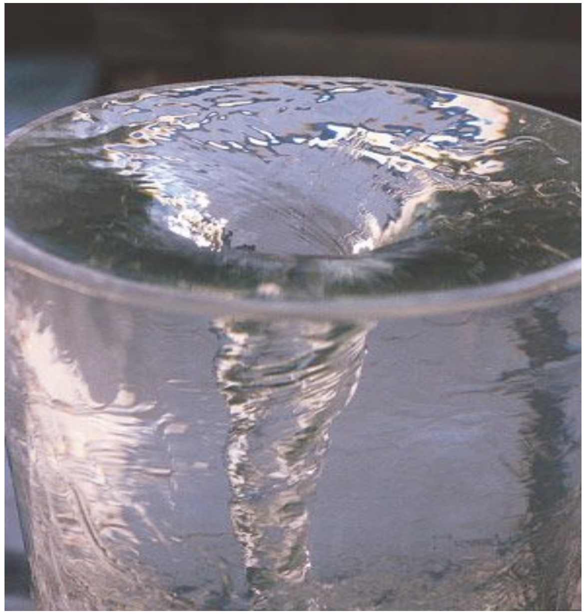
AdS/CFT and Condensed Matter, Heraklion, May 2009

Quantum Dynamics of Superfluid Vortices

Assa Auerbach, Technion, Israel

1. Gross Pitaevskii Field Theory
2. Magnus dynamics
3. 2D Coulomb Plasma
4. "Quantum Electrodynamics" of 2D vortices
5. Charge-vortex duality in transport
6. Vortex mass and dynamics on the lattice

simple bosons -
interesting excitations



References:

1. Class Notes
2. AA, D. Arovas, S. Ghosh, PRB 74, (2006)
3. D. Arovas, AA, PRB 78, (2008)
4. Netanel Lindner, AA, D. Arovas, PRL 101, (2009)

What is superfluidity/superconductivity?

Interacting Bosons
Bose field coherent states
Persistent current - rigidity

Gross Pitaevskii theory

Boson Coherent States Path Integral

$$Z = \int \mathcal{D}^2\varphi \exp\left[\frac{1}{\hbar} S[\varphi]\right]$$

$$S = \int dt \left(i \int d^d x \varphi^* \partial_t \varphi - \mathcal{H}^{GP}[\varphi] - \mu N[\varphi] \right)$$

$$\mathcal{H}^{GP} = \int d^d x \, \frac{-\hbar^2}{2m} \varphi^*(\mathbf{x}) \left(\vec{\nabla} + \frac{i2\pi}{\Phi_0} \mathbf{A} \right)^2 \varphi(\mathbf{x}) - \mu\rho_0 + \frac{g}{2}\rho_0^2$$

Electromagnetic field $\nabla \times \mathbf{A}^{em} = \mathbf{B}$

Rotating frame: $\nabla \times \mathbf{A}^{cor} = -\frac{2m}{q}\omega\hat{\mathbf{z}}$

1. Saddle point energy

$$\dot{\varphi} = 0 \qquad \frac{\delta \mathcal{H}^{GP}}{\delta\varphi} = 0$$

2. Semiclassical dynamics

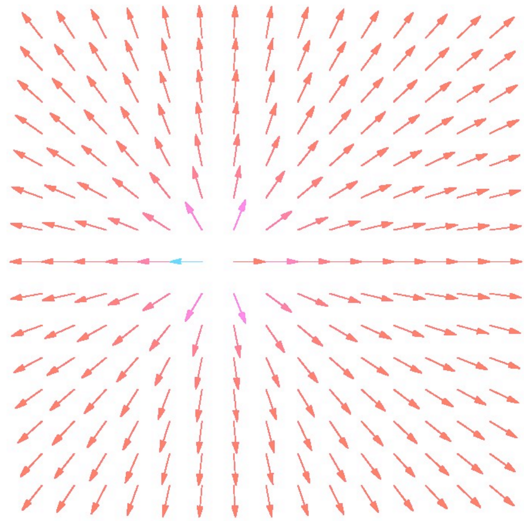
$$\left.\frac{\delta S}{\delta \varphi}\right|_{\varphi^{sc}(t)} = 0$$

Vortex Solution

$$\left(K(\boldsymbol{A}) + V - \mu + g\left|\widetilde{\varphi}(\boldsymbol{x})\right|^2\right)\widetilde{\varphi}(\boldsymbol{x}) = 0$$

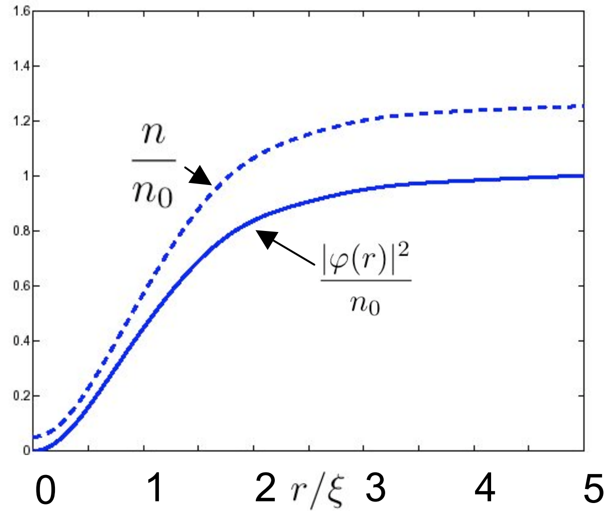
$$\lim_{|\boldsymbol{x}|\to\infty} |\tilde{\varphi}| \to \sqrt{n_0}$$

coherence lengthscale $\xi = \dfrac{\hbar}{\sqrt{g n_0 m}}$



One flux quantum in the system

Vortex (approx.) solution: $\widetilde{\varphi} \approx \dfrac{\sqrt{n_0}\, r\, e^{i\phi}}{\sqrt{r^2 + \xi^2}}$



Fluctuations parameter: $\dfrac{\delta n}{n_0} = \dfrac{1}{4\pi n_0 \xi^2}$

small parameter of Bogoliubov theory

Vortex Interactions: 3D magnetostatics

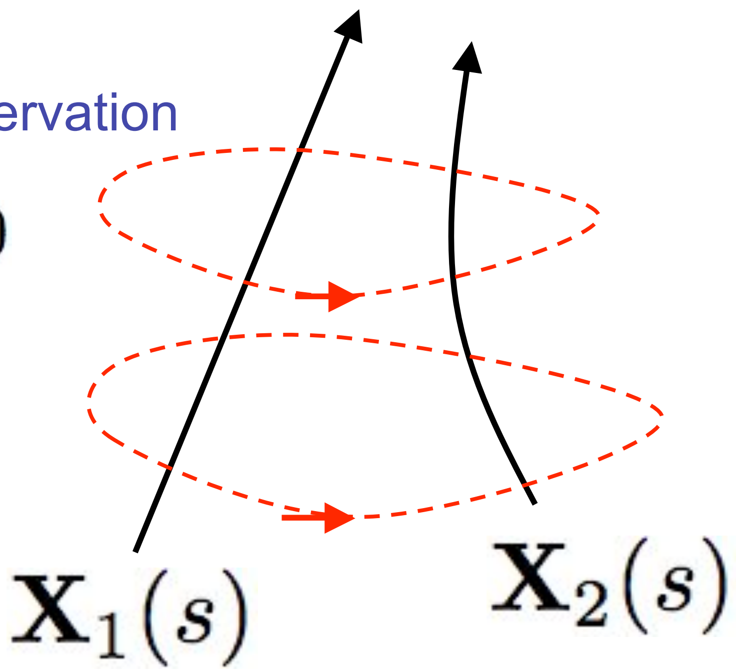
quantized circulation around each vortex $\oint dl \cdot \mathbf{v}_i = \frac{\hbar}{m} k_i$

many vortices $\mathbf{v}(\mathbf{x}) = \sum_i \mathbf{v}_i(\mathbf{x}, \mathbf{X}_i(s))$

$$E[\varphi] = \frac{1}{2} \int d^d x m \rho(\mathbf{x}) |\mathbf{v}(\mathbf{x})|^2 = E_{vor}[\mathbf{X}_1, \mathbf{X}_2, \ldots]$$

static current conservation

$$\nabla \cdot \mathbf{v}(\mathbf{x}) = 0$$



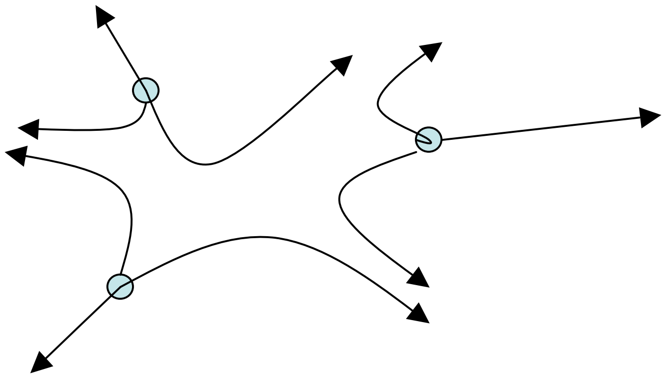
analogy:
interacting lines of
quantized currents

2D Electrostatics

$$\mathbf{E} \equiv \rho \hat{\mathbf{z}} \times \mathbf{v} = 2\lambda_i \frac{\hat{\mathbf{r}}_i}{|\mathbf{r}_i|} \qquad \lambda_i = k_i \frac{\rho_0 \hbar}{2m} \quad \text{(CGS)}$$

vortex interactions = 2D Coloumb $\quad V(\mathbf{r}_1 - \mathbf{r}_2) = -\frac{q^2}{2\pi} \log |\mathbf{r}_1 - \mathbf{r}_2|$

$$H^{\mathrm{cl}} = \epsilon_c \left[-\sum_{i \neq j} \log |\mathbf{R}_i - \mathbf{R}_j| + \pi \phi_0^{-1} B \sum_i \mathbf{R}_i^2 \right]$$



