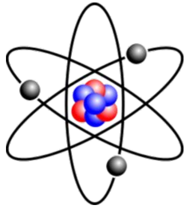
Nucleus from String Theory



Takeshi Morita

University of Crete

Ref) hep-th 1103.5688

+work in progress

based on collaboration with Koji Hashimoto (RIKEN)

1. Introduction and Motivation

◆ Hierarchy of our world

QCD

quark

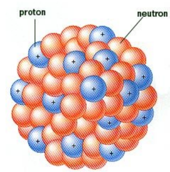
gluon

Hadron

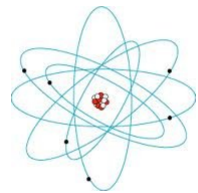
baryon

meson

Nucleus



Atom



QCD (+ QED)

QED

One goal of particle physicists may be
a construction of nucleus from QCD.

Nuclear force between 2 baryons

Lattice gauge theory

1. Introduction and Motivation

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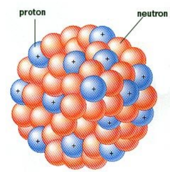

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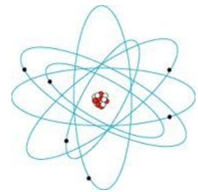

QCD (+ QED)

QED

Can Holography solve this problem?

Nuclear force between 2 baryons

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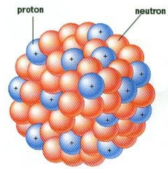

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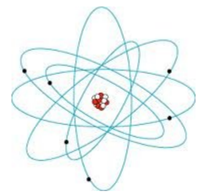

QCD (+ QED)

QED

part of nuclear force

Sakai-Sugimoto model

Nuclear force between 2 baryons

Lattice gauge theory

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QCD | Hadron | Nucleus | Atom

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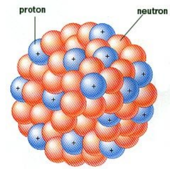
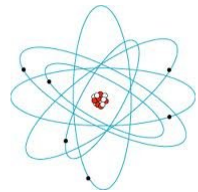




Bound state of

large number of baryons

↓

Nucleus in string theory

QCD (+ QED)

QED

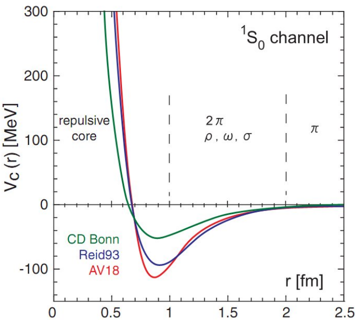
Sakai-Sugimoto model

Lattice gauge theory

Nuclear force between 2 baryons

1. Introduction and Motivation

- Two baryon potential



Sakai-Sugimoto   One pion exchange

- In nuclear physics, a lot of models are required to explain experiments.
- The existence of the bound state in the Sakai-Sugimoto model encourages the understanding of the nuclear physics from a simple model.

1. Introduction and Motivation

◆ Basic Ideas

Sakai-Sugimoto model
QCD → Chiral lag. + Skyrme term + ・・・

Skyrme model
Chiral lag. + Skyrme term → baryon = soliton → nucleus = multi-soliton → difficult

String theory
soliton = D-brane → nucleus = multi-D-brane → easier

⇒ baryon = soliton = D-brane (D4 brane) cf. Baryon vertex (Witten)

effective theory of N' D-branes = U(N') gauge theory

(N' baryon system) (Nuclear Matrix Model, Hashimoto-Iizuka-Yi)

↓ Large N' limit reduces the calculation.

Bound state → Nucleus?

1. Introduction and Motivation

◆ Properties of the“Nucleus”

- Holographic (non-SUSY) QCD + N’ baryon vertex + large N’ limit
  → Nucleus (the bound state of N’ baryons) always exists. (Universal)

- Exhibit a Nuclear Density saturation:
  (radius of nucleus of mass N’) $\propto N'^{1/3}$

- In the Sakai-Sugimoto model, the radius is close to the experimental data.

But….

- Singular baryon distribution at the surface
  (1/N’correction might resolve it??)
- Attractive potential between two baryons has not been found.
- Bound energy has not been evaluated.

