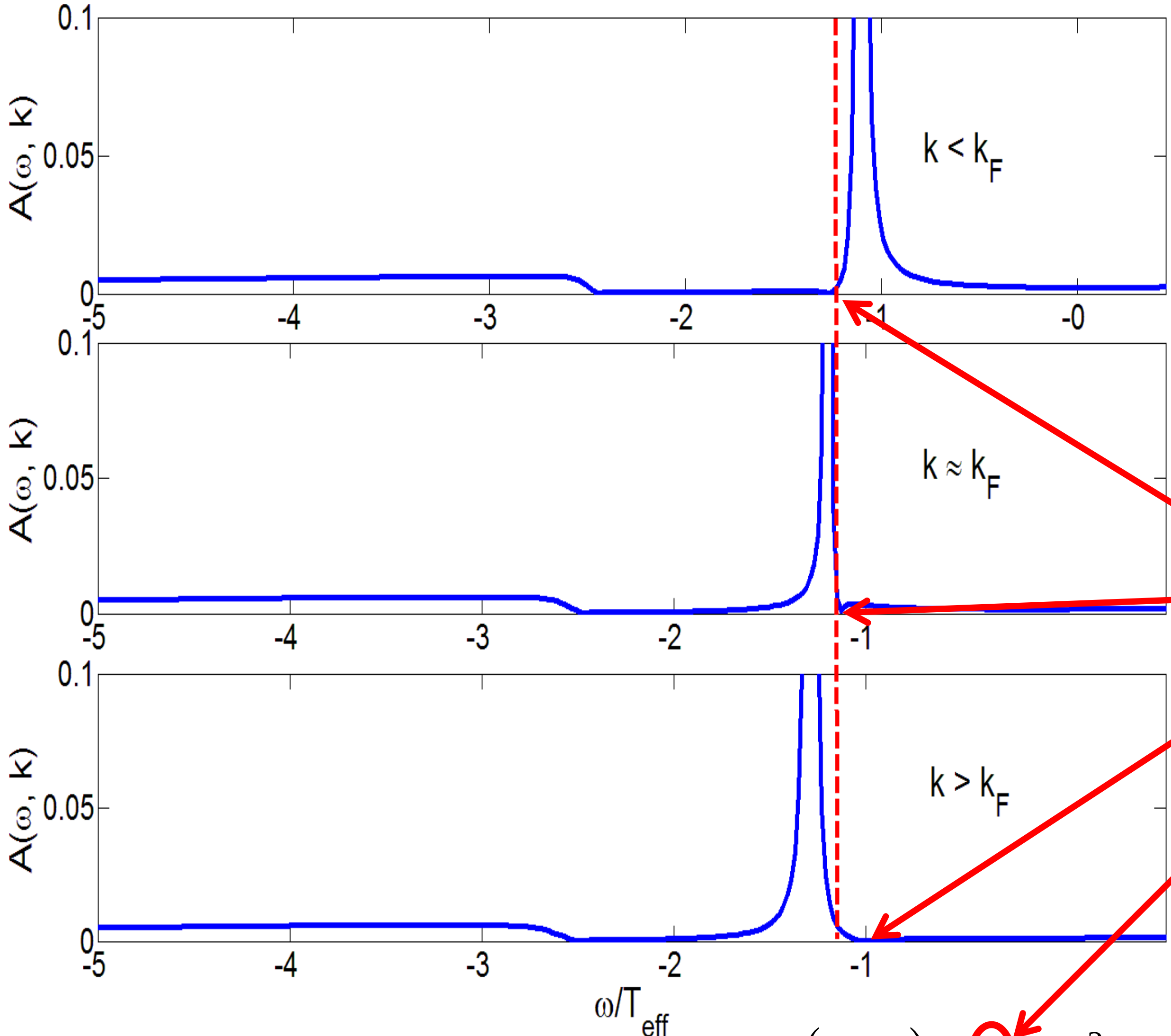
Identifying (E_F, k_F)



$$A(E_F, k) = Z\delta(k - k_F)$$

$$\Sigma''(\omega, k) = 0 + \partial^2\Sigma''/\partial\omega^2\Big|_{\omega=E_F}(\omega - E_F)^2 + \ldots$$

Identifying (E_F, k_F)

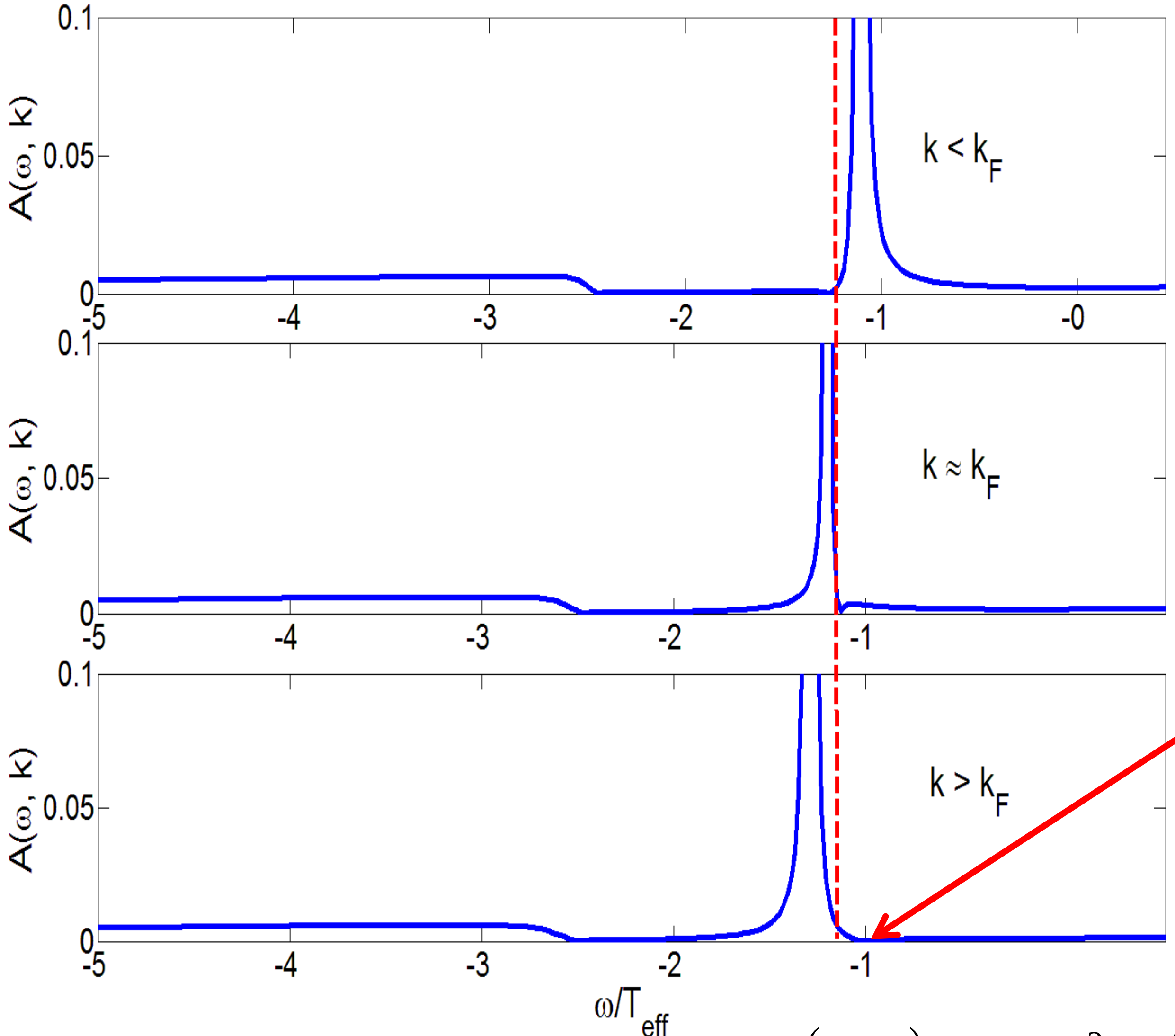


$$A(E_F, k) = Z\delta(k - k_F)$$

Dip in the spectrum: zero density of states at E_F for $k \neq k_F$

$$\Sigma''(\omega, k) = 0 + \partial^2\Sigma''/\partial\omega^2\Big|_{\omega=E_F} (\omega - E_F)^2 + \ldots$$