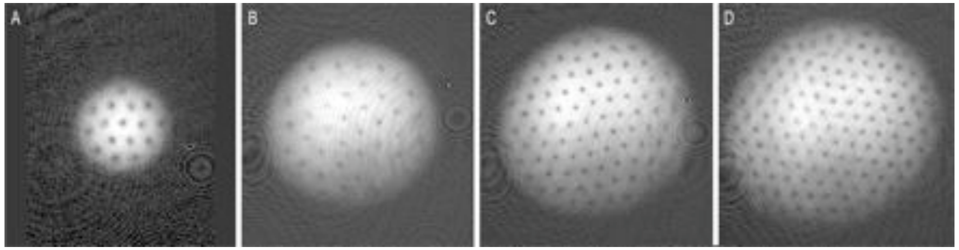
2D One component Coulomb Plasma

1. Incompressible, vortex density = flux density B
2. Ground state = triangular lattice
3. Melting temperature is independent of B

Realizations of superfluid vortices

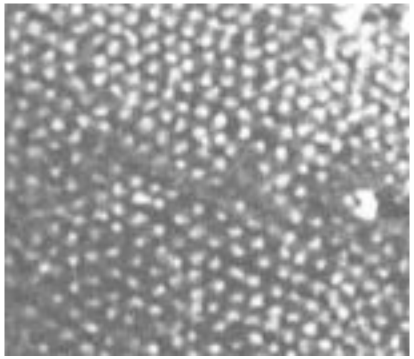
1. Rotating BEC 's

J. R. Abo-Shaeer et. al., Science 2001

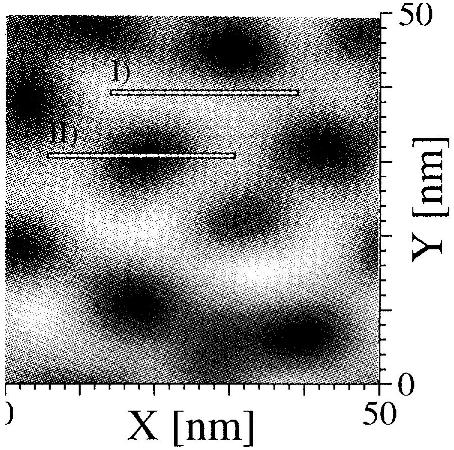


2. Superconductors in a magnetic field

Nb-Mo



YBCO, Fischer '95

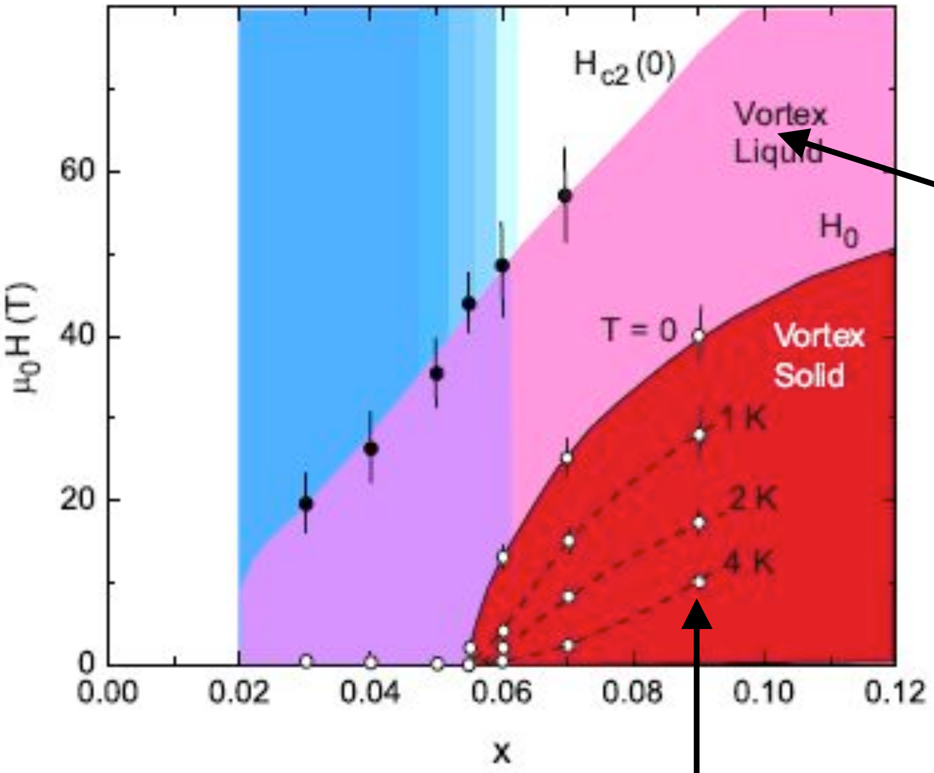


Quantum melting in cuprates

Low temperature vortex liquid in $La_{2-x}Sr_xCuO_4$

Nature Physics 3, 311-314 (2007)

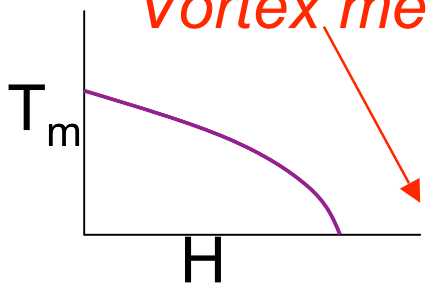
Lu Li[1], J. G. Checkelsky[1], Seiki Komiya[2], Yoichi Ando[2], and N. P. Ong[1*]



A quantum phase:

Vortex condensate?

Vortex metal?



T_m decreases with field. (Non classical)

Magnus *Force* $\mathbf{E}^{mag} = \hat{\mathbf{z}} \times \mathbf{j}$

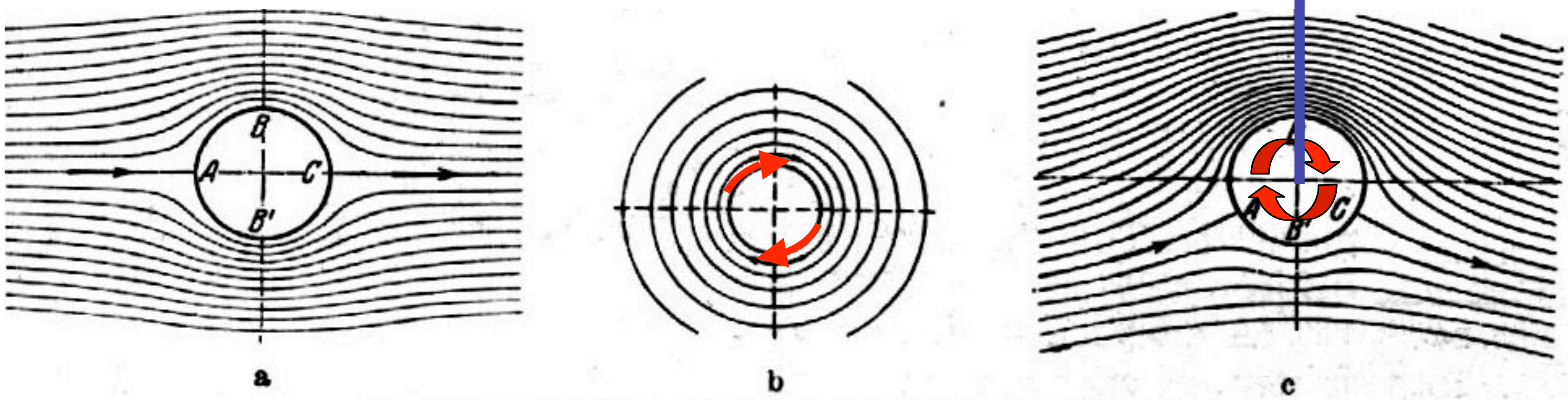
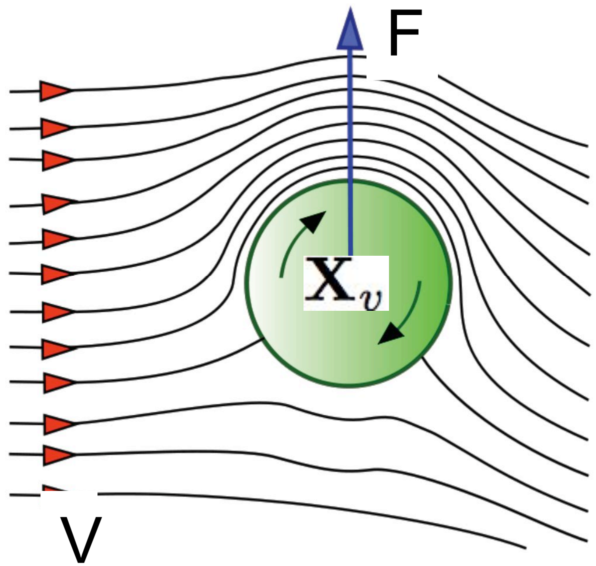
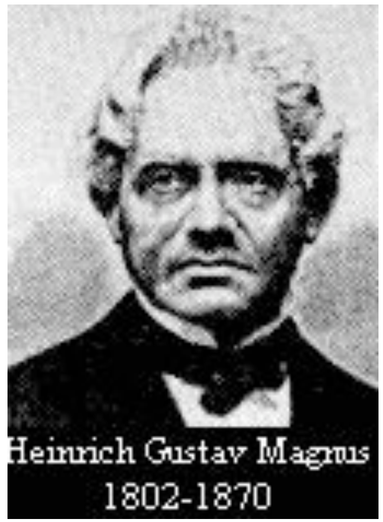