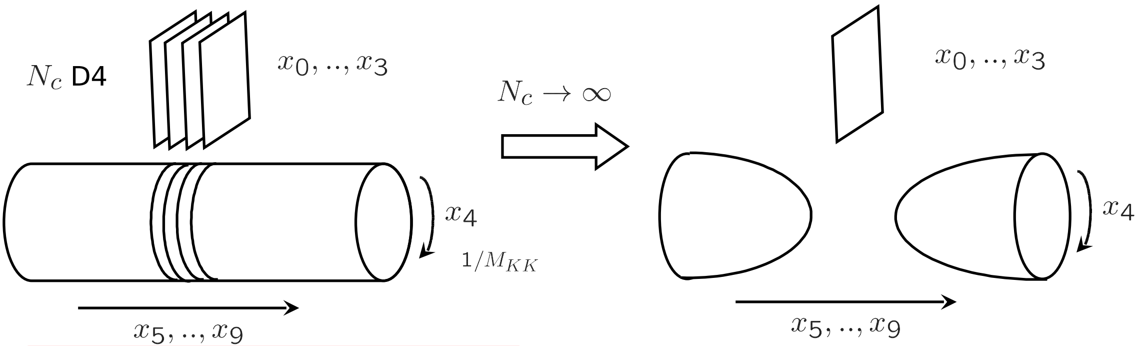
Plan of this talk

1. Introduction and Motivation
2. Sakai-Sugimoto model and Baryon
3. Nuclear Matrix Model
4. Baryon bound state
5. Conclusions

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

・ Holographic 4d pure YM (Witten 1998)



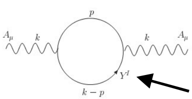
x_4 : AP boundary condition for fermions

→ mass $\propto M_{KK}$ → breaks supersymmetry

AdS-Soliton $\simeq$ confinement geometry

$\lambda_4 \ll 1$: 5d SYM → 4d pure YM ⟷ $\lambda_4 \gg 1$: gravity description OK

$M_{KK} \gg \mu$



KK modes

Relevant massless modes

$U(N_c)$ gauge field A_μ (D4)

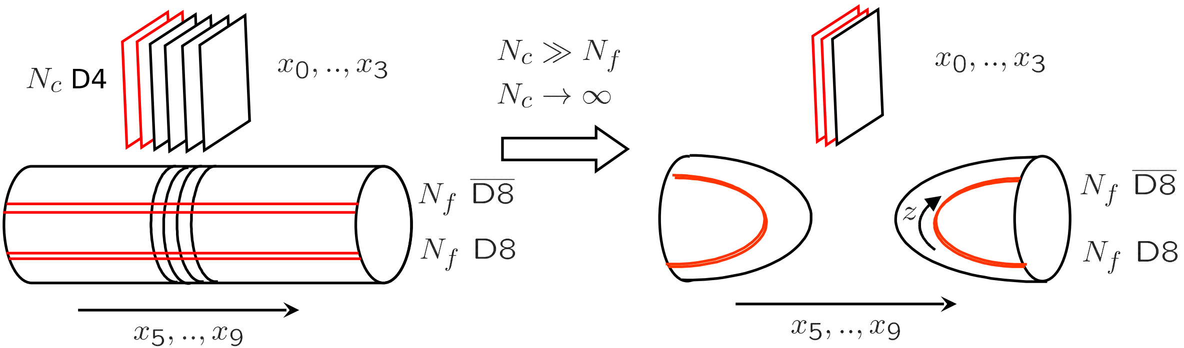
Although no overlapping regime exists,

several qualitative agreements were found.