Identifying (E_F, k_F)



$$A(E_F, k) = Z\delta(k - k_F)$$

Finite temperature effect: dip position shifted by $\delta E_F \propto T$

$$\Sigma''(\omega, k) = 0 + \partial^2 \Sigma'' / \partial \omega^2 \Big|_{\omega = E_F} (\omega - E_F)^2 + \ldots$$

Self-energy structure

- $\Sigma = \Sigma' + i\Sigma''$
- Σ' - E_F renormalization, effective mass renormalization
- Σ'' - Dip at E_F in EDC, peak broadening

$$\Sigma''(\omega,k) = \delta \times (\omega - E_F)^2 + \ldots$$

δ – QP width

No constant term in $\mathrm{d}\Sigma''/\mathrm{d}\,\omega$

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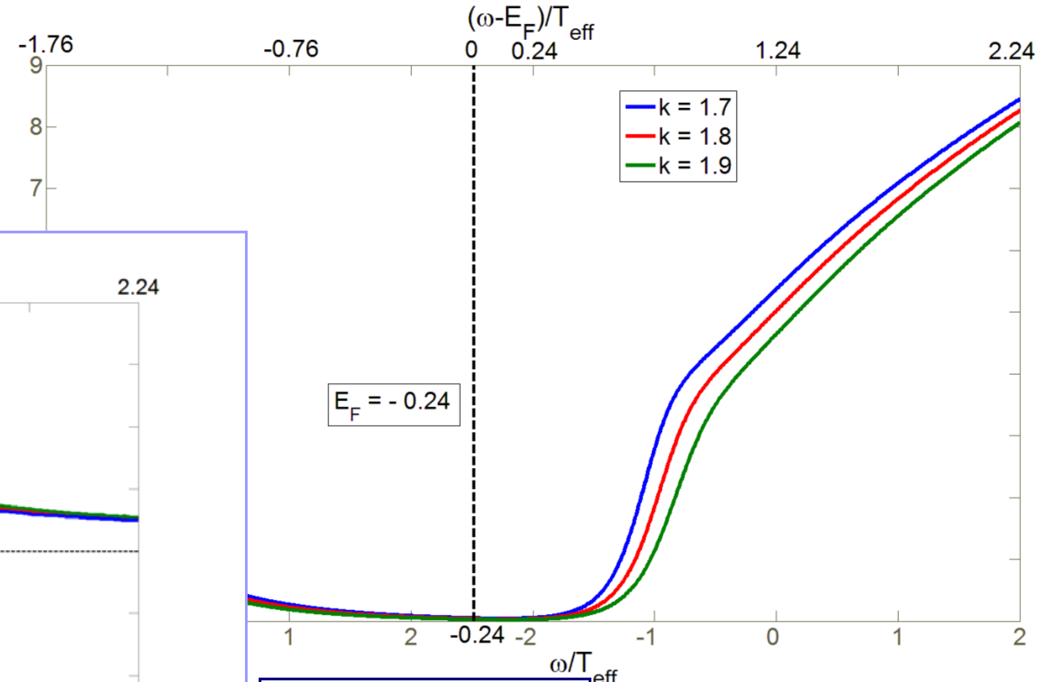
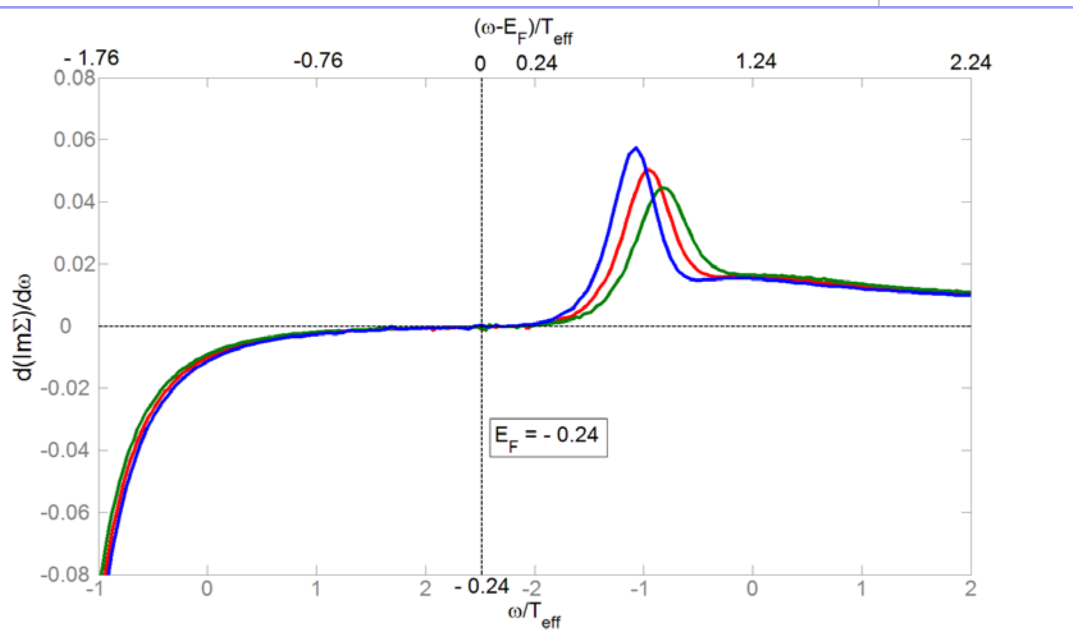
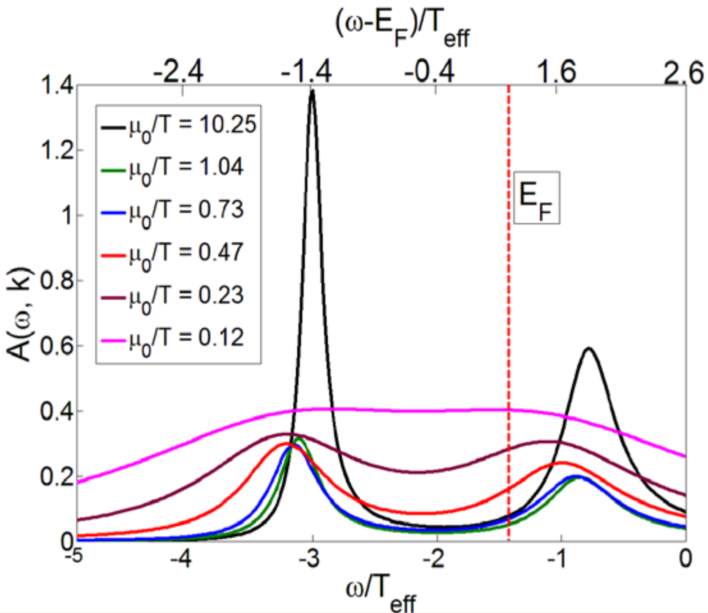