Fig. 249 Stream lines (about a circular cylinder at rest) for three different flows



$$\vec{F} = -\vec{\nabla} E(\mathbf{X}_v) = n\vec{V} \times \vec{\kappa}_v$$

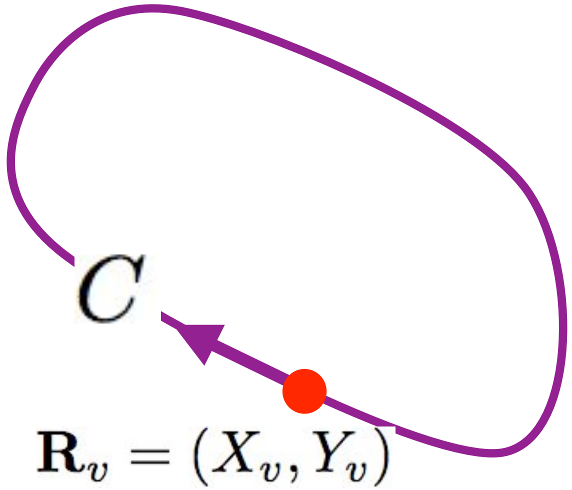


Magnus semiclassical ***dynamics***

$$U^{\text{adiab}}[C] = \int \mathcal{D}\mathbf{R_v}(t) \exp\left\{ i \int_0^t dt' \, \arg[\langle \Phi_{\mathbf{R_v}} | \frac{d}{dt'} | \Phi_{\mathbf{R_v}} \rangle] - \mathcal{H}(\mathbf{R_v}(t')) \right\}$$

$$= e^{i \int_0^t dt' \left(2\pi n_0 X_v \dot{Y}_v - \mathcal{H}[\mathbf{R_v}] \right)}$$

2π X number of bosons enclosed by C



Analogy:

a vortex behaves as a charge in a strong magnetic field of magnitude n_b

Vortex follows equipotential contours,
i.e. "Goes With The Flow"

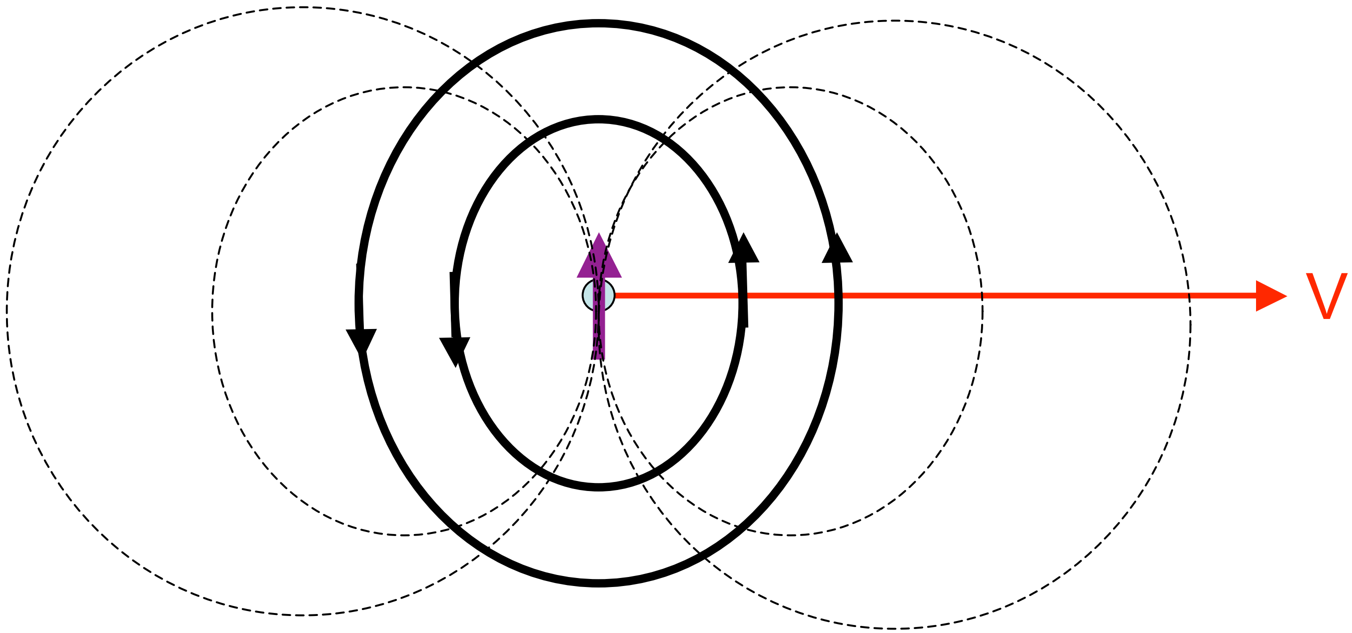
<u>Warning:</u> SC dynamics are valid only for

1. smooth potentials

2. no Umklapp scattering of a lattice

Vortex Motion

vorticity current $$\mathcal{J}_v(\mathbf{x}) = \sum_i k_i \mathbf{V}_i \delta(\mathbf{x} - \mathbf{X}(t))$$



a moving vortex creates a stress field $$\mathbf{\Sigma} \equiv m\dot{\mathbf{v}}(\mathbf{x}) = \hbar\vec{\nabla}\dot{\phi} = h\hat{\mathbf{z}} \times \mathcal{J}_v$$

Charge-Vortex duality in transport

1. Charge transport equation $\mathbf{j}^{\alpha} = \sum_{\beta} \sigma^{\alpha\beta} E^{\beta}$ $\rho \equiv \sigma^{-1}$

2. Vortices transport equation $\mathcal{J}_v^{\alpha} = \sum_{\beta} \sigma_v^{\alpha\beta} \varepsilon_v^{\beta}$

Magnus field on a vortex $\varepsilon_v = \frac{h}{q} \hat{\mathbf{z}} \times \mathbf{j}$

Vortex Induced stress field $\mathbf{E} = \frac{1}{q} \Sigma = \frac{h}{q} \hat{\mathbf{z}} \times \mathcal{J}_v$

$$\sigma_v^{\alpha\beta} = \left(\frac{h}{q} \right)^2 \rho^{\alpha\beta}$$

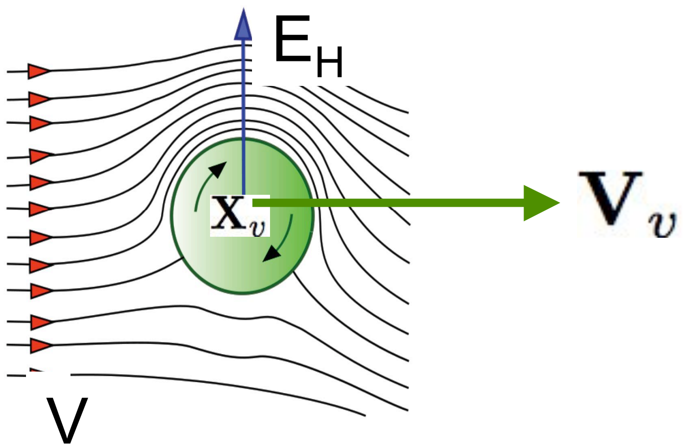
vortex conductivity = charge resistivity

Semiclassical transport theory

Semiclassical (smooth potential) dynamics, *without tunneling*:

a. Vortices follow equipotential contours: $\rho_{xx} = 0$
b. With no potential, vortices 'Go with the Flow'
 and the Hall resistance (by Galiliean symmetry) is 'classical':

$$\rho_{xy} = -\frac{B}{n_0 q c}$$



QED Theory of Phase Fluctuations

Arovas and AA, PRB 78 (2008)

phase modes behave as 2+1D photons, minimally coupled to the vortex density and current

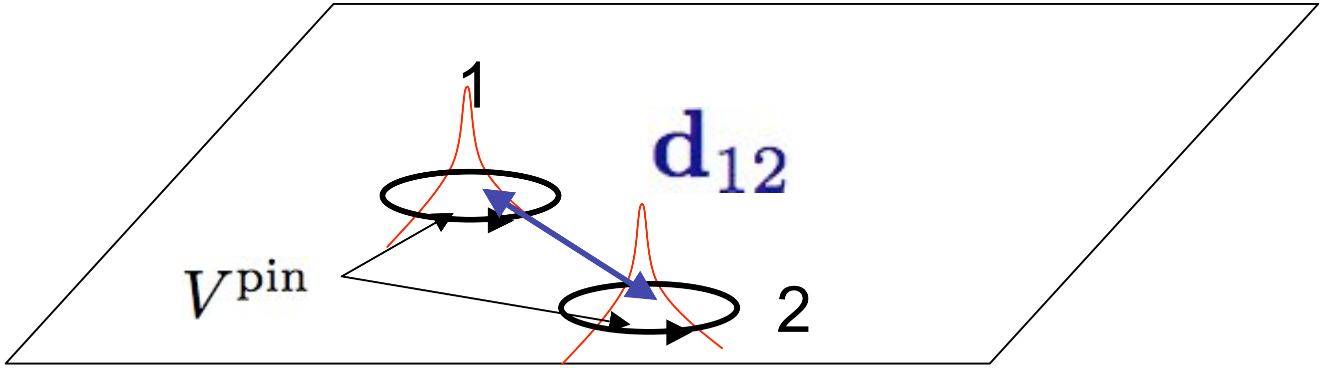
$$\mathcal{L}_{\mathrm{E}} = \frac{1}{2} m c^2 n_0 \left\{ \mathcal{E}^2 + (\mathcal{B}-1)^2 + \frac{1}{4}(\xi \nabla \mathcal{B})^2 \right\} + 2\pi i \hbar n_0 \mathcal{J}^\mu \mathcal{A}_\mu$$

$$\mathcal{J}^\mu(x,t) = \sum_i q_i \begin{Bmatrix} c \\ \dot{X}_i \end{Bmatrix} \delta[x - X_i(t)],$$

The phase ‘photons’ produce retarded interactions between vortices, and produce a vortex self energy (mass).