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

・ Sakai-Sugimoto model

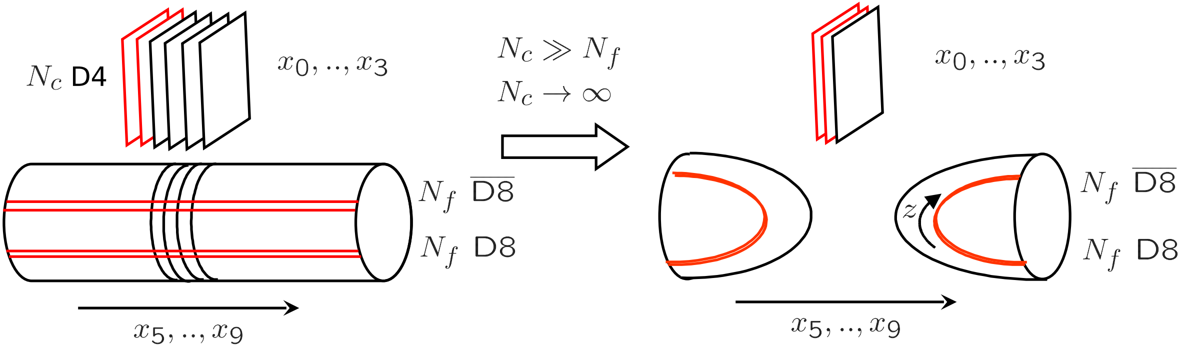


	0	1	2	3	(4)	5	6	7	8	9
D4	-	-	-	-	-					
D8/anti-D8	-	-	-	-		-	-	-	-	-

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

- Sakai-Sugimoto model



Symmetry:

$U(N_c) \times U(N_f)_L \times U(N_f)_R$

Relevant massless modes:

$U(N_c)$ gauge field A_μ (D4)

$U(N_f)_L$ gauge field $A_\mu^{(L)}$ (D8)

$U(N_f)_R$ gauge field $A_\mu^{(R)}$ ($\overline{\text{D8}}$)

$U(N_c) \times U(N_f)_L$ bi-fundamental quark q_L^f (D4-D8)

$U(N_c) \times U(N_f)_R$ bi-fundamental quark q_R^f (D4-$\overline{\text{D8}}$)

⟶ Similar to QCD

Effective theory on D8

Symmetry: $U(N_f)$

Relevant massless modes:

$U(N_f) \times U(N_f)$ scalar

⟶ Interpreted as pion

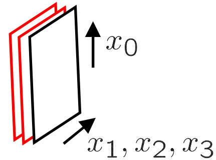
⟶ Similar to chiral lag.

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Effective theory on D8:

Chiral lag. + Skyrme term + massive vector mesions

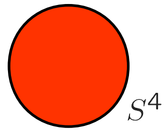


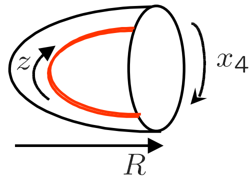
$$S = S_{\rm YM} + S_{\rm CS}\ ,$$

$$S_{\rm YM} = -\kappa \int d^4x dz\ {\rm tr} \left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2 \right]$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int_{M^4\times\mathbb{R}} \omega_5^{U(N_f)}(\mathcal{A})\ .$$

$$h(z) = (1+z^2)^{-1/3}\ , \quad k(z) = 1+z^2$$





	0	1	2	3	(4)	z	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-

Effective theory on D8 → 9d gauge theory

→ 5d gauge theory

Ignore S^4 dependence by hand

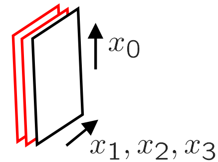
(QCD irrelevant modes)

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Effective theory on D8:

Chiral lag. + Skyrme term + massive vector mesions



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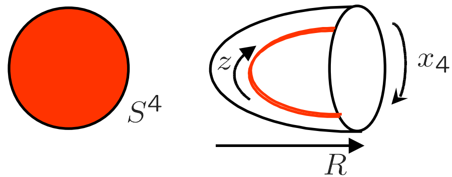


・ 4 dimensional fields

$$\begin{cases} A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu) \psi_n(z) \\ A_z(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu) \phi_n(z) \end{cases}$$

$B_\mu^{(n)}$: massive vector meson

$\varphi^{(0)}$: massless pion ($\varphi^{(n)}$ $(n>0)$ can be gauge away.)

No quarks in this model. → How to describe the baryon?

Hint: Skyrme model

Chiral lag. + Skyrme term (Pion effective action) → baryon = soliton

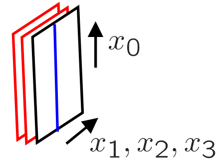
We can expect that the baryons in this model would be also described as solitons.

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Effective theory on D8:

Chiral lag. + Skyrme term + massive vector mesions



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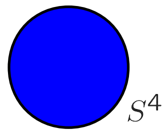

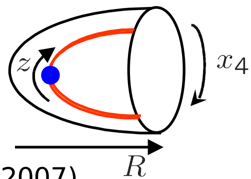

◆ single baryon soliton (Hata-Sakai-Sugimoto-Yamato 2007)

We can construct a soliton localized as follows.