$\Delta = 1.40$

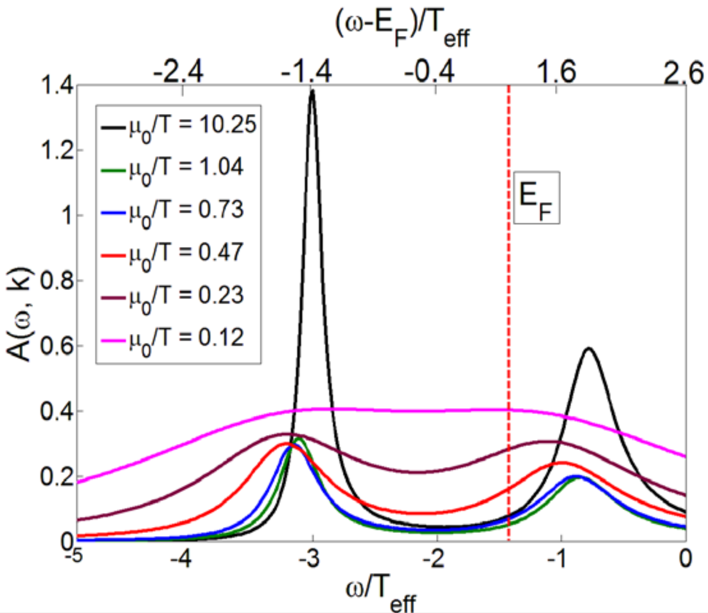
$\mu_0 / T = 30.9$

Temperature dependence



$$\Delta = 1.05$$

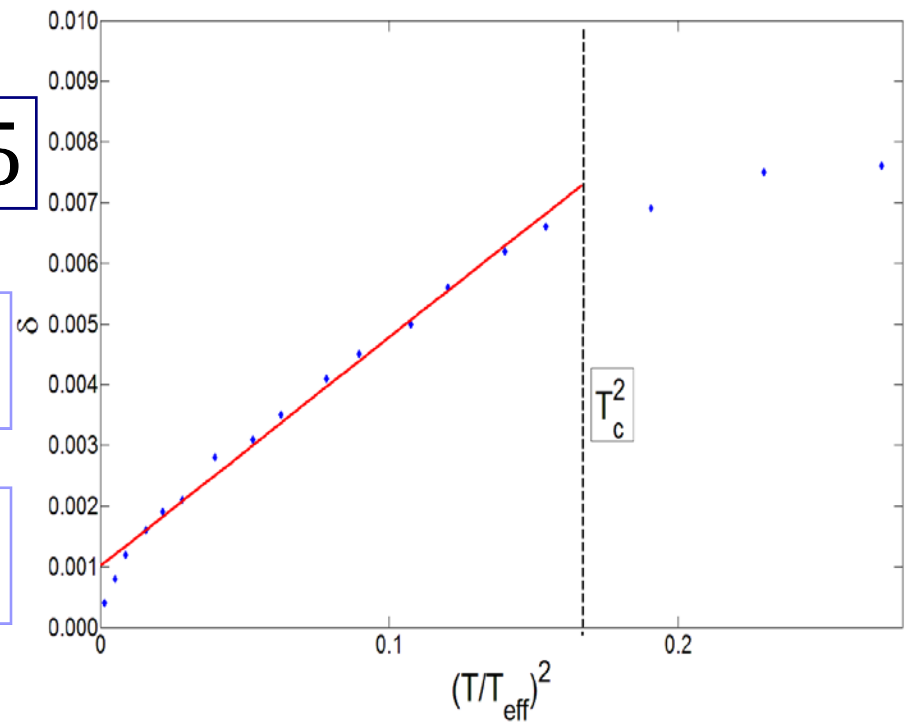
Temperature dependence



$$\Delta = 1.05$$

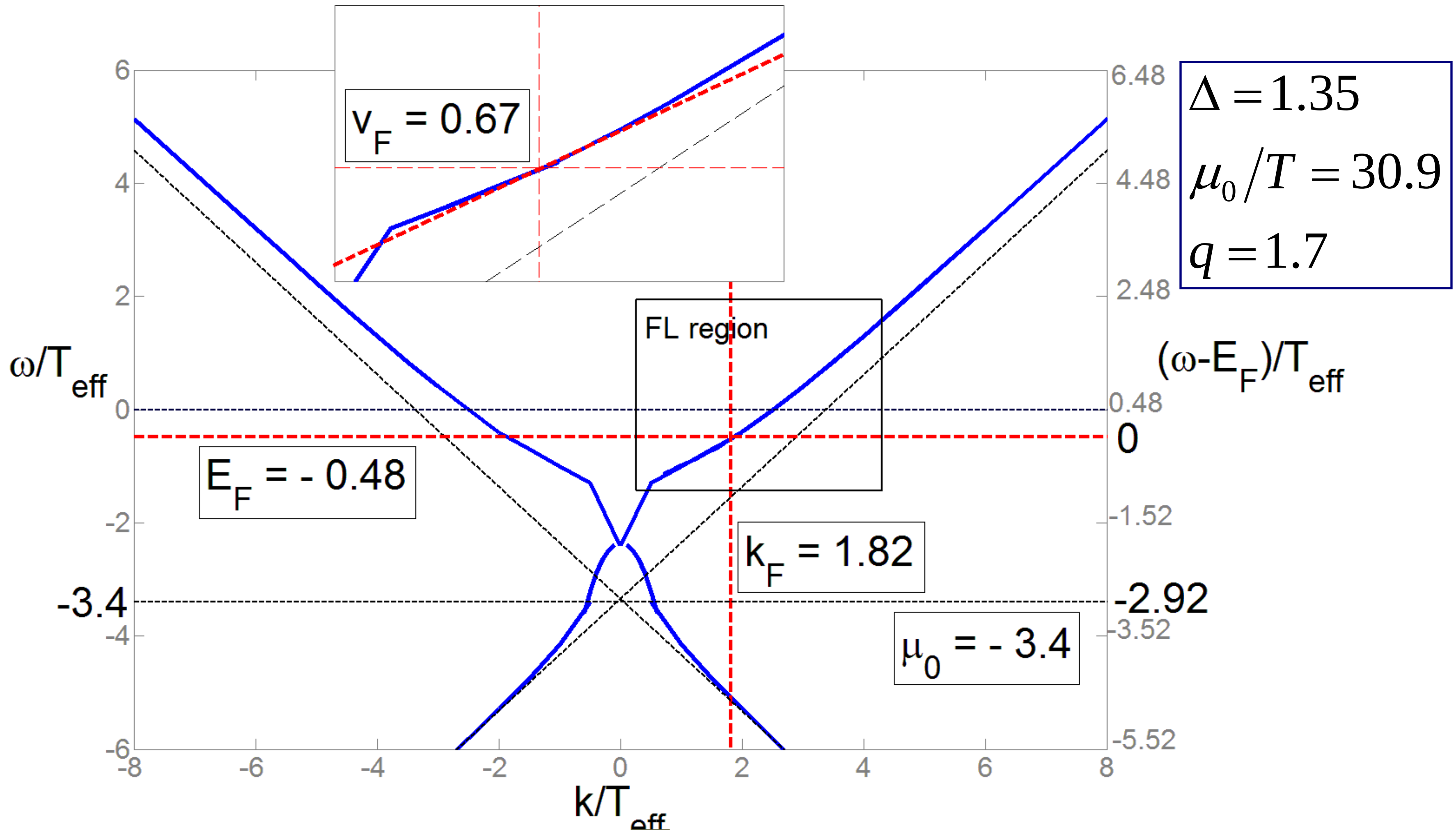
Peak width δ vs. temperature
$\delta \propto T^2$

Notice sharp critical temperature
$T_c^2 \approx 0.16 \Rightarrow (\mu/T)_c \approx 4$



