Momentum dissipation and charge transport in holography

Richard Davison, Leiden University

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Based on:
1306.5792 [hep-th] by RD,
1311.2451 [hep-th] by RD, K. Schalm, J. Zaanen,
and work by many others.

Overview: Some history

- Holography: some strongly coupled quantum field theories can be rewritten as classical theories of gravity in higher-dimensional spacetimes.

- Solutions of the gravitational theory correspond to equilibrium states of the field theory (e.g. black holes = thermal states) and excitations of these gravitational solutions encode the transport properties of the dual field theory.

- Try to learn general lessons, to help understand the transport properties of real, strongly interacting thermal states (e.g. the quark-gluon plasma).

Policastro, Son, Starinets, Herzog, etc.... 2001+

Overview: AdS/CMT

- There are many other real systems whose transport properties are not understood e.g. some strongly correlated electron systems.

- Holographic toy models of these states are charged black holes: dual to field theory states with a non-zero charge density.

- These states exhibit emergent quantum criticality at low energies. They transform very simply under rescalings of space and time.

- Holography lets us study the physics of quantum critical states of matter, in a controlled and simple way.

Overview: Transport in AdS/CMT

- Conceptually, the simplest transport property is the electrical conductivity $\sigma\left(\omega, T\right)$. It is also relatively easy to measure.

- But for the holographic theories just described

$$\sigma_{DC} \to \infty \qquad\qquad \sigma\left(\omega\right) \sim \delta\left(\omega\right)$$

- This is because these theories have a conserved momentum: a small current cannot dissipate, because it carries momentum.

- To get a realistic answer, we have to incorporate a mechanism by which the charge can dissipate momentum. In this talk, I will describe simple ways to do this.

Outline of this talk

- Basic technology and properties of holographic theories
- Explicit translational symmetry breaking and massive gravity
- A simple mechanism for resistivity=entropy
- Conclusions

Basic technology of holography I

- A gravitational theory will have an action $S\left[g_{\mu\nu}, A_\mu, \phi, \ldots\right]$

- Each field in the gravitational theory encodes the dynamics of an operator in the dual field theory:

Gravity field

$$g_{\mu\nu}\left(r,t,\vec{x}\right) \longleftrightarrow T^{\mu\nu}\left(t,\vec{x}\right)$$

Field theory operator

$$A_\mu\left(r,t,\vec{x}\right) \longleftrightarrow J^{\mu}\left(t,\vec{x}\right)$$

$$\phi\left(r,t,\vec{x}\right) \longleftrightarrow \mathcal{O}_\phi\left(t,\vec{x}\right)$$

- Gauged symmetries in the gravitational theory correspond to global symmetries in the field theory:

Diffeomorphism invariance: $\partial_\mu T^{\mu\nu} = 0$

U(1) gauge invariance: $\partial_\mu J^{\mu} = 0$

Basic technology of holography II

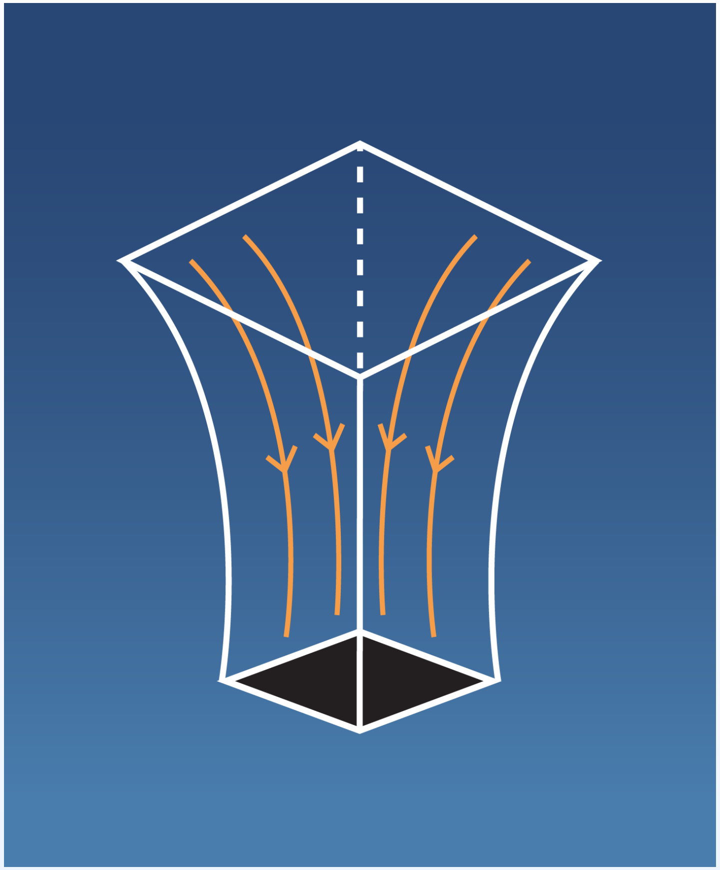
extra co-ordinate r = energy scale

gravity

$$\mathrm{AdS}_4 \quad + \quad A_t\,(r=\infty)$$

Solve
Einstein's
equations

Near-horizon geometry



field theory

$$\mathrm{CFT}_3 \quad + \quad \mu$$

RG flow

IR physics

Emergent and local quantum criticality

- The specific IR physics depends on the gravitational action. Generically, the near-horizon metric is covariant under a scaling symmetry $t \to \lambda t, \qquad \vec{x} \to \lambda^{\frac{1}{z}} \vec{x}$ z: dynamical critical exponent

- In the IR, there is emergent quantum criticality, which can violate hyperscaling.

- In holography, the simplest examples have $z \to \infty$. These exhibit local quantum criticality. The dual geometries are conformal to $\mathrm{AdS}_2 \times \mathrm{R}^2$.

- For $z \to \infty$, the IR physics is approximately momentum-independent, and low energy excitations exist at all momenta.

Linear response from holography

- The linear response of these field theories to perturbations is controlled by the linear excitations around the gravitational solutions $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu},$

$$A_\mu \to A_\mu + \delta A_\mu, \quad \text{etc.}$$

- From these, we compute retarded Greens functions: the response of an operator to a small external source

$$G_{\mathcal{O}\mathcal{O}}(\omega) \sim \frac{\delta\langle\mathcal{O}(\omega)\rangle}{\delta\mathcal{J}(\omega)} \sim \left.\frac{\partial_r\phi(r,\omega)}{\phi(r,\omega)}\right|_{r\to\infty}$$

gravity calculation

- Using a Kubo formula, it is simple to determine electrical conductivity from δA_x

$$\sigma(\omega) = \frac{i}{\omega} G_{J^x J^x}(\omega) = \frac{\delta\langle J^x(\omega)\rangle}{\delta E_x(\omega)}$$

Basic properties of holographic theories

- These states are quite different from those composed of long-lived quasiparticles. They are highly collective, and we deal directly with the collective currents of the system: $T^{\mu\nu}, \ J^{\mu}$ etc.

- The intrinsic relaxation times are short: $\vec{J}$ wants to decay quickly but it can't. It carries momentum, which is conserved

$$\partial_\mu T^{\mu\nu} = 0 \quad \longrightarrow \quad \partial_t \vec{P} = 0$$

- To get realistic transport, we need to dissipate it e.g. by breaking translational invariance.

- In general this is very hard. It is very instructive to work with simple cases where we can clearly identify what is happening.

A simple theory of massive gravity I

- The starting point: momentum conservation is enforced by the diffeomorphism invariance of gravity. The simplest way to remove this is to give a mass to the graviton e.g. $\mathcal{L}_m = \sqrt{-g} m^2 f_{\mu\nu} g^{\mu\nu}$
  $f_{xx} = f_{yy} = 1$
- In fact, a more complicated action was studied first:

$$S = \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + m^2 \left[\left(\mathrm{Tr}\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right) \right] \right) \qquad \mathcal{K}^\mu_\alpha \mathcal{K}^\alpha_\nu = g^{\mu\alpha} f_{\alpha\nu}$$

- It has a simple solution with isotropy and translational invariance:

$$ds^2 = \frac{L^2}{r^2} \left(-f(r) dt^2 + dx^2 + dy^2 + \frac{dr^2}{f(r)} \right) \qquad A_t(r) = \mu \left(1 - \frac{r}{r_0} \right)$$

$$f(r) = 1 - m^2 r^2 - (1 - m^2 r_0^2 + \frac{1}{4} \mu^2 r_0^2) \frac{r^3}{r_0^3} + \frac{1}{4} \mu^2 r_0^2 \frac{r^4}{r_0^4}$$

Vegh (2013)

- Numerical calculations show that σ_{DC} is finite.

A simple theory of massive gravity II

- This theory breaks diffeomorphism invariance in such a simple way that it is easy to learn a lot about what is happening.

- Near the horizon, the geometry is still $\mathrm{AdS}_2 \times \mathrm{R}^2$. It is a marginal deformation: the effect of m is to change the length scale of AdS_2

- The mass term has a much more important effect: the breaking of diffeomorphism invariance creates new dynamical degrees of freedom. One of these couples to δA_x.

- In the field theory, a new operator couples to J_x and its dimension controls the scaling behaviours of σ :

$$\sigma_{DC} \sim T^0 \qquad\qquad \sigma\left(\omega, T=0\right) \sim \omega^0$$

Drude peak in massive gravity

- For small frequencies and graviton masses: $(m/\mu)^2 \ll \omega/\mu \ll 1$

RD (2013)

$$\sigma(\omega) = \frac{\sigma_{\rm DC}}{1 - i\omega\tau} \qquad \sigma_{DC} \sim \tau \qquad \tau = \frac{4\pi(\epsilon + P)}{s}\frac{1}{m^2}$$

- This is just a Drude peak! But Drude's theory is based on long-lived quasiparticles with lifetime τ, and these are not present here.