Vortex Tunneling in the continuum

spatially varying potentials can induce vortex tunneling



Vortex tunneling rate:

$$t_v = V^{\mathrm{pin}} \, |\langle \Phi_0(1) | \Phi_0(2) \rangle|$$

Bogoliubov theory yields: AA, D.P. Arovas, S. Ghosh, Phys Rev B 74, 2006

$$t_v = V^{\mathrm{pin}} \exp\left(-\frac{\pi}{2} n_0 d_{ij}^2\right) \left(1 + \mathcal{O}(g)\right) \equiv \frac{\hbar^2}{M_v d_{ij}^2}$$

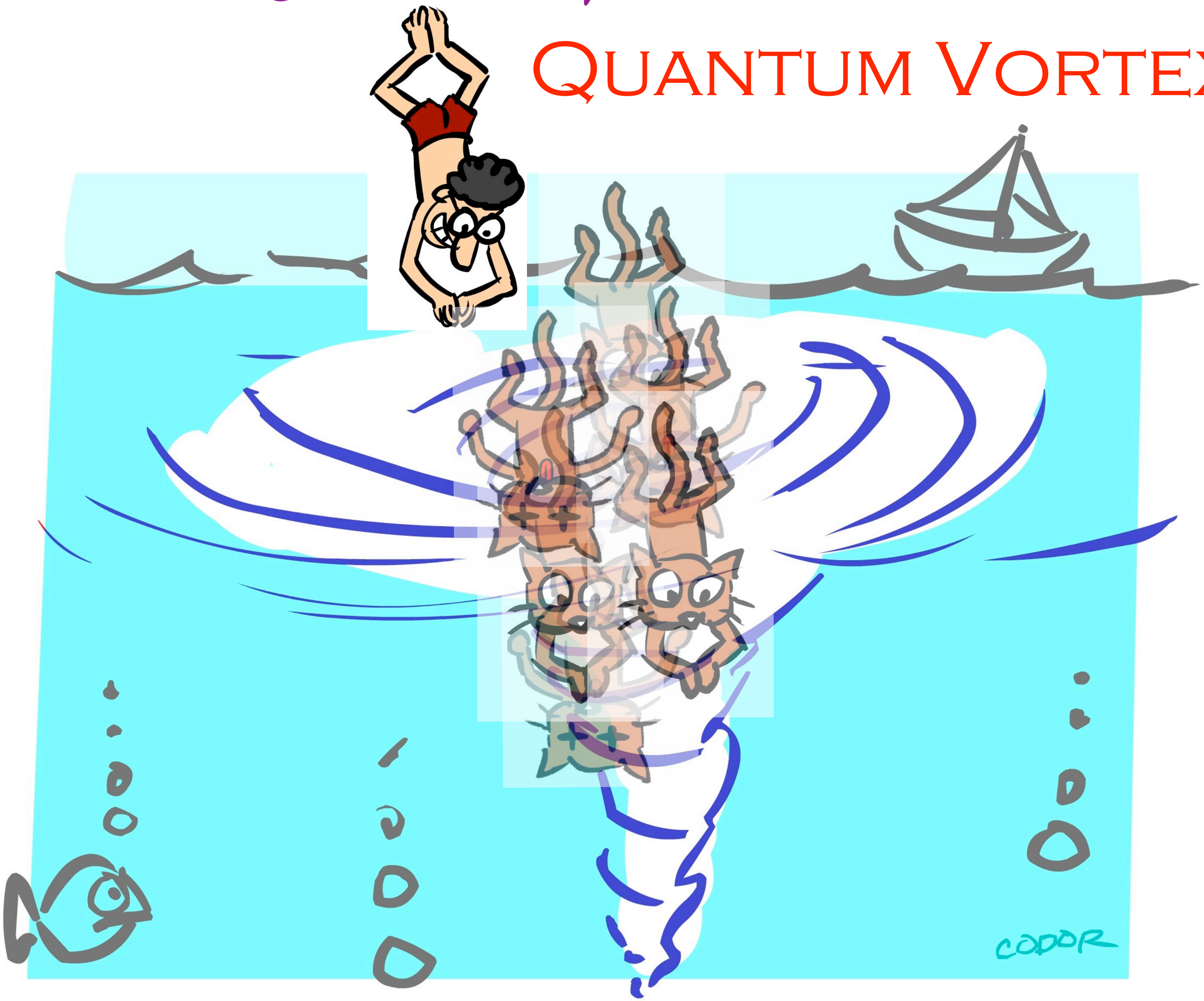
effective 'mass'

Summary: vortices in 2D Continuum

1. Vortices localize on equipotential contours and drift with the superflow.
2. GP equation cannot treat short range potential effects.
3. Short range potentials delocalize vortices (make them ‘’quantum’’).

Dick Codor's impression...

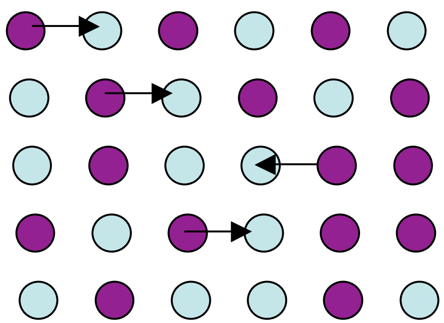
Quantum Vortex



Hard core bosons on a square lattice

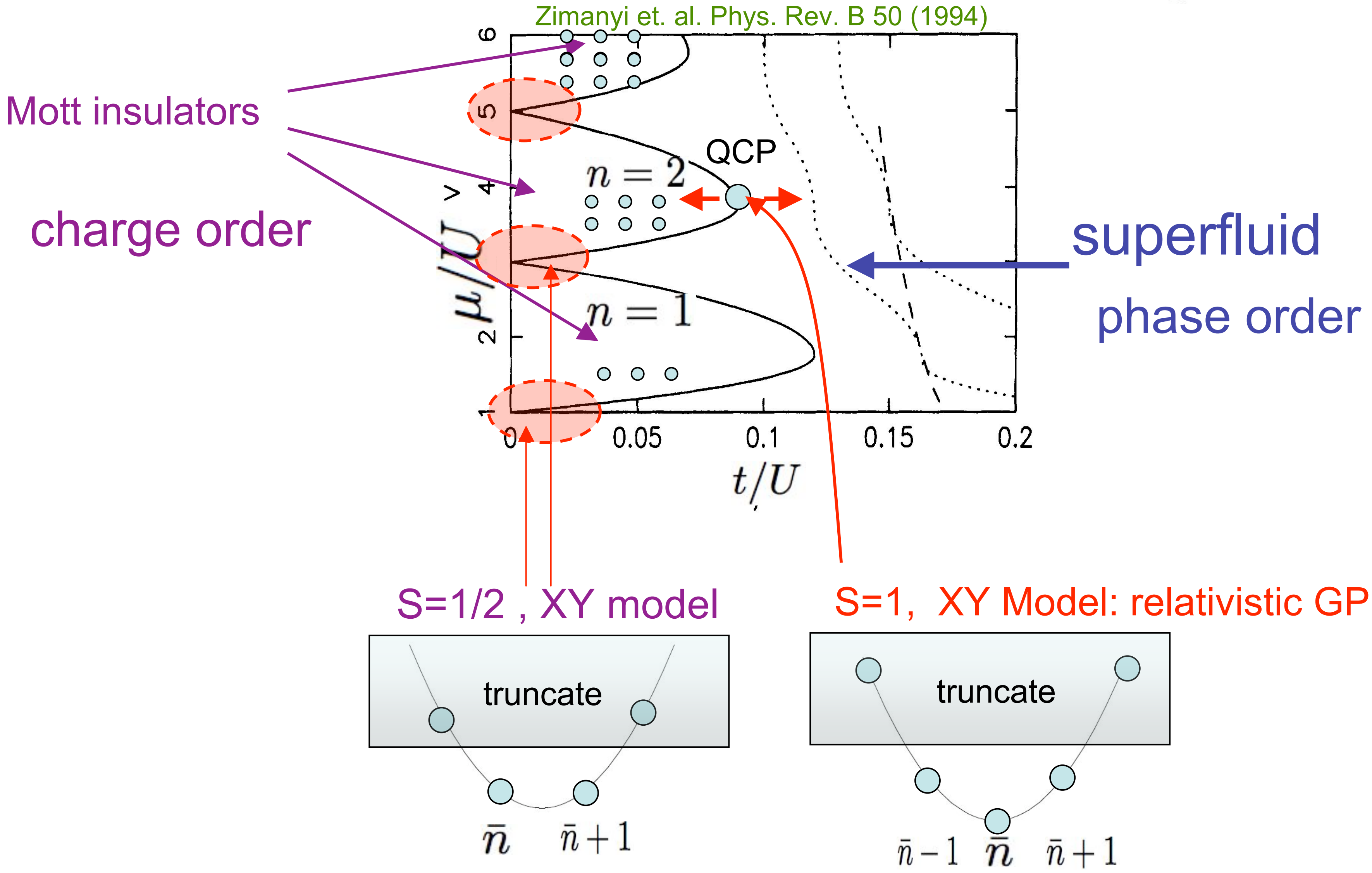
Netanel Lindner, AA, D. Arovas, PRL 101, (2009)

1. Vortices have a light mass, vortex solid can melt into a Quantum Vortex Liquid at 0.006 vortices per site.
2. Hall conductivity (Magnus action) reverses sign at half filling, with an associated vanishing temperature scale.
3. Vortices at half filling carry local spin 1/2 doublets. (V-spins)
4. QCP at half filling?