	0	1	2	3	(4)	z	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-
soliton	-						-	-	-	-

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Effective theory on D8:

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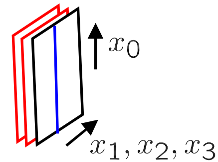

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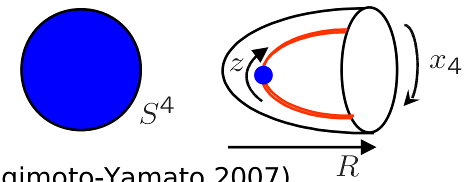


◆ single baryon soliton (Hata-Sakai-Sugimoto-Yamato 2007)

$N_f = 2$ $\lambda \gg 1$ Instanton like solution exists.

$$\mathcal{A} = A + \frac{1}{\sqrt{2N_f}}\widehat{A}$$

SU(N_f) U(1)

$$\begin{cases} A_M(x) = -if(\xi)\, g\partial_M g^{-1} & (M = 1,2,3,z) \\ \widehat{A}_0 = \dfrac{1}{8\pi^2 a}\dfrac{1}{\xi^2}\left[1 - \dfrac{\rho^4}{(\rho^2+\xi^2)^2}\right] \end{cases}$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\ , \quad \xi = \sqrt{(\vec{x}-\vec{X})^2 + (z-Z)^2}$$

$$g(x) = \frac{(z-Z) - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi}\ , \quad Z = 0$$

$$\rho^2 = \frac{1}{8\pi^2 a}\sqrt{\frac{6}{5}} \qquad \kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c$$

"Moduli"space: $\mathcal{M} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$

Owing to the potential, actual moduli is only $\mathbb{R}^3$. (X^1, X^2, X^3)

Mass: $M \simeq M_0 + \sqrt{\frac{2}{15}}N_c + \frac{1}{4}\sqrt{\frac{5}{6}}\frac{(l+1)^2}{N_c} + \frac{2(n_\rho + n_z)+2}{\sqrt{6}}$ ← After quantizing the collective coordinates.

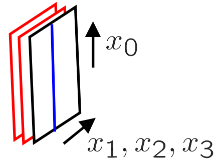
$$M_0 = 8\pi^2\kappa$$

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Effective theory on D8:

Chiral lag. + Skyrme term + massive vector mesions



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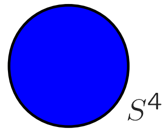

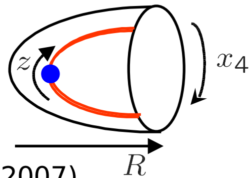

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$$\rho^2 = \frac{1}{8\pi^2 a} \sqrt{\frac{6}{5}} \qquad \kappa = \frac{\lambda N_c}{216\pi^3} \equiv a \lambda N_c$$

Baryon number=Instanton number: $\displaystyle \frac{1}{8\pi^2} \int_B {\rm tr}\, F^2 = n$ $\qquad B \simeq \mathbb{R}^4 \ \ (x^1, x^2, x^3, z)$

Coupling to U(1): $\displaystyle S_{CS}^{D8} \simeq \frac{N_c}{8\pi^2} \int_{\mathbb{R} \times B} a\, {\rm tr}\, F_{\rm cl}^2 \simeq n N_c \int_{\mathbb{R}} a$ ← n baryon solution has $n N_c \ U(1)$ charge as expected.

$(\widehat{A}_0 = \widehat{A}_{0\rm cl} + a)$

2. Sakai-Sugimoto model and Baryon

◆ Sakai-Sugimoto Model (Sakai-Sugimoto 2004)

Mass: $$M \simeq M_0 + \sqrt{\frac{2}{15}} N_c + \frac{1}{4}\sqrt{\frac{5}{6}} \frac{(l+1)^2}{N_c} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}}$$

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\ (l=1)$	$\boxed{940^+}$	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta\ (l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

$\boxed{\quad}$: input

Experimental data

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\ (l=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\ (l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

Not bad (??)

However, in this parameter, meson masses do not agree well.