Dispersion relation

Quasiparticle dispersion : $E - E_F = v_R(k - k_F) + O((k - k_F)^2)$

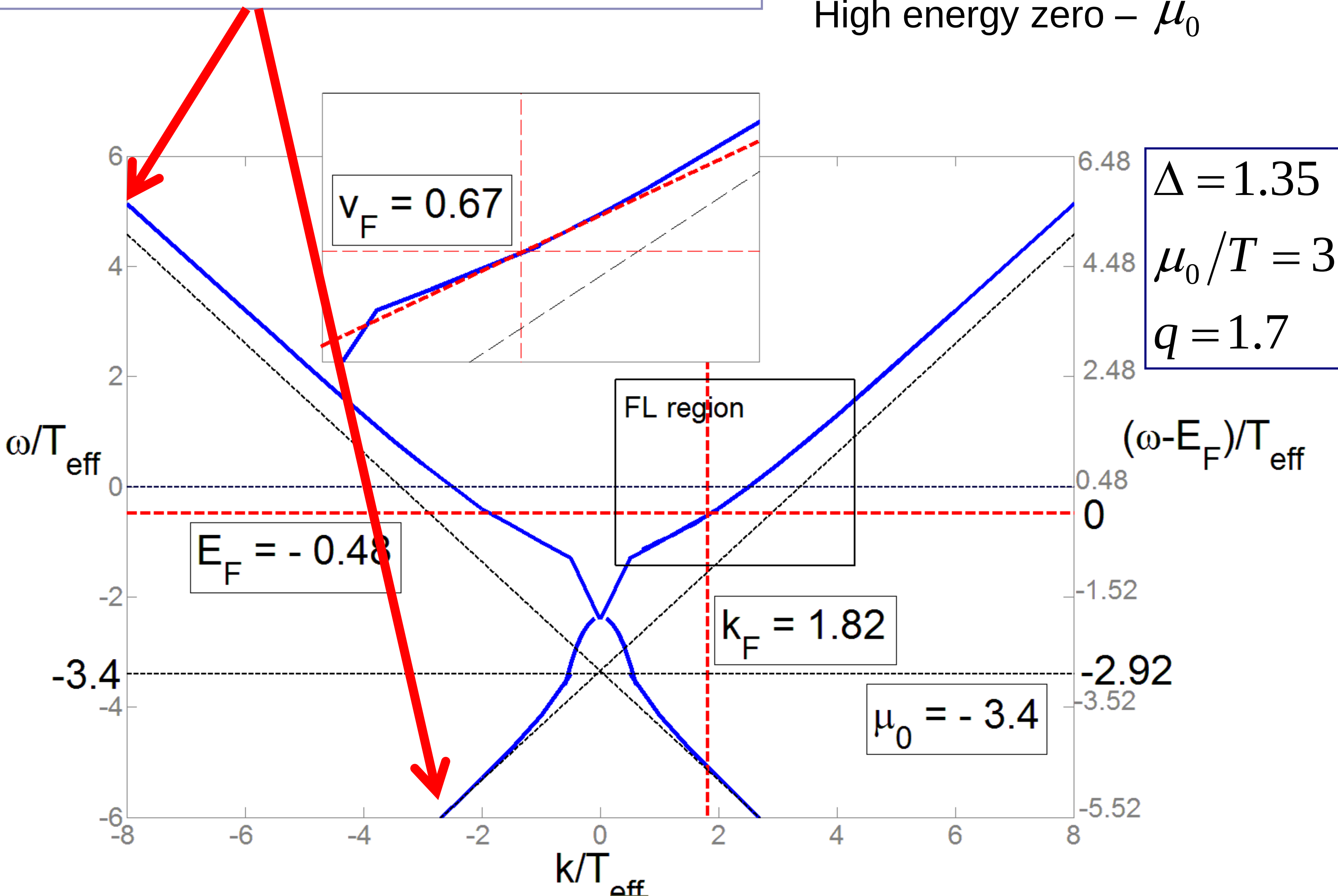


Dispersion relation

Bare (Lorentz) dispersion $\omega = k$ at high ω

Low energy zero – E_F
High energy zero – μ_0

$\Delta = 1.35$
$\mu_0 / T = 30.9$
$q = 1.7$



Dispersion relation

E_F is ***not*** at AdS zero – emergence of a new scale!

Low energy zero – E_F
High energy zero – μ_0

$\Delta = 1.35$
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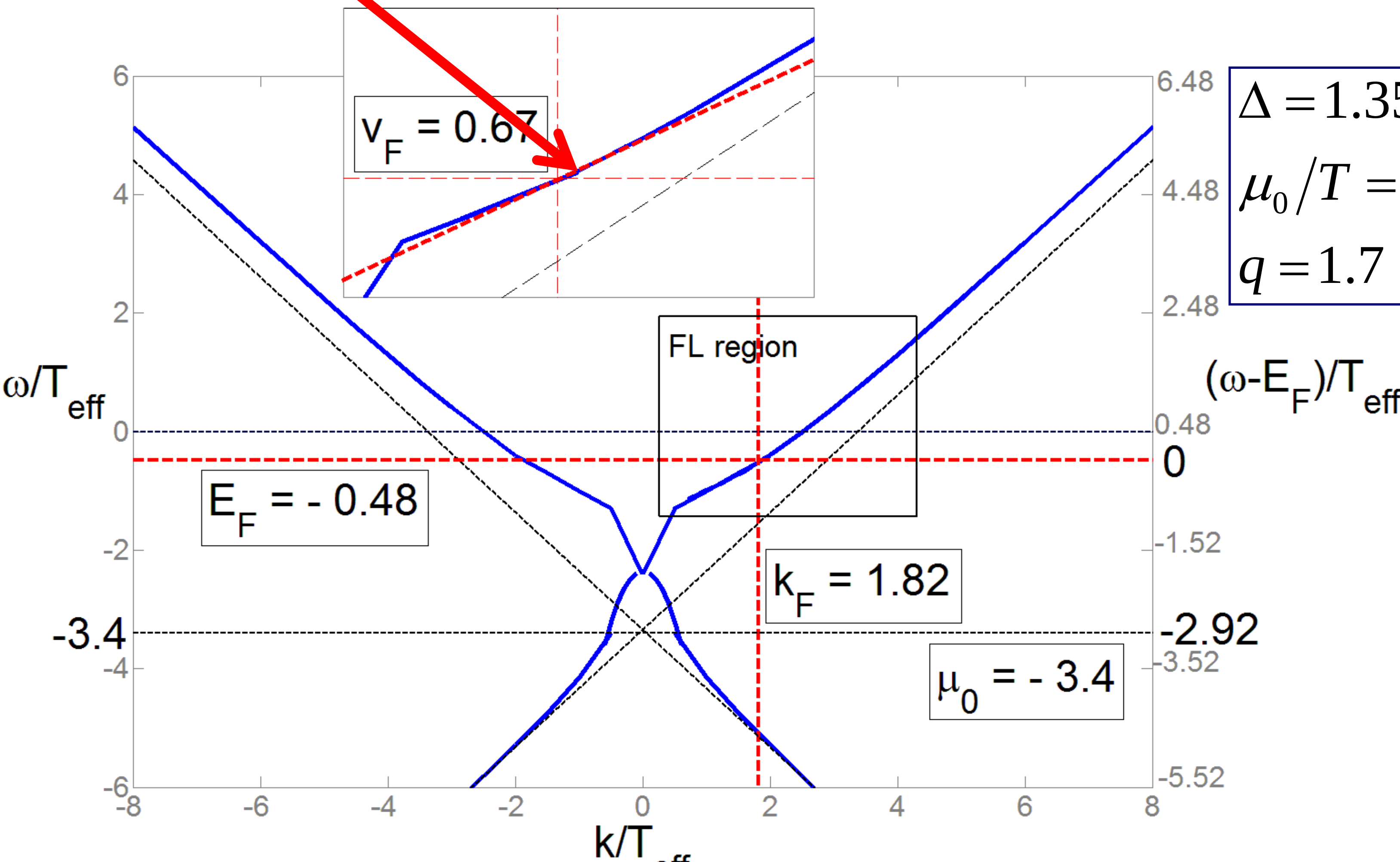
$q = 1.7$