- At long distances and low energies, we can deduce a simple effective theory of what is going on. The main effect of the graviton mass is to make the total momentum of the state dissipate at the rate

$$\partial_t \vec{P} = -\Gamma \vec{P} \qquad \Gamma \equiv \tau^{-1} = \frac{s}{4\pi(\epsilon + P)} m^2 \sim m^2 T^0$$

- This momentum dissipation rate controls the conductivity.

What is going on in the field theory? I

- The coupling to new degrees of freedom due to the graviton mass produces the desired effect: it causes momentum to dissipate in the dual field theory and gives finite σ_{DC}.

- But what is really going on? Consider the simpler mass term $\mathcal{L}_m = \sqrt{-g} m^2 f_{\mu\nu} g^{\mu\nu}$. This has the same solution and equations for δA_x i.e. the same conductivity.

- Rewrite the fixed reference metric in terms of scalar fields ("Stuckelberg trick"): $$m^2 f_{\mu\nu} = \delta_{ij} \partial_\mu \varphi^i \partial_\nu \varphi^j$$

- This restores diffeomorphism invariance at the price of introducing new degrees of freedom (scalar fields).

What is going on in the field theory? II

- The resulting action is much more reasonable:

Andrade, Withers (2013)

$$S = \int d^4x\sqrt{-g}\left[\mathcal{R} + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \sum_{i=1}^{2}\partial_\mu\varphi^i\partial^\mu\varphi^i\right]$$

- The new massless scalar fields have equations of motion with simple solutions that explicitly break translational invariance

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi^i\right) = 0 \qquad \varphi^i = mx^i$$

x^i: field theory spatial co-ordinates

- The equations for δA_x are the same as in the previous theory of massive gravity, and therefore so is the conductivity.

- There is an effective graviton mass from coupling to a scalar field with a source that breaks translational invariance. The new degrees of freedom are excitations of the scalar fields.

Generalise: what are the key features?

- There are two mathematical features that make things so simple:

  1. Although the scalar fields explicitly break translational invariance, their derivatives are independent of x^i. Thus the gravitational $T^{\mu\nu}$ is independent of x^i, and so is the metric.

  2. The equations of motion for δA_x are so simple that there is a universal expression for σ_{DC} that depends on the near-horizon gravitational solution. Blake, Tong (2013)
  i.e. one does not have to explicitly embed this near-horizon geometry into an AdS spacetime, or to solve the equations for δA_x explicitly.

Generalise: many metals & insulators

- We were working with the simplest action with a charged black hole solution: Einstein-Maxwell theory.

scalar field present in string theory

- Can generalise this to an Einstein-Maxwell-dilaton theory (plus scalars to break translational symmetry), and classify the possible near-horizon solutions i.e. possible IR effective field theories.

- The equations for δA_x retain the simplicity, and so it is simple to read off σ_{DC}: $\sigma_{DC} \sim T^{\alpha}, \quad \sigma\left(\omega, T=0\right) \sim \omega^{\beta}, \quad \alpha \neq \beta$

Gouteraux (2014), Donos, Gauntlett (2014)

- They can be conductors (coherent or incoherent) or "insulators". Power laws determined by the exponents characterising the scaling symmetries of the IR physics (e.g. z).

More realistic examples

- These states break translational invariance in a featureless way.

- The tools developed here are useful in more realistic examples: the main effect of translational symmetry breaking is to generate an effective mass for the graviton, which controls σ_{DC} .

- If the scalar has a periodic or spatially random source, the equations of motion for δA_x retain the simplicity of the toy model, at leading order in the strength of the source.

  Blake, Tong, Vegh (2013), Lucas, Sachdev, Schalm (2014)

- In these cases, it is easy to calculate σ_{DC} from the effective graviton mass, which now depends on the characteristics of the lattice or disorder that is turned on.

Hydrodynamics and resistivity=entropy

- The holographic theories provide quantum critical phases with power law resistivities ρ_{DC}. The power typically depends on the various exponents $z,\ \theta,\ \mathrm{etc.}$ controlling the IR physics.

- In some cases, it is possible to identify a more physical reason for these results. In particular, some of these holographic theories have the intriguing result $\rho_{DC}\left(T\right) \sim s(T)$.

- We can identify a mechanism responsible for this, which is due to universal properties of holographic theories.

- The holographic theories are examples of cases where this mechanism exists, but it can exist independently of holography.