2. Sakai-Sugimoto model and Baryon

◆ Summary of this section

$$S = S_{\mathrm{YM}} + S_{\mathrm{CS}} \ ,$$
$$S_{\mathrm{YM}} = -\kappa \int d^4x dz \ \mathrm{tr} \left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2 \right]$$
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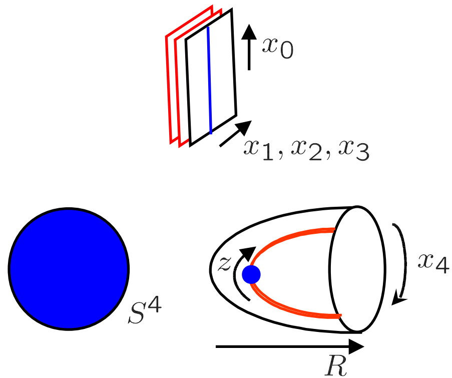

Baryon = Instantol like Soliton in 5dYM

⇩

String: Soliton = D-brane

Nuclei Matrix Model

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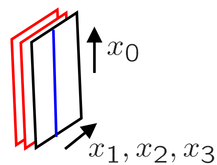
3. Nuclear Matrix Model

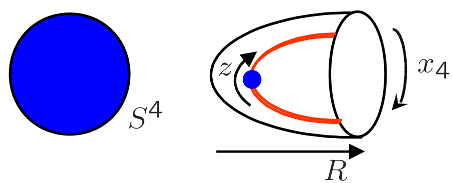
◆ Baryon vertex in Sakai-Sugimoto model (cf. Witten 1998)

	0	1	2	3	(4)	z	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-
soliton	-						-	-	-	-

This Soliton must be D4-brane.

Consistent with Witten's baryon vertex.





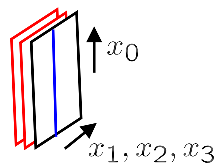
3. Nuclear Matrix Model

◆ Baryon vertex in Sakai-Sugimoto model (cf. Witten 1998)

	0	1	2	3	(4)	z	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-
D4'	-						-	-	-	-

This Soliton must be D4-brane.

Consistent with Witten's baryon vertex.



cf. Dp/Dp+4

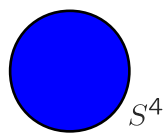
Two descriptions of the system are possible.

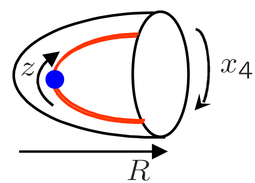
Dp+4: Dp is described as solitons.

Dp: Dp corresponds to

the collective coordinates of the solitons.

cf. ADHM equation





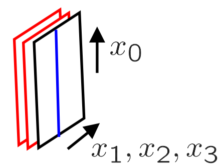
→ We can expect that alternative description of the baryon is possible by using D4 brane.

3. Nuclear Matrix Model

◆ Effective action on D4’ brane (Hashimoto-Iizuka-Yi 2010)

	0	1	2	3	(4)	z	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-
D4’	-						-	-	-	-

Effective theory on D4’ is a matrix quantum mechanics. ← Ignore S^4 dependence again



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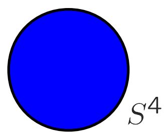
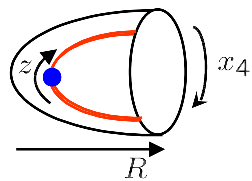




→ We can expect that alternative description of the baryon is possible by using D4 brane.

3. Nuclear Matrix Model

◆ Effective action on D4' brane (Hashimoto-Iizuka-Yi 2010)

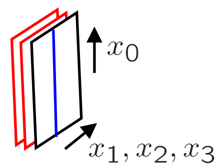
A baryon system → U(A) gauge theory:

$$\begin{aligned} S = C_1 \int dt \Big[& \left(D_0 X^M\right)^2 - \frac{2}{3} M_{KK}^2 \left(X^4\right)^2 + D_0 \bar{w}_f^{\dot\alpha} D_0 w_{\dot\alpha f} - \frac{1}{6} M_{KK}^2 \bar{w}_f^{\dot\alpha} w_{\dot\alpha f} \\ & + \frac{1}{8C_2} \mathrm{Tr}_A [X_M, X_N][X_M, X_N] - \frac{1}{4C_2}\left(2\bar{w}_f^{i\dot\alpha} w_{jf\dot\beta}\bar{w}_g^{j\dot\beta} w_{\dot\alpha i g} - \bar{w}_f^{i\dot\alpha} w_{jf\dot\alpha}\bar{w}_g^{j\dot\beta} w_{\dot\beta i g}\right) \\ & - \frac{2}{C_2}(\bar\sigma_{MN})^{\dot\alpha}{}_{\dot\beta} X^M X^N \bar{w}_f^{\dot\beta} w_{f\dot\alpha}\Big] + N_c \int dt\, \mathrm{Tr}_A A_0. \end{aligned}$$