List of Fermi liquid tests

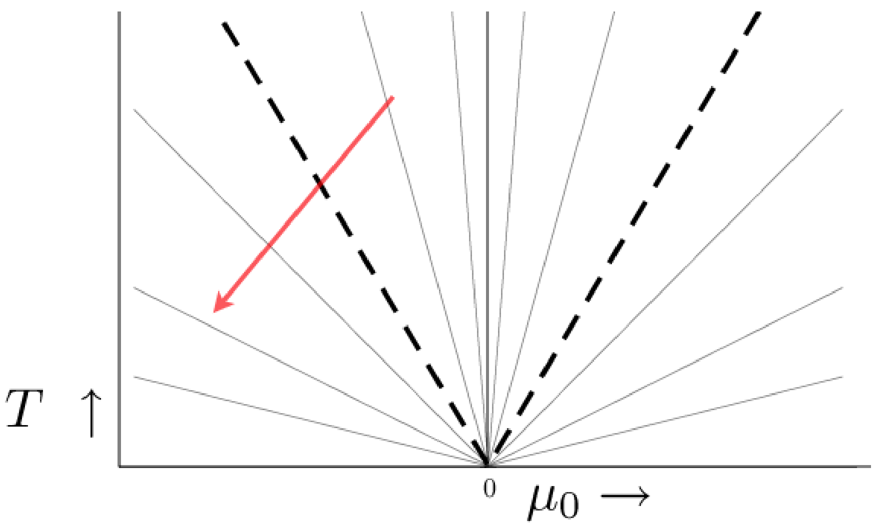
- Quasiparticle peaks ✓
- Zero density of states at the Fermi surface: $A(E_F, k_F) = 0$ ✓
- Analytical structure of self-energy ✓
- Quadratic scaling of QP peak widths with temperature ✓
- Linear dispersion of QP excitations ✓

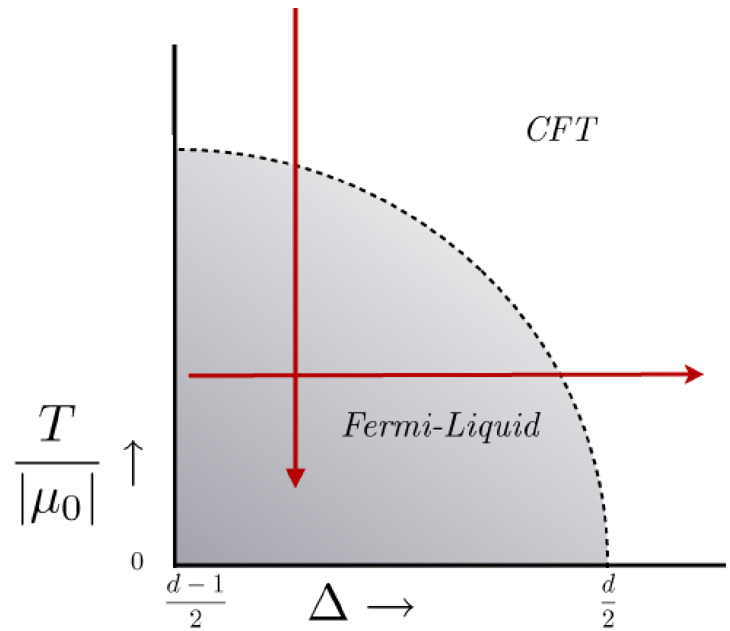
Outline of the AdS/CFT results

- Two independent parameters: μ_0/T and Δ
- Quantum criticality suggests two regimes – conformal and Fermi liquid

Δ fixed:
low μ_0/T - CFT region
high μ_0/T - FL region

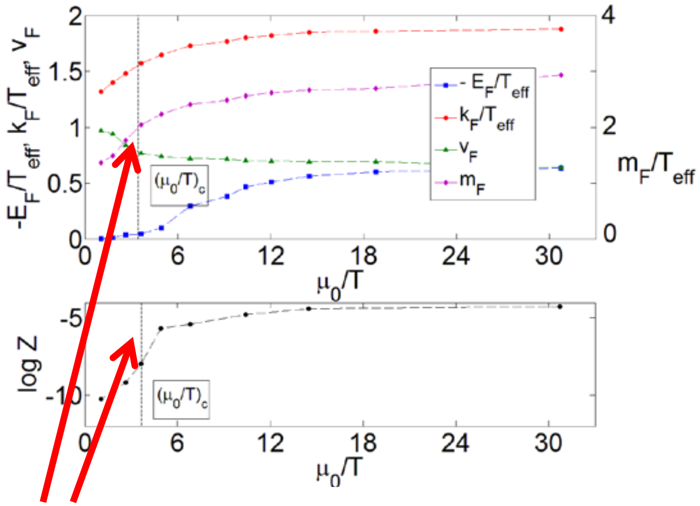
μ_0/T fixed:
low Δ - FL region
high Δ - CFT region





Charge density dependence

$\Delta = 1.25$



Critical value at $(\mu/T)_c \approx 5$
$\approx$ consistent with δ scaling

Charge density dependence

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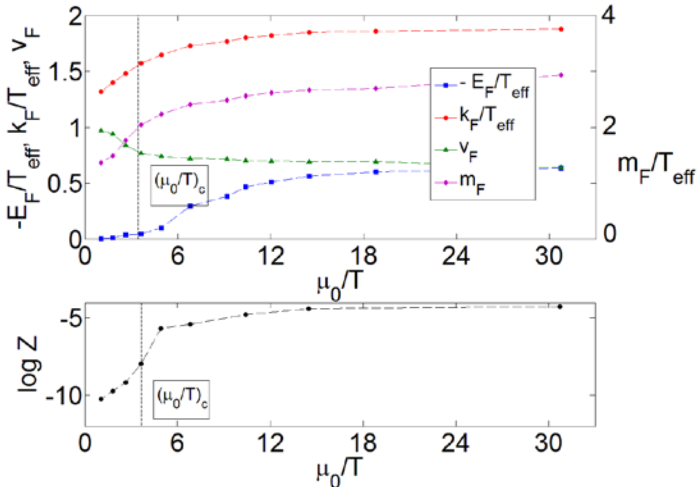
Linear growth of $\mu_0, -E_F, m_F$ in k_F



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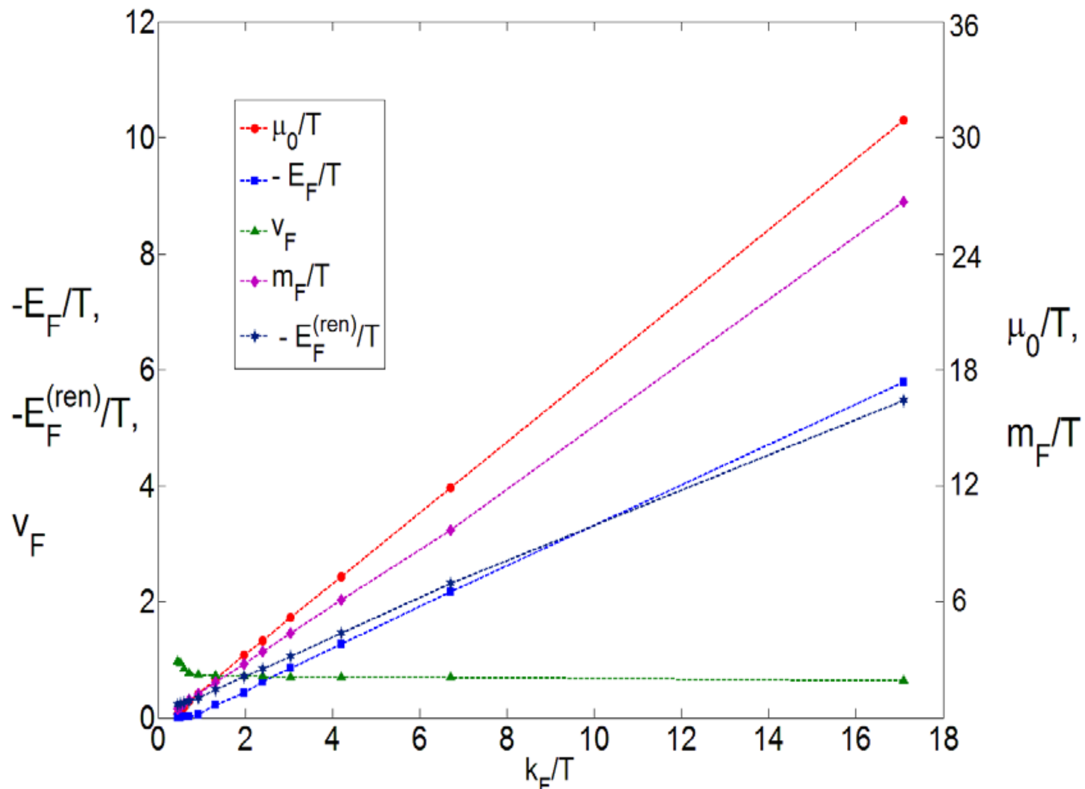