Lattice bosons

Bose Hubbard Model $\mathcal{H} = -t\sum_{ij} a_i^{\dagger} a_j + U\sum n_i{}^2 - \mu\sum_i n_i$



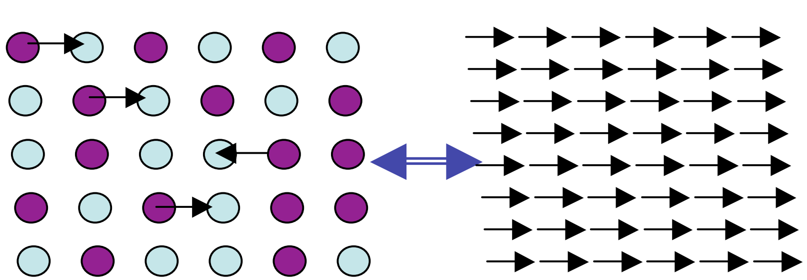
Hard Core Bosons

Hard core bosons $(a^\dagger)^2 = 0$

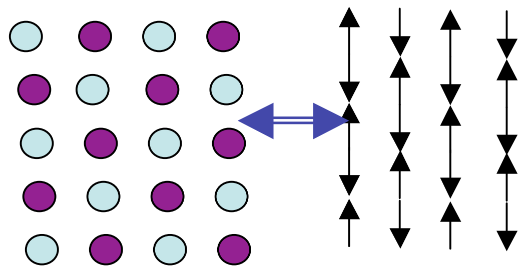
$$\mathcal{H} = \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} -\frac{t}{2}\left(e^{i\int_{\mathbf{r}}^{\mathbf{r}'} d\mathbf{l}\cdot\mathbf{A}}\; a^\dagger_{\mathbf{r}} a_{\mathbf{r}'} + \text{h.c.} \right) + V n_{\mathbf{r}} n_{\mathbf{r}'}.$$

S=1/2, XXZ model

$$= \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} \frac{-t}{2}\left(e^{i\int_{\mathbf{r}}^{\mathbf{r}'} d\mathbf{l}\cdot\mathbf{A}} S^+_{\mathbf{r}} S^-_{\mathbf{r}'} + \text{h.c.} \right) + V S^z_{\mathbf{r}} S^z_{\mathbf{r}'}$$

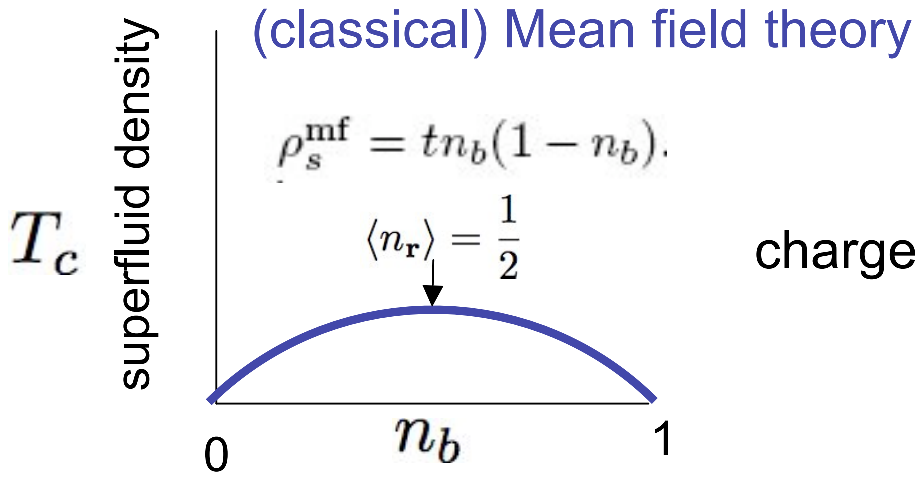


superfluid
$V < t$



half filling
charge density wave
$V > t$

Properties of XXZ model



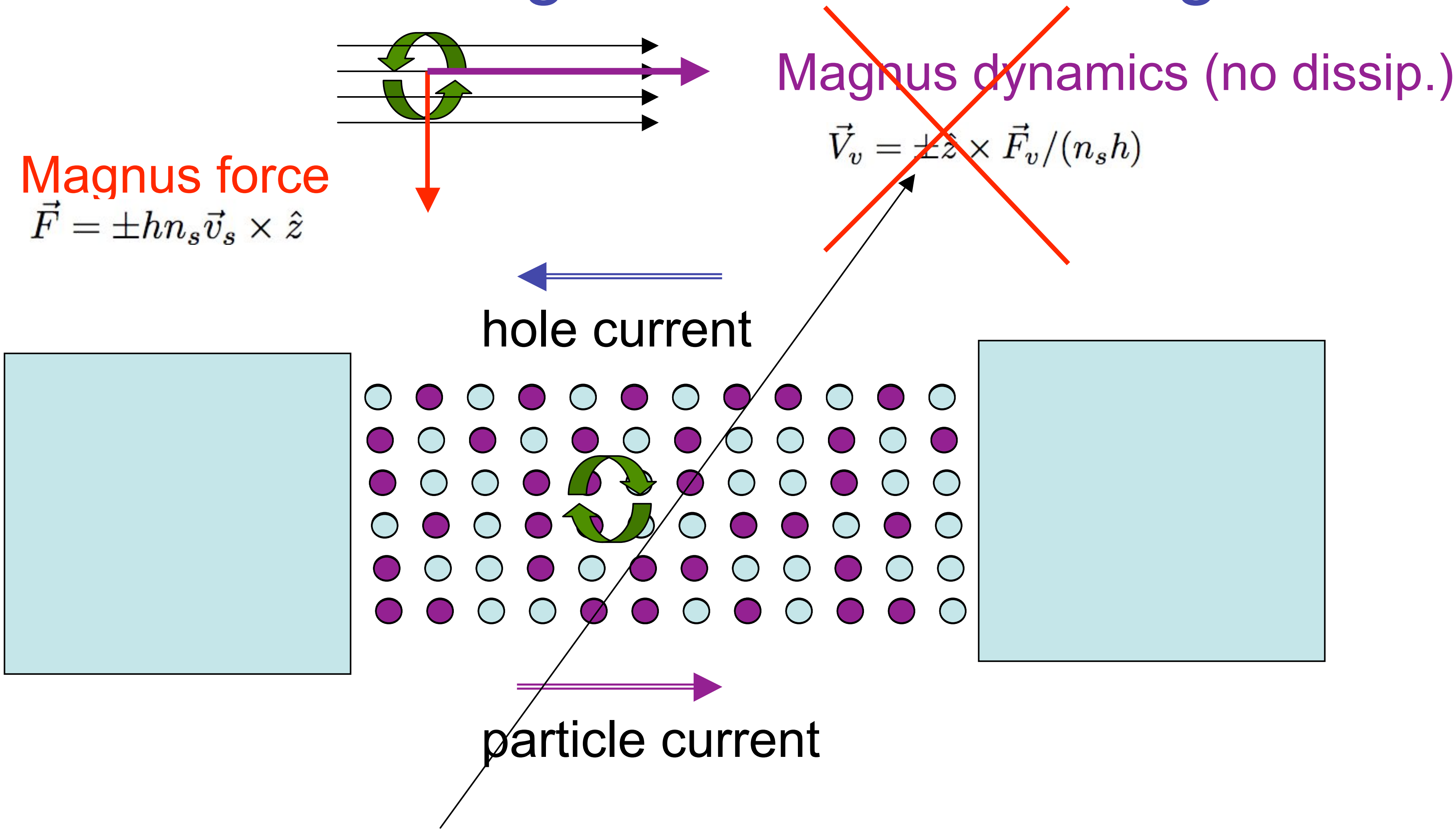
charge conjugation $\quad C \equiv \exp\left(i\pi \sum_{\bf r} S^x_{\bf r}\right)$

$$C^\dagger \mathcal{H}[{\bf A}, n_b] C = \mathcal{H}[-{\bf A}, 1 - n_b]$$

$$\sigma_H(n_b, T) = -\sigma_H(1 - n_b, T)$$

1. Half filling has maximal stiffness
2. At half filling the Hall conductivity vanishes
3. Hall conductivity is antisymmetric about half filling

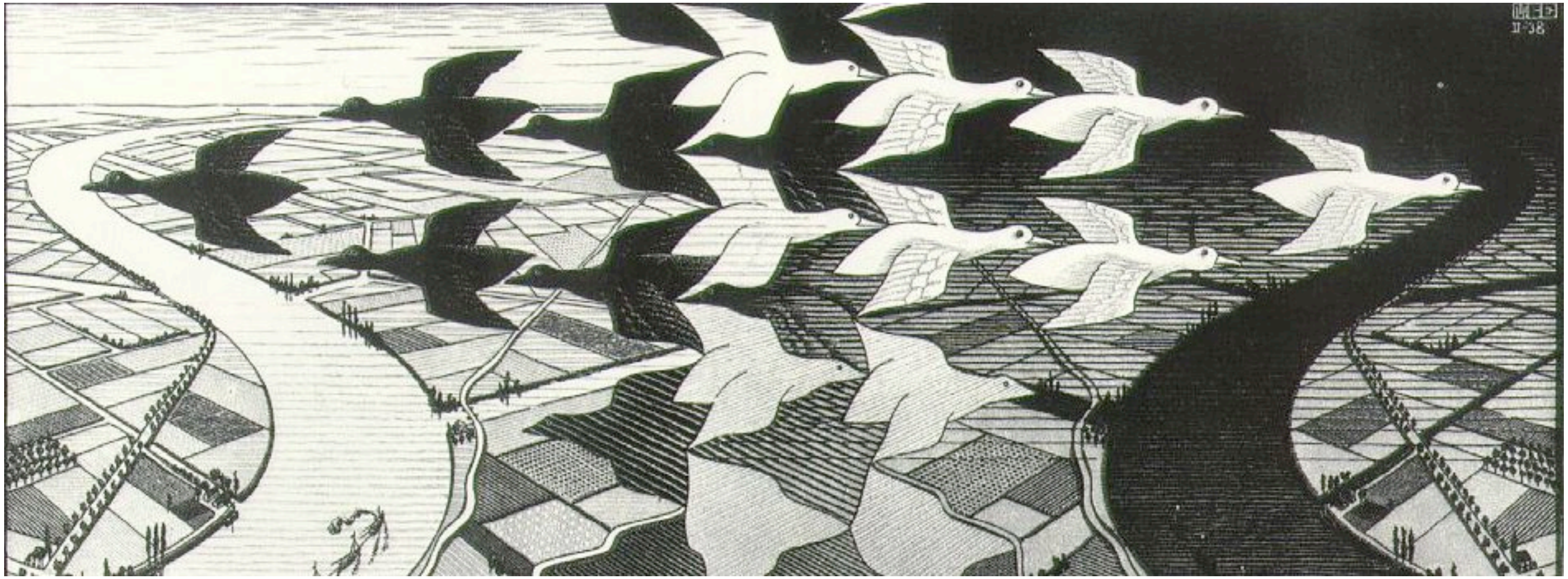
The strangeness of Half Filling



PH symmetry:

no inertial drift (vanishing Hall conductivity)

Escher, Day and Night, 1938



To study vortex quantum dynamics:

1. Vortices can be introduced into the ground state with an external magnetic field.

2. Vortex effective Hamiltonian can be extracted by fitting the exact spectrum to an effective hopping model.

3. Hall conductivity is evaulated using Chern numbers of the gauged S=1/2 quantum XXZ model a finite torus.

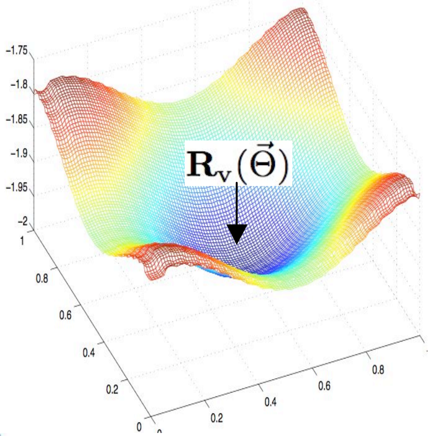
Extracting the vortex hopping rate

Vortex Harper hamiltonian

$$H^{\mathrm{v}}_{\mathbf{R},\mathbf{R}'} = -\frac{t_v}{2}\sum_{\boldsymbol{\eta}} e^{iA^d_{\boldsymbol{\eta}}}\,\delta_{\mathbf{R}',\mathbf{R}+\boldsymbol{\eta}} + U_N(\mathbf{R})\,\delta_{\mathbf{R},\mathbf{R}'}$$

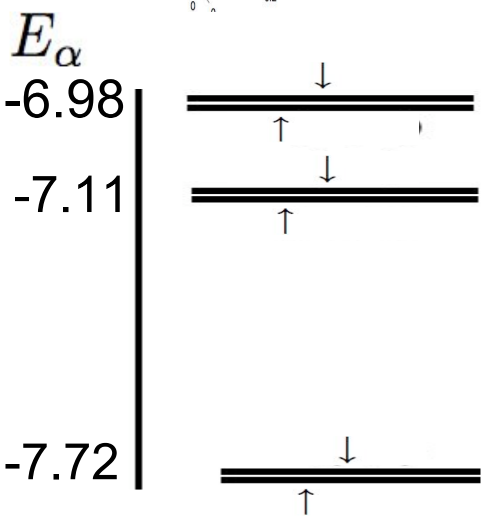
magnus field = boson density

vortex confining potential

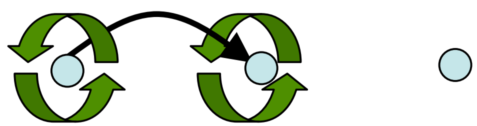


$$U(\mathbf{r}) = \frac{K}{2L^2}(\mathbf{r}-\mathbf{R}_v)^2$$

variational calculation $K \simeq 39.2\, t n_b(1-n_b)$



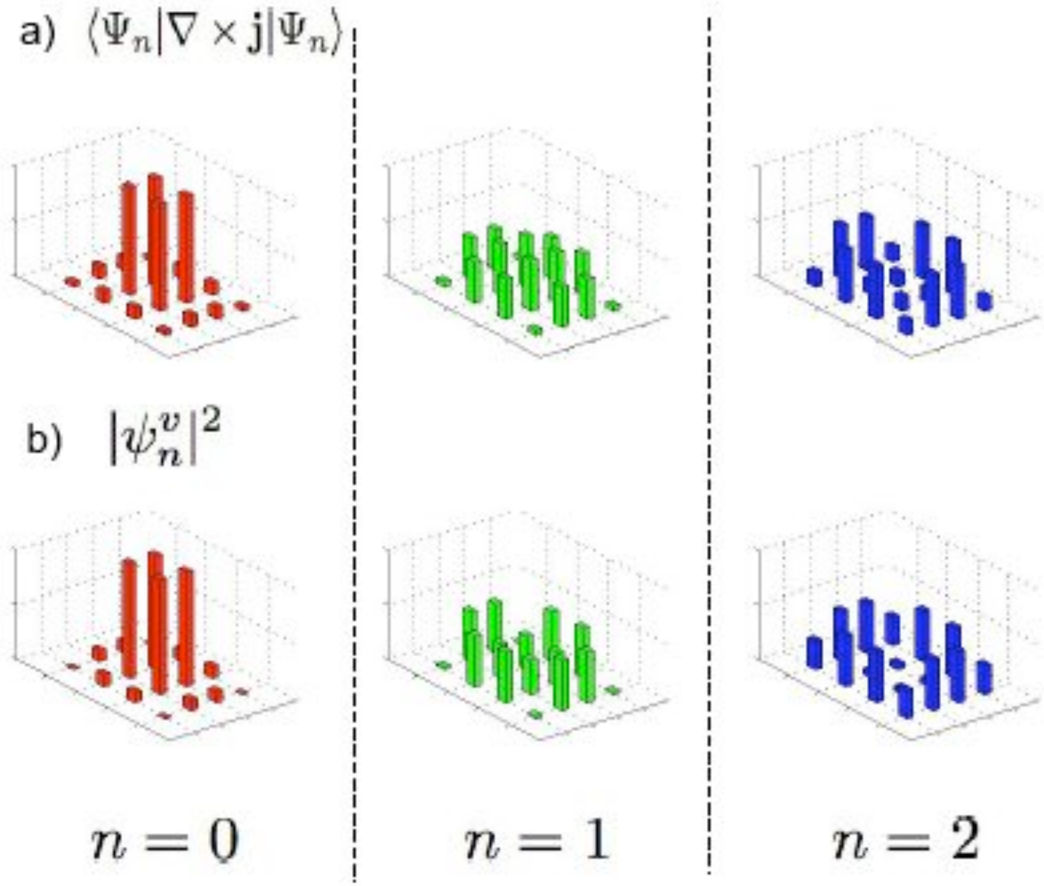
Fitting the Vortex hopping to exact diag.



$$t_{\mathrm{v}} = \frac{\hbar^2}{M_{\mathrm{v}} a^2}$$

$$t_{\mathrm{v}}(n_b,0) \;=\; t - 12.6\left(n_b - \frac{1}{2}\right)^2 + 1264\left(n_b - \frac{1}{2}\right)^4$$

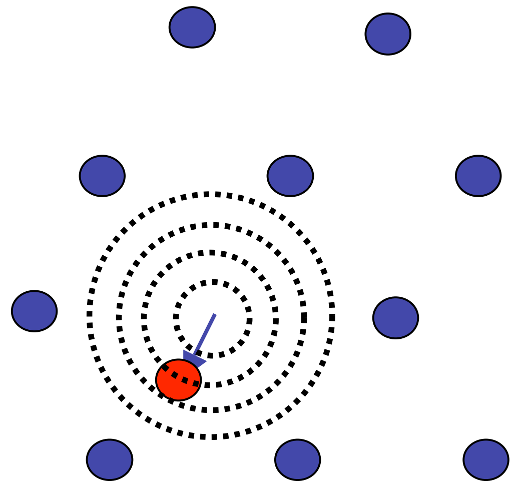
Vortex mass = boson mass at half filling single vortex wavefunctions



exact diag.

Harper model

Quantum Melting



Multivortex hamiltonian = Bose coloumb liquid

$$\mathcal{H}^{\mathrm{mv}} = \sum_{i,s=\uparrow\downarrow} \frac{\mathbf{p}_i^2}{2M_{\mathrm{v}}} + \frac{\pi t}{4}\sum_{i\neq j} \log(|\mathbf{r}_i - \mathbf{r}_j|) - \frac{n_{\mathrm{v}}\pi^2 t}{4}\sum_i |\mathbf{r}_i|^2 + \mathcal{H}^{\mathrm{ret}}(\omega).$$

Magro and Ceperley: Wigner solid melts at $r_s = 12$

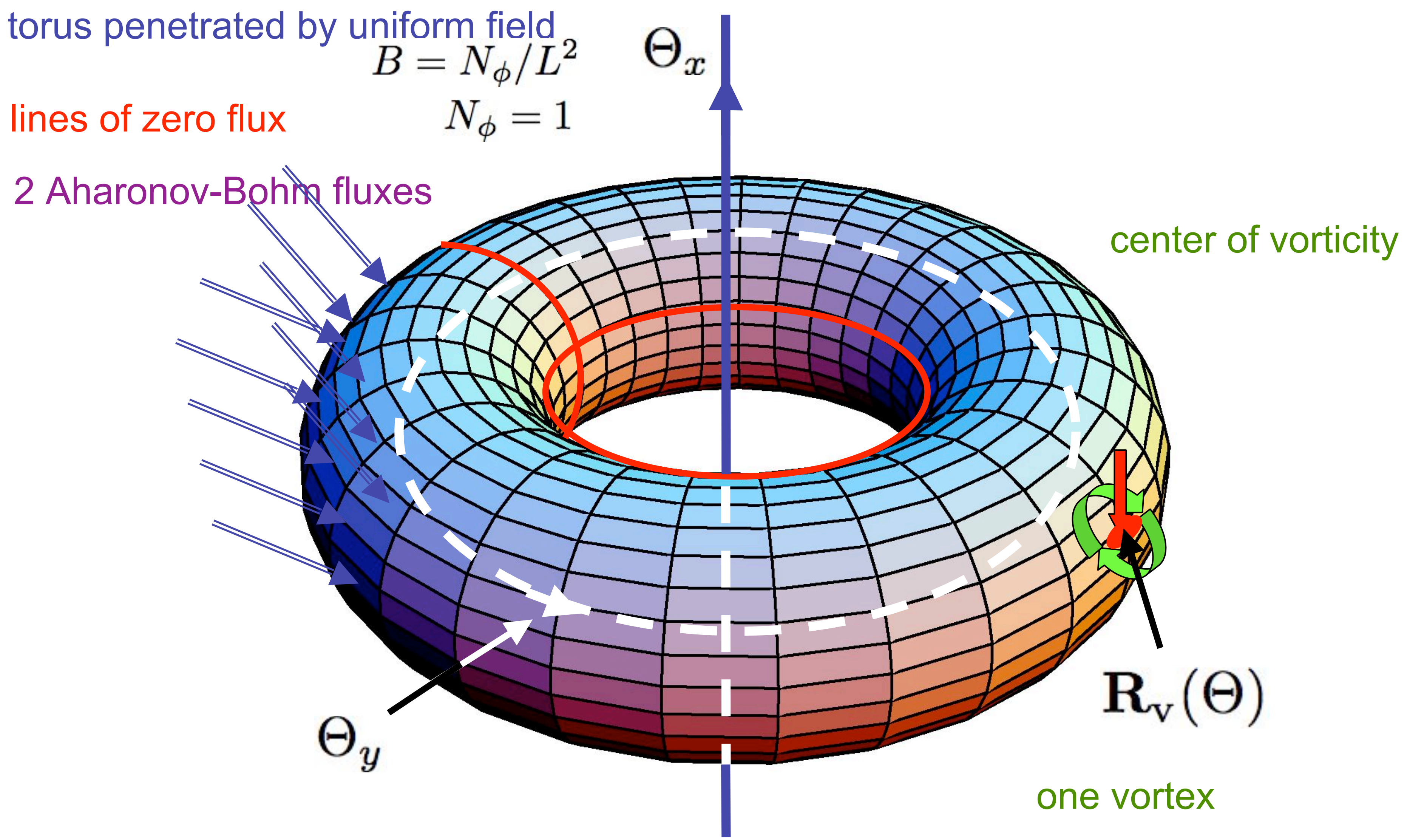
$$r_s^{-2} = \pi n_{\mathrm{v}} a_0^2. \qquad a_0 = \left(\frac{\hbar^2}{\pi t M_{\mathrm{v}}}\right)^{1/2}$$