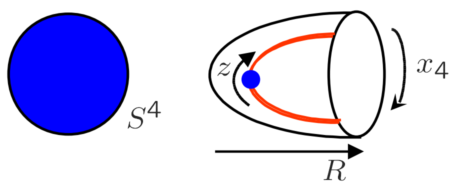
$$C_1 = \lambda N_c M_{KK}/54\pi, \quad C_2 = 3^6\pi^2/4\lambda^2 M_{KK}^4$$

Matrices:

$$\begin{cases} X_M \ (M = 1,2,3,z): \ A \times A \text{ adjoint matrix} \\ \qquad\qquad\qquad A_0 \ : \ A \times A \text{ adjoint matrix} \\ w^i_{\dot\alpha f} \ (\dot\alpha = 1,2): \ A \times N_f \text{ bi-fundamental matrix} \end{cases}$$



These matrices indeed correspond to the collective coordinates of the soliton.



3. Nuclear Matrix Model

◆ Effective action on D4' brane (Hashimoto-Iizuka-Yi 2010)

$A=1,\ N_f=2$ case (After integrating A_0)

$$
\begin{aligned}
H &\equiv P_i^{\dot\alpha}\dot w_i^{\dot\alpha} + \bar P_{\dot\alpha}^i \dot{\bar w}_{\dot\alpha}^i - L \\
&= \frac{\lambda N_c M_{\rm KK}}{54\pi}\Bigg[\partial_0 \bar w_i^{\dot\alpha}\partial_0 w_{\dot\alpha}^i + \frac{1}{6}M_{\rm KK}^2 \bar w_i^{\dot\alpha} w_{\dot\alpha}^i \\
&\quad + \frac{\lambda^2 M_{\rm KK}^4}{3^6\pi^2}\left[4w_1^i(w_2^i)^* w_2^j (w_1^j)^* + (w_1^i(w_1^i)^*)^2 + (w_2^i(w_2^i)^*)^2 - 2w_1^i(w_1^i)^* w_2^j(w_2^j)^*\right] \\
&\quad + \frac{1}{4\bar w_i^{\dot\alpha} w_{\dot\alpha}^i}\left(\left(\frac{54\pi}{\lambda M_{\rm KK}}\right)^2 + \left(\bar w_i^{\dot\alpha}\partial_0 w_{\dot\alpha}^i - \partial_0 \bar w_i^{\dot\alpha} w_{\dot\alpha}^i\right)^2\right)\Bigg].
\end{aligned}
$$

Classical solution in $A=1,\ N_f=2$ case

$X_M\ (M=1,2,3)$: free

$X_z = 0$

$$
w_{\dot\alpha}^{i=1} = \begin{pmatrix}\rho \\ 0\end{pmatrix}_{\dot\alpha}, \quad w_{\dot\alpha}^{i=2} = \begin{pmatrix}0 \\ \rho\end{pmatrix}_{\dot\alpha} \qquad \rho = 2^{-1/4}3^{7/4}\sqrt{\pi}\lambda^{-1/2}M_{\rm KK}^{-1}
$$

Quantized mass:

$$
M = M_0 + \sqrt{\frac{(l+1)^2}{6} + \frac{N_c^2}{6}} + \frac{2(n_\rho + n_Z)+2}{\sqrt{6}}
$$

Result from the soliton

$$
M = M_0 + \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2} + \frac{2(n_\rho + n_z)+2}{\sqrt{6}}
$$

$$
\rho^2 = \sqrt{\frac{5}{4}}\,\rho_{\rm soliton}^2
$$

They are close but slightly different. The reason is unclear…

3. Nuclear Matrix Model

◆ Summary of this section

A baryon system → U(A) gauge theory:

$$S = C_1 \int dt \Big[\left(D_0 X^M \right)^2 - \frac{2}{3} M_{KK}^2 \left(X^4 \right)^2 + D_0 \bar{w}_f^{\dot{\alpha}} D_0 w_{\dot{\alpha} f} - \frac{1}{6} M_{KK}^2 \bar{w}_f^{\dot{\alpha}} w_{\dot{\alpha} f}$$
$$+ \frac{1}{8C_2} \mathrm{Tr}_A [X_M, X_N][X_M, X_N] - \frac{1}{4C_2} \left(2 \bar{w}_f^{i\dot{\alpha}} w_{jf\dot{\beta}} \bar{w}_g^{j\dot{\beta}} w_{\dot{\alpha} ig} - \bar{w}_f^{i\dot{\alpha}} w_{jf\dot{\alpha}} \bar{w}_g^{j\dot{\beta}} w_{\dot{\beta} ig} \right)$$
$$- \frac{2}{C_2} (\bar{\sigma}_{MN})^{\dot{\alpha}}{}_{\dot{\beta}} X^M X^N \bar{w}_f^{\dot{\beta}} w_{f\dot{\alpha}} \Big] + N_c \int dt \, \mathrm{Tr}_A A_0 .$$

The baryons can be evaluated by this matrix model.

1. Introduction and Motivation
2. Sakai-Sugimoto model and Baryon
3. Nuclear Matrix Model
4. Baryon bound state
5. Conclusions

4. Baryon bound state

◆ Large A limit (Hashimoto-TM 2011)

$$S = C_1 \int dt \Big[\left(D_0 X^M\right)^2 - \frac{2}{3} M_{KK}^2 \left(X^4\right)^2 + D_0 \bar{w}_f^{\dot{\alpha}} D_0 w_{\dot{\alpha} f} - \frac{1}{6} M_{KK}^2 \bar{w}_f^{\dot{\alpha}} w_{\dot{\alpha} f}$$
$$+ \frac{1}{8C_2} \mathrm{Tr}_A [X_M, X_N][X_M, X_N] - \frac{1}{4C_2} \left(2 \bar{w}_f^{i\dot{\alpha}} w_{jf\dot{\beta}} \bar{w}_g^{j\dot{\beta}} w_{\dot{\alpha} ig} - \bar{w}_f^{i\dot{\alpha}} w_{jf\dot{\alpha}} \bar{w}_g^{j\dot{\beta}} w_{\dot{\beta} ig} \right)$$
$$- \frac{2}{C_2} (\bar{\sigma}_{MN})^{\dot{\alpha}}{}_{\dot{\beta}} X^M X^N \bar{w}_f^{\dot{\beta}} w_{f\dot{\alpha}} \Big] + N_c \int dt \, \mathrm{Tr}_A A_0 .$$

$A \to \infty$

Matrices:

$$\begin{cases} \left.\begin{array}{r} X_M \ (M = 1,2,3,z): \ A \times A \text{ adjoint matrix} \\ A_0 : \ A \times A \text{ adjoint matrix} \end{array}\right\} \text{dominant} \\ w^i_{\dot{\alpha} f} \ (\dot{\alpha} = 1,2): \ A \times N_f \text{ bi-fundamental matrix} \quad \text{irrelevant} \end{cases}$$

Reduces to a bosonic BFSS model.

$$S = C_1 \int dt \, \mathrm{Tr}_A \Big[\left(D_0 X^M\right)^2 - \frac{2}{3} M_{KK}^2 \left(X^4\right)^2 + \frac{1}{8C_2} [X_M, X_N][X_M, X_N] \Big].$$

canonical normalization

$$S = \int dt \ \mathrm{tr}_A \left(\frac{1}{2} (D_t Y^I)^2 - \frac{m_Y^2}{2} (Y^z)^2 + \frac{g_0^2}{4} [Y^I, Y^J]^2 \right),$$

4. Baryon bound state

◆ Large A limit (Hashimoto-TM 2011)

Q. What is the most stable state of this model?

$$S = \int dt \ \mathrm{tr}_A \left(\frac{1}{2} \sum_{I=1}^{4} (D_t Y^I)^2 - \frac{m_Y^2}{2} (Y^z)^2 + \frac{g_0^2}{4} [Y^I, Y^J]^2 \right) ,$$

A. Bound state (Luscher 1983)

He showed that all the eigen values are trapped by the potential.

His proof is general and it ensures the existence of nuclei!

However this argument does not tell us the details of the configuration...