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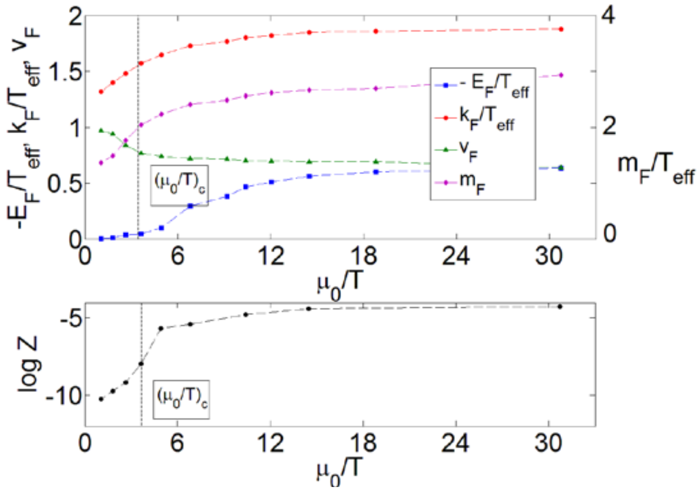

Linear growth of $\mu_0, -E_F, m_F$ in k_F

Notice that $E_F^{(ren)} \approx E_F$

Critical value at $(\mu/T)_c \approx 5$
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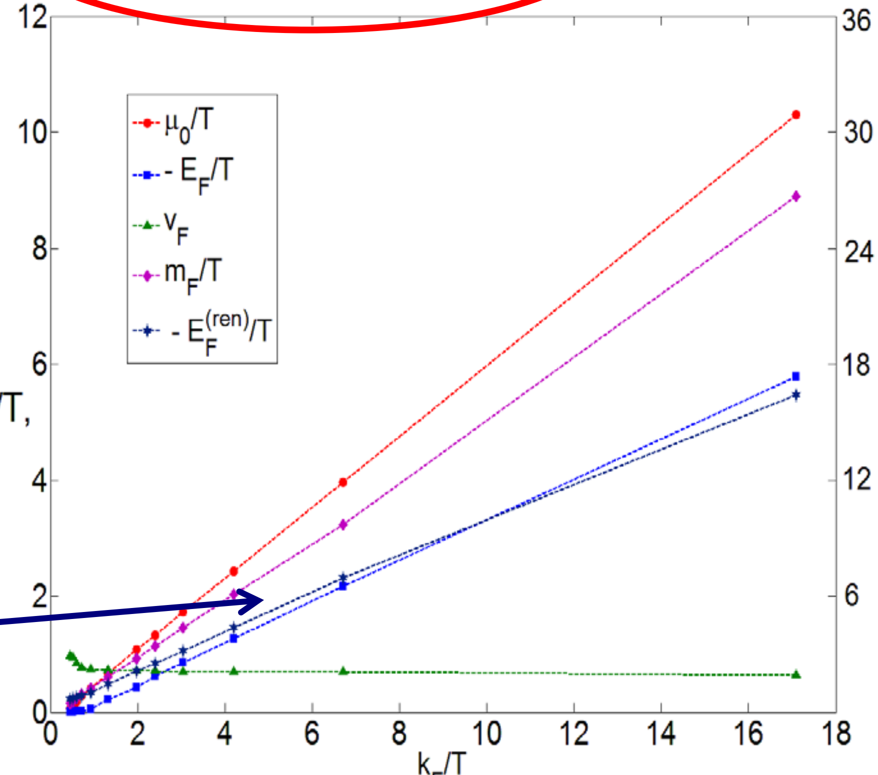


Renormalized Fermi energy:

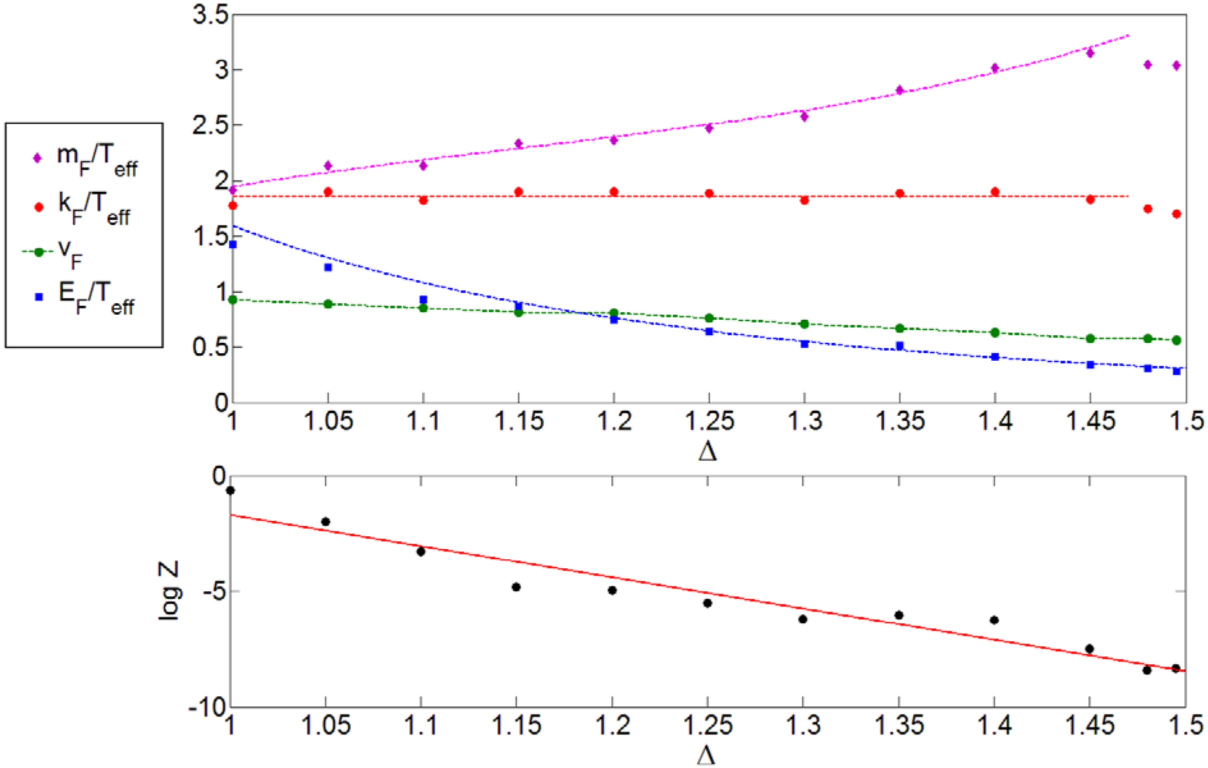
$$E = k_F^2/2m_F + (k - k_F)/m_F + \ldots$$

$$E_F^{(ren)} = k_F^2/2m_F$$

Coupling strength dependence

- $k_F \approx$ const. for all $\Delta \Rightarrow$ ***Luttinger theorem?***

- Increasingly heavy fermions for high Δ (masses are in units of T_{eff} !)