Therefore, the vortex lattice should quantum melt at

$$n_{\mathrm{v}}^{\mathrm{cr}} \leq \left(6.5 - 7.9\frac{V}{t}\right) \times 10^{-3} \text{ vortices per site.}$$

Quantum Vortex liquid: not Bose condensed!

The Gauged Torus



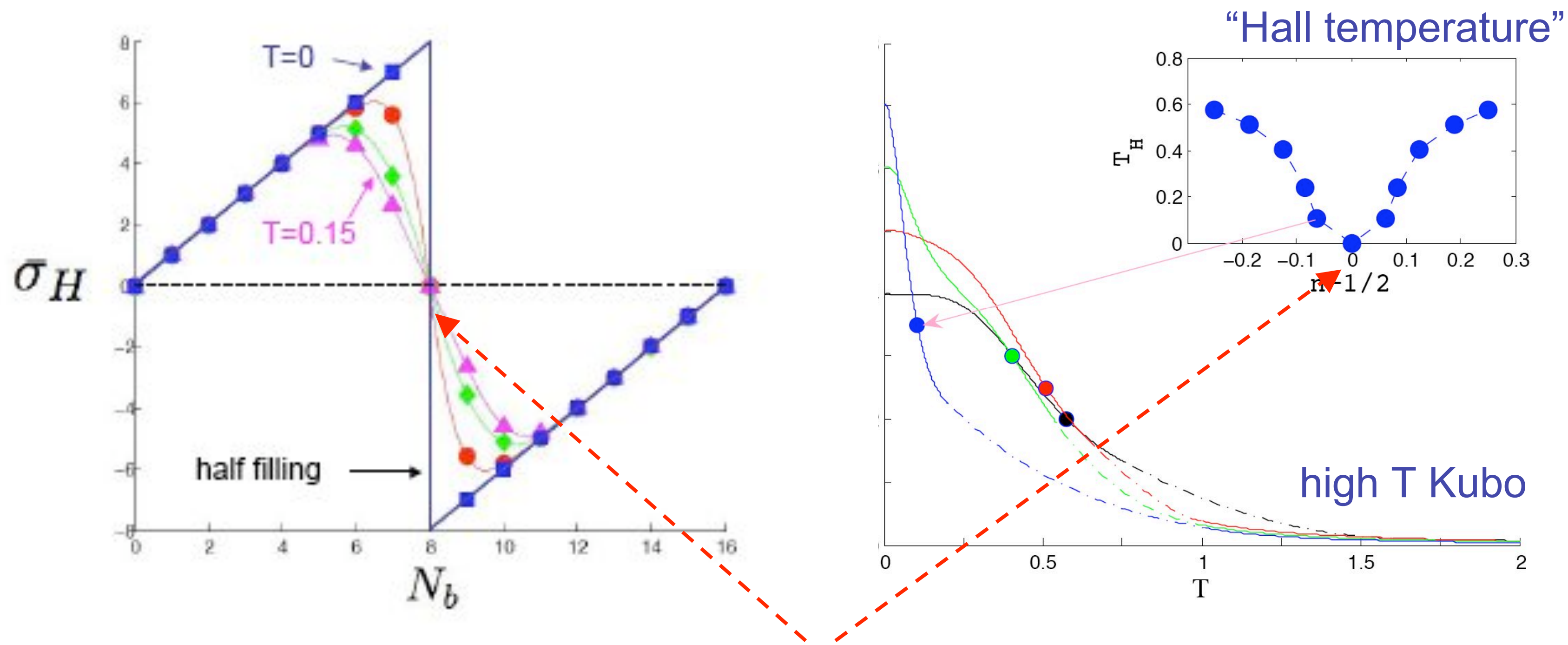
Thanks to Gil Refael

Hall conductivity

Thermally averaged Chern numbers

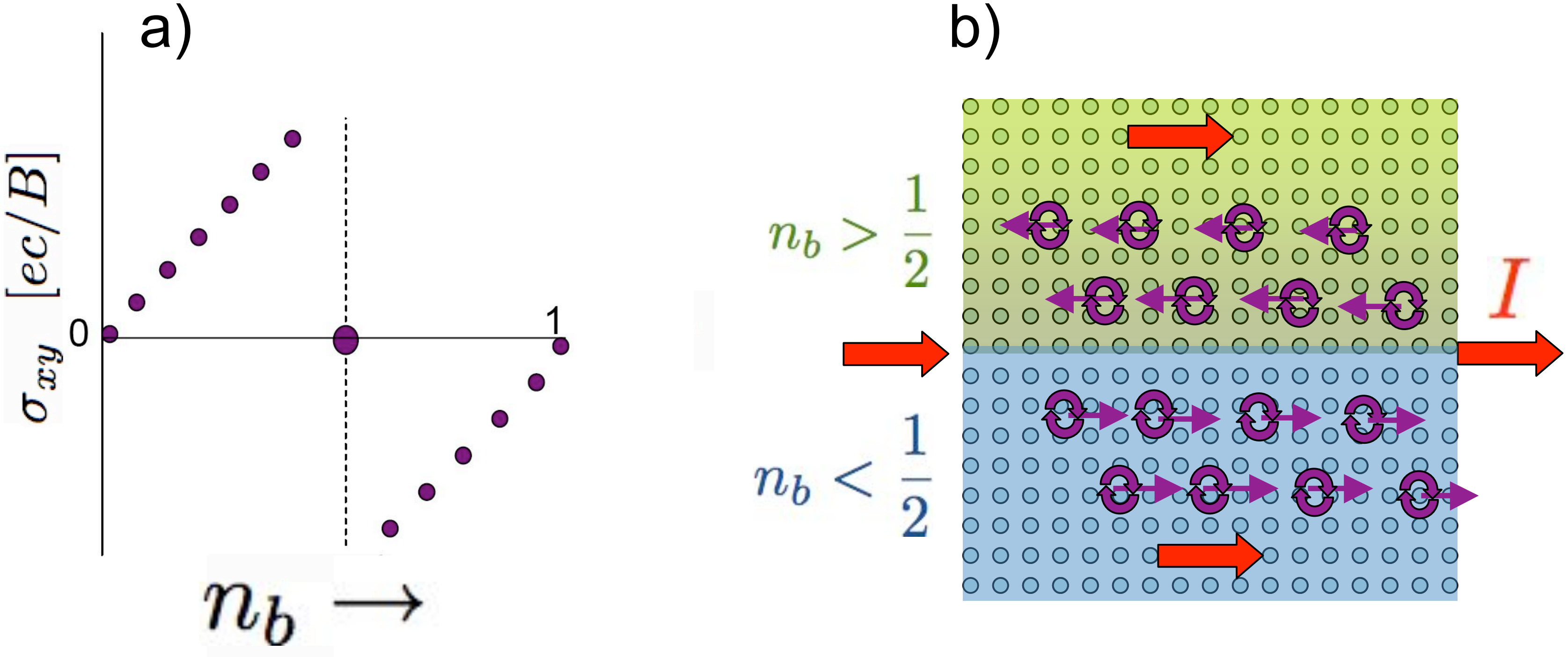
$$\sigma_{\rm H}(n_b,T) \;=\; \frac{1}{4\pi}\sum_{n=0}^{\infty}\int_0^{2\pi}\int_0^{2\pi} d^2\Theta \;\; \frac{e^{-E_n/T}}{Z} \;\; {\rm Im}\left\langle \frac{\partial\psi_n}{\partial\Theta_x}\middle|\frac{\partial\psi_n}{\partial\Theta_y}\right\rangle$$

Thouless, Kohmoto, Nightingale, Den-Nijs, PRL (82).
Y. Avron, R. Seiler and B. Shapiro, Nucl. Phys. B (86).