4. Baryon bound state

◆ Large A limit (Hashimoto-TM 2011)

Q. What is the most stable state of this model?

$$S = \int dt \, \mathrm{tr}_A \left(\frac{1}{2} \sum_{I=1}^{4} (D_t Y^I)^2 - \frac{m_Y^2}{2} (Y^z)^2 + \frac{g_0^2}{4} [Y^I, Y^J]^2 \right) ,$$

→ If the model is

$$S = \int dt \, \mathrm{tr}_A \left(\frac{1}{2} \sum_{I=1}^{D} (D_t Y^I)^2 + \frac{g_0^2}{4} [Y^I, Y^J]^2 \right) ,$$

We can exactly solve the model in a large D limit. $A, D \to \infty$, with fixed $\tilde{\lambda}_A = g_0^2 DA$

(Mahato-Mandal-TM 2009)

Q. Can we apply this approximation to finite D and m_Y case?

A. In $m_Y = 0$ case, the 1/D expansion works even D=2 qualitatively.

→ We can assume D=3 is large and the contribution of Y^z as 1/D correction

Or
- If $m_Y \gg \tilde{\lambda}_A^{1/3}$, we can treat D as 3 by integrating Y^z.
- If $m_Y \ll \tilde{\lambda}_A^{1/3}$, we can treat D as 4.

→ The approximation may not be so bad.

4. Baryon bound state

◆ Large A limit (Hashimoto-TM 2011)

★ D4 charge distribution (Taylor, Raamsdonk 1999)

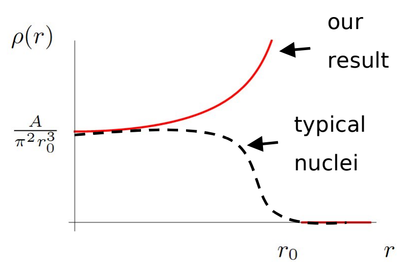
$$\rho(x) = \frac{1}{(2\pi)^D} \int d^D k \, e^{-ik\cdot x} \langle \mathrm{tr}_A \exp[ik \cdot X] \rangle$$

baryon distribution = D4 charge distribution

We evaluated this quantity at 0 temperature by using the 1/D expansion.

$$\rho(r) = \int \prod_{\hat{i}=4}^{D} dx^{\hat{i}} \frac{1}{(2\pi)^D} \int d^D k \, e^{-ik\cdot x} \langle \mathrm{tr}_A \exp(ik \cdot Y) \rangle_{T=0}$$

$$= \begin{cases} \dfrac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} & (r < r_0) \\ 0 & (r_0 < r) \end{cases} \qquad r_0 \equiv (2A/\tilde{\lambda}_A^{1/3})^{1/2}$$



⟶ Singular at the surface...... 1/A correction might resolve it (?).

⟶ $\sqrt{r^2_{\mathrm{mean}}}\Big|_{T=0} = \frac{3^{5/2}\pi^{2/3}}{2^{5/6}5^{1/6}} \frac{A^{1/3}}{M_{\mathrm{KK}}(N_c\lambda^2)^{1/3}} \sim 0.7A^{1/3}$ [fm] ⟷ experiment $1.0A^{1/3}$ [fm]

inputs $m_\rho \sim 776$ [MeV]

$g_{\pi NN} \sim 13.2$

4. Baryon bound state

◆ Universality

- Non-supersymmetric holography
- Baryon vertex (D brane)
- Large baryon number

The degree of freedom of the open string on the baryon vertex is dominant.

The effective action would be a similar matrix model

$$S = C_1 \int dt \mathrm{Tr}_A \Big[\big(D_0 X^M \big)^2 + \frac{1}{8C_2} [X_M, X_N][X_M, X_N] \Big].$$

C_1, C_2: depend on the details of the model but would not depend on A.

$$\sqrt{r^2_{mean}} \propto A^{1/3}$$

This result is model independent.

4. Baryon bound state

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This result is model independent.

Conclusion

- We found a stable baryon bound state in the large baryon number case.
- Some properties are similar to the real nuclei.
- Owing to the large-A limit, our results would be universal in holographic QCD.

Future directions

- Two body problem (full path-integral of the matrix model may be necessary.)
- Bound energy (1/N correction may be necessary.)
- 1/A correction

  → flavor dependence, rotating nuclei, resolution of the singularity

  Numerical calculation?
- Equation of state of High density baryons → Neutron star, super nova, accelerator