$\mu_0/T = 30.9$

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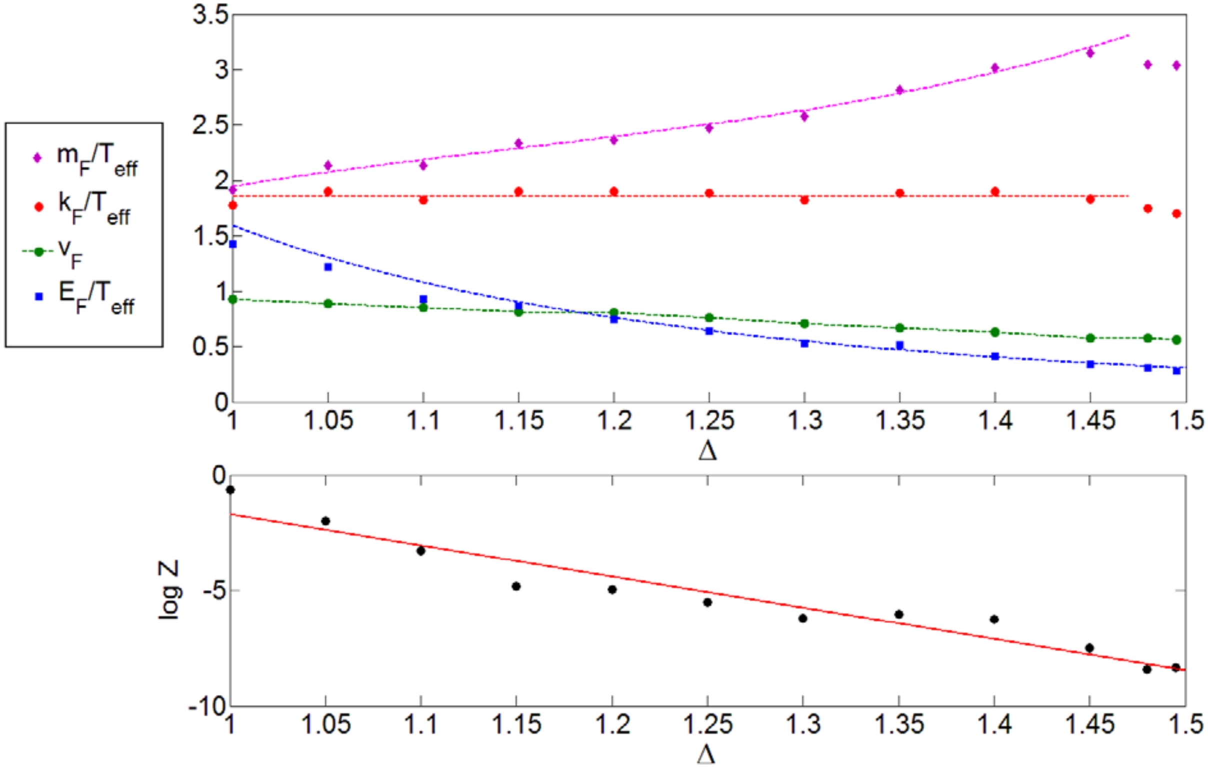

$\Delta = 3/2$ transition:

- k_F deviates from the constant Luttinger value

- v_F , m_F stay finite

- reemergence of Lorentz invariance

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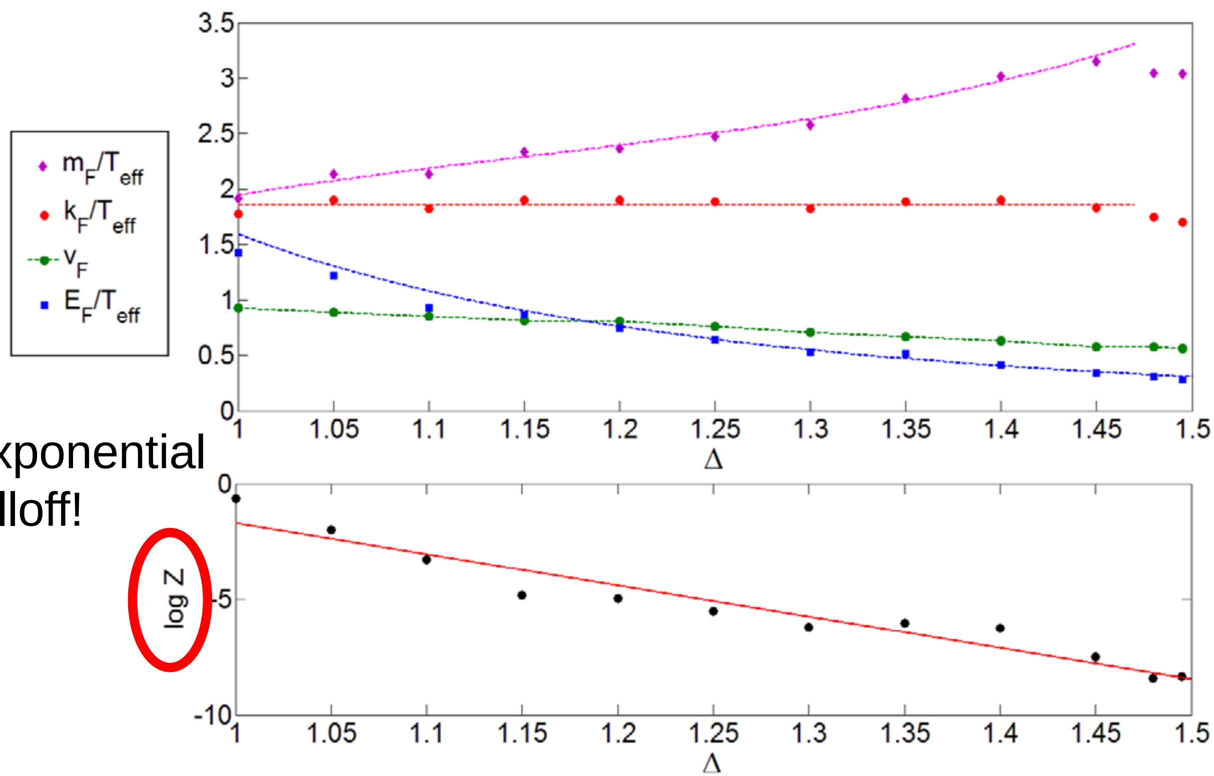

Exponential falloff!

$\Delta = 3/2$ transition:

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$\mu_0/T = 30.9$

Transition at $\Delta = 3/2$

- ***No symmetry breaking*** – purely statistical transition
- Exponential behavior of field strength renormalization:

$$Z = \exp\left(-\alpha\left(\Delta - \Delta_c\right)\right)$$

- Suggests strong momentum dependence of Σ'' (unusual):

$$\left.\frac{\partial \Sigma''}{\partial \omega}\right|_{\Delta \to 3/2} \approx -\frac{m_F}{k_F}\left.\frac{\partial \Sigma''}{\partial k}\right|_{\Delta \to 3/2} = \exp\left(-\alpha\left(\Delta - 3/2\right)\right)\left(1 + \ldots\right)$$