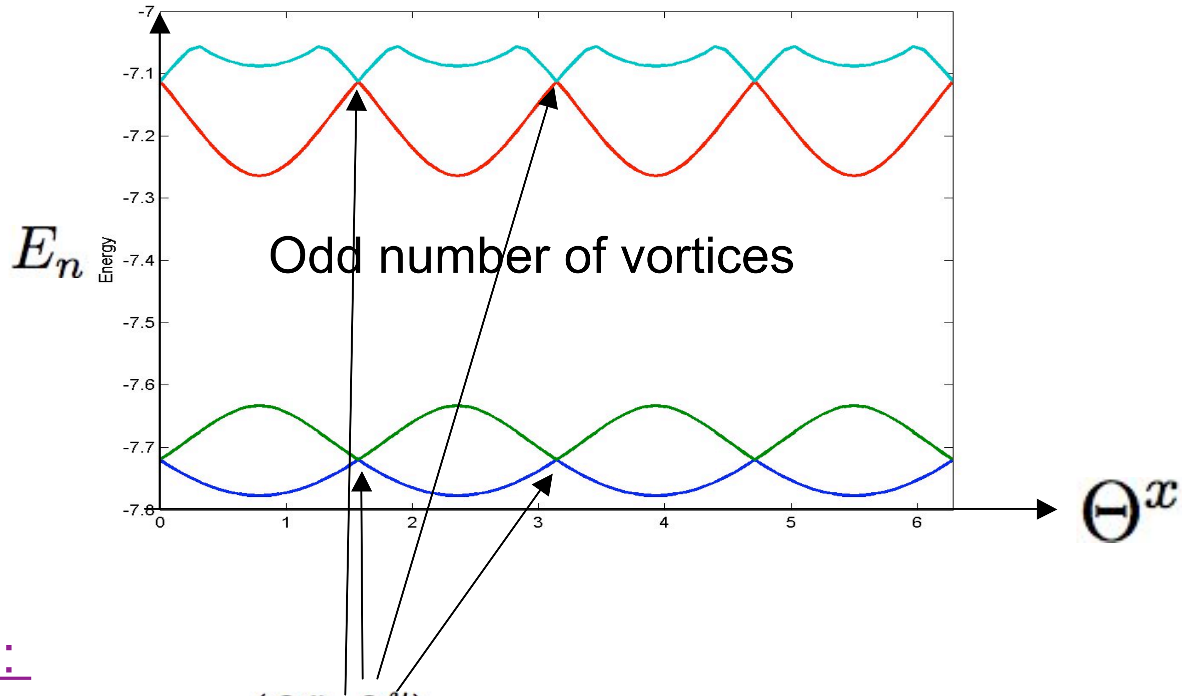
Quantum Critical point?

Magnus field reversal



Vortex degeneracies

Netanel accidentally found the spectrum of a vortex at half filling to be ‘infested’ with two-fold degeneracies.....



Theorem:

1. Degeneracies (Θ_i^x, Θ_i^y) correspond vorticity center on all lattice sites.
2. At degeneracies: all the states are even-fold degenerate (doublets).

Proof:

construct a non commuting algebra of symmetries

The $\mathcal{P}i$ operators

$$\Pi^x = P^x[\mathbf{R}_v] \cdot C \cdot U^x[\mathbf{A}]$$
$$\Pi^y = P^y[\mathbf{R}_v] \cdot C \cdot U^y[\mathbf{A}]$$

1. reflection about $\mathbf{R}_v$

2. charge conjugation $C = e^{i\pi \sum_{\mathbf{r}} S^x_{\mathbf{r}}}$

3. gauge transformation $U^\alpha = e^{i\sum_{\mathbf{r}} \chi^\alpha(\mathbf{r};\mathbf{R_v}) S^z_{\mathbf{r}}}$

1. ***point group**** symmetries $\left[\mathcal{H}[\Theta], \Pi^x[\mathbf{R}_v]\right] = \left[\mathcal{H}[\Theta], \Pi^y[\mathbf{R}_v]\right] = 0$

2. For odd vorticity $\Pi^x \Pi^y = (-1)^{N_\phi} \Pi^y \Pi^x \equiv i\Pi^z$

=> *All states are doubly degenerate*

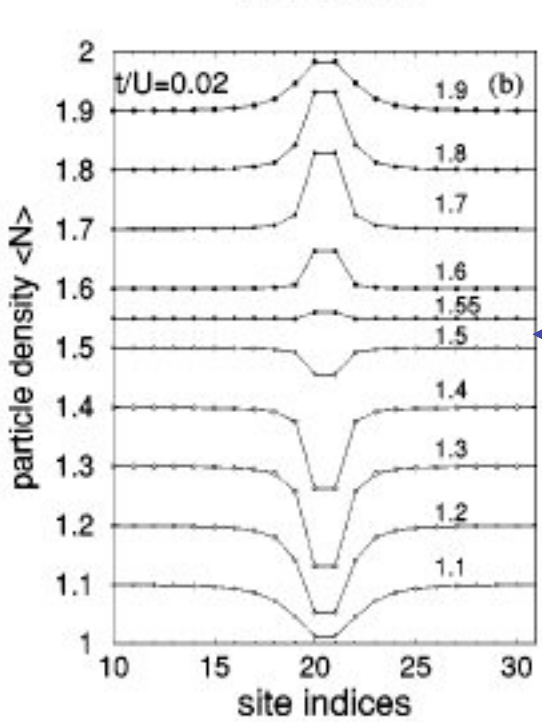
$$\Pi^y \Pi^x |E_n, \pi^x\rangle = -\Pi^x \Pi^y |E_n, \pi^x\rangle \Rightarrow \Pi^y |E_n, \pi^x\rangle = |E_n, -\pi^x\rangle$$

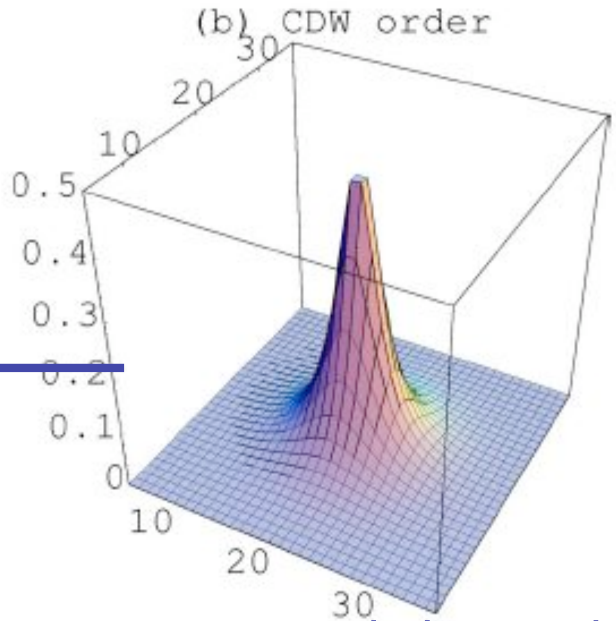
*For one vortex, there are no translational symmetries on finite tori

charge distributions

Congjun Wu,[1] Han-dong Chen,[2] Jiang-piang Hu,[3] and Shou-Cheng Zhang[1]

Phys Rev A 69 (2004)



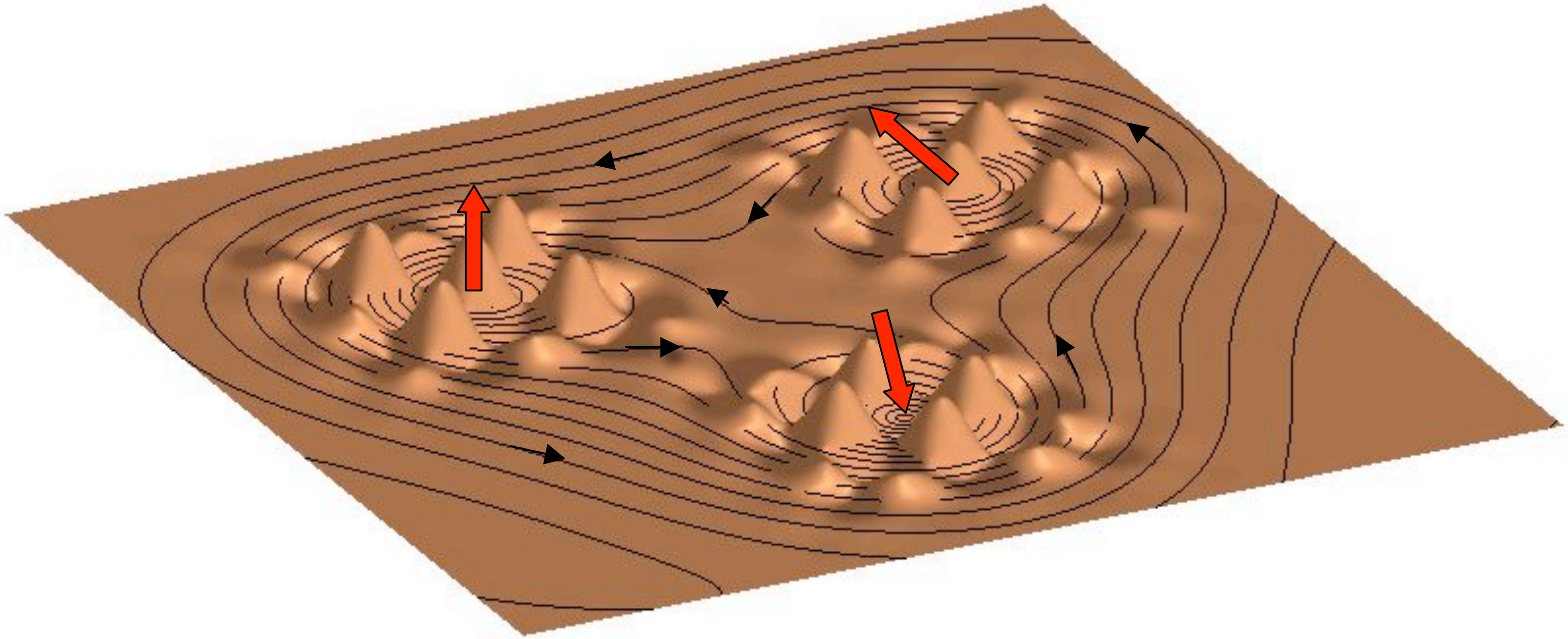


v-spin up

staggered charge density
at half filling

Multiple species of <u>vortex condensates</u> were discussed by:
Lannert, Fisher, Senthil, PRB (01)
Tesanovic, PRL 93 (2004),
Balents, Bartosch, Burkov, Sachdev, and Sengupta, PRB 71, 144508 (2005).

Illustration of 3 vortices with v-spin



Implications of v-spins:

1. order: CDW (supersolid) in the vortex lattice
2. Low temperature entropy of v-spins

Summary

arXiv:0810.2604 :

Vortices on 2D lattices (hard core bosons limit)

1. Have a light mass, vortex solid can melt into a uncondensed Quantum Vortex Liquid at 0.006 vortices per site.
2. Hall conductivity (Magnus action) reverses sign at half filling, with an associated vanishing temperature scale.
3. Vortices at half filling carry local spin 1/2 doublets. (V-spins)
4. QCP at half filling?

$n_b = 1/2$