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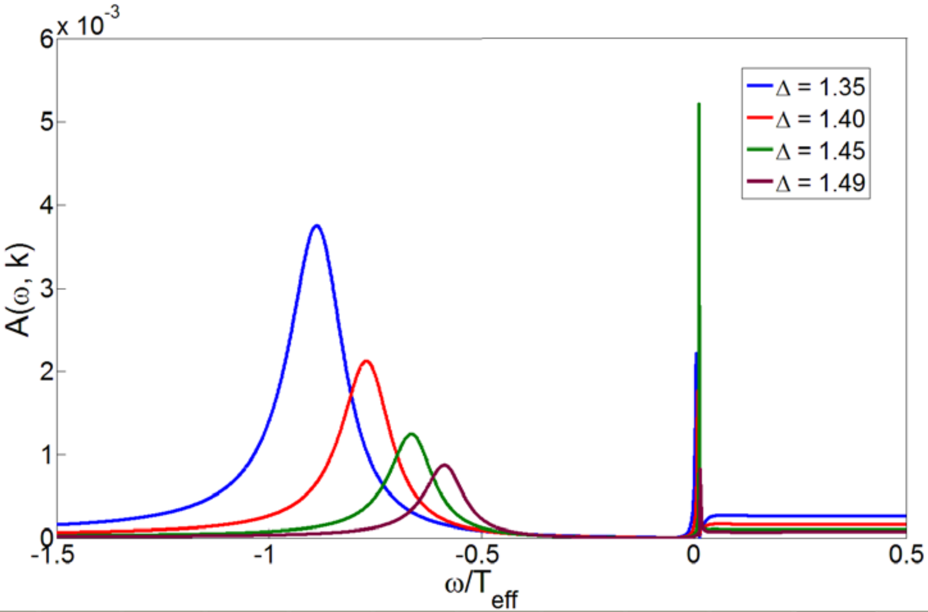
- Seen in high-T_c materials (Randeria et al 2005)
- Dynamical restoration of Lorentz invariance at leading order
- The model – Fradkin et al 2005 (arXiv:cond-mat/0508747)
- Fermi liquid/nematic Fermi fluid transition?

Summary

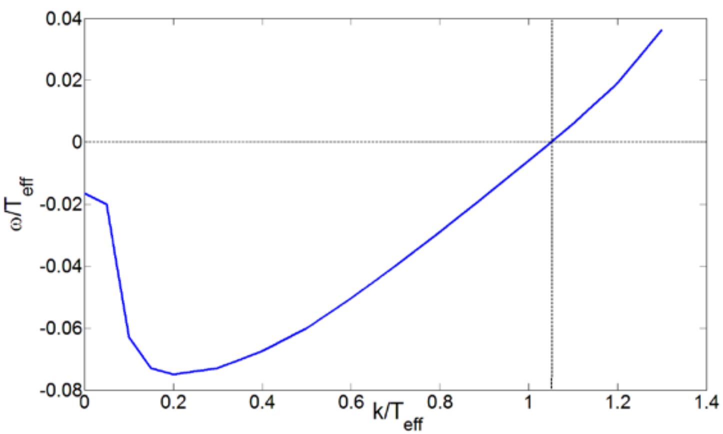
- ***Emergence of Fermi liquid from quantum critical state***
- ***AdS takes care of the Fermi signs***
- ***Nontrivial emergent properties: Luttinger theorem, Fermi momentum scale***
- ***Novel non-Fermi-Liquid state at the critical dimension*** Δ_c

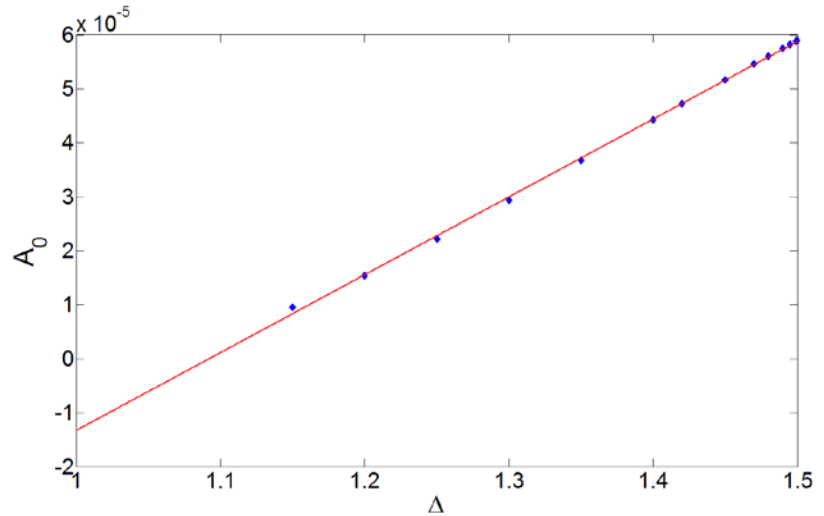
Appendix

A closer look at the $\omega \approx 0$ excitation



$\omega \approx 0$ peak intensity vs. conformal dimension: $A_0 \propto \Delta$





Finite temperature ($q = 1.65$) dispersion:

$\omega_* = a_1(k - k_*) + a_2(k - k_*)^2 + O((k - k_*)^3)$

A closer look at the $\omega \approx 0$ excitation

- First examined by Liu, McGreevy & Vegh 2009, arXiv:0904:2477v1
- Nondispersive mode at $T = 0 \implies$ non-QP behavior:

$$\omega_* = a_2 (k - k_*)^2 + O\left((k - k_*)^3\right)$$

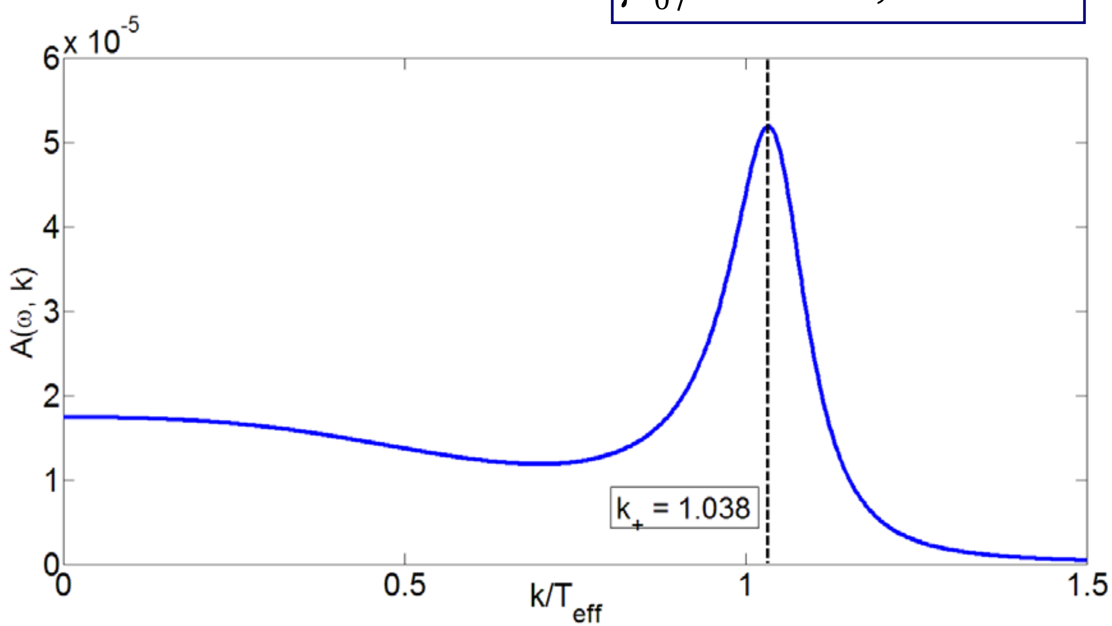
- Nonlocal in MDC

  $\Downarrow$

  localized in real space
- Critical vibration mode at (ω_*, k_*) ?
- k_* is ***not*** the Fermi momentum!
  - no QP dispersion!
  - no gapless modes near (ω_*, k_*)
  - asymmetric and non-Dirac delta peak:

$$A(\omega - \omega_*, k - k_*) \approx \left[\log\left((\omega - \omega_*)/(k - k_*)^2\right) + \text{const.}\right]^{-1}$$

$\mu_0/T = 1.65, \Delta = 1.49$



Perspective for future work

- Study the quasinormal modes of the gravity theory – where is the k_F scale to be seen?
- Apply the method by Susskind et al
- Consider full dyonic RN black hole $\Longrightarrow$ susceptibilities, Landau quantization... Quantum Hall states?
- Physical interpretation of the $\Delta = 3/2$ transition and the emergent non-FL state
- Try to catch new phenomenology by adding further terms to the gravitational theory: chiral current (antiferromagnetic behavior?),