“Almost conserved” momentum I

- If we weakly break translational invariance, so that $\vec{P}$ lives for a long time (it is “almost conserved”), it controls the decay rate of $\vec{J}$ at late times, and the conductivity is proportional to Γ, the rate at which momentum dissipates.

- Suppose this dissipation is caused by a coupling to a periodic source for an irrelevant operator in the IR: $\delta H = V \int dx e^{ik_L x} \mathcal{O}(x)$ The dissipation rate will be small and we can work perturbatively.

- At leading order, Γ is determined by the number of low energy states at k_L in the translationally invariant theory: it is these that couple to the lattice once it's turned on.

"Almost conserved" momentum II

- Using the memory matrix formalism, this intuition is confirmed:

$$\Gamma = \frac{V^2 k_L^2}{\chi_{\vec{P}\vec{P}}} \lim_{\omega \to 0} \frac{\mathrm{Im} G_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega}\Bigg|_{V\to 0}$$

- For coupling to a spatially random source, one should integrate over momenta

$$\Gamma = \frac{V^2}{\chi_{\vec{P}\vec{P}}} \int d^2k \; k^2 \lim_{\omega \to 0} \frac{\mathrm{Im} G_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega}\Bigg|_{V\to 0}$$

- And in both cases, $\rho_{DC}(T) \sim \Gamma(T)$.

- Note that this argument exists independently of holography, but is particularly useful in these cases since momentum is the only long-lived quantity.

Hartnoll, Kovtun, Muller, Sachdev (2007)
Hartnoll, Herzog (2008)
Hartnoll, Hofman (2012)

Hydrodynamics in holography

- So, in some cases, the resistivity is controlled by the properties of the translationally invariant state.

- One of the most generic features of translationally invariant holographic theories is that they behave hydrodynamically at sufficiently low energies and long distances.

- These collective states are "almost perfect fluids": they reach local thermal equilibrium in the shortest possible time, due to their minimal viscosity

$$\eta = \frac{\hbar}{k_B}\frac{s}{4\pi}$$

Kovtun, Son, Starinets (2004)
Iqbal, Liu (2008)

- This is very different than in quasiparticle theories where η is very large due to the weak interactions e.g. in a Fermi liquid, $\eta \sim 1/T^2$

Weak disorder and hydrodynamics

- In a hydrodynamic state, the correlation functions of $T^{\mu\nu},\ J^{\mu}$ at low energies and long distances are fixed.

- So if we couple a (relativistic, conformal) hydrodynamic state to random sources of energy density and charge density (i.e. random disorder)

$$\rho_{DC} \sim \frac{\mathcal{V}_{T^{tt}}^2}{\sigma^2}\int dk k\left(\eta k^2 + \ldots\right), \quad \rho_{DC} \sim \frac{\mathcal{V}_{J^t}^2}{\sigma^2}\int dk k\Big(\frac{1}{\sigma_Q}\left[2\frac{\sigma^2}{\epsilon+P} - \left(\frac{d\sigma}{d\mu}\right)_T\right]^2 + k^2\frac{\sigma^2}{(\epsilon+P)^2}\eta + \ldots\Big),$$

charge density

"universal conductivity"

provided the disorder is irrelevant.

- And so if hydrodynamics applies at small distances $\sim \mu^{-1}$, random disorder will produce a viscous contribution to the resistivity: $\rho_{DC}\left(T\right) \sim \eta\left(T\right)/l^2$.

RD, Schalm, Zaanen (2013)
see also: Andreev, Kivelson, Spivak (2011)

Resistivity=entropy and the cuprates

- A simplified way to think about it: in a hydrodynamic state, momentum diffuses at a rate $D \sim \eta$. If it interacts with impurities a distance l apart, its lifetime is $\tau^{-1} \sim D/l^2$ and so the momentum dissipation rate is $\Gamma \sim \tau^{-1} \sim \eta/l^2$

- Locally critical ($z \to \infty$) holographic theories are well-described by hydrodynamics with $\eta = s/4\pi$ and couple weakly to random sources of energy and charge density:

$$\rho_{DC}(T) \sim s(T)$$

see also Anantua et. al. (2012)

- At finite z, disorder is relevant at low energies. Perturbation theory breaks down at low T, where $\rho_{DC}(T) \sim T^{2/z} s(T)$

Lucas, Sachdev, Schalm (2014)

How realistic is this?

- In the strange metal phase of the cuprates, $\rho_{DC}\left(T\right) \sim s\left(T\right)???$ They are both linear in T at optimal doping, and appear to change in a similar way as doping is reduced.

- A possible resolution: strong interactions in the cuprates cause the electrons to quickly form a collective, hydrodynamic state with minimal viscosity, before momentum is dissipated. Weak interactions with disorder then give a linear resistivity. At T=0, resistivity vanishes as a perfect (non-dissipative) fluid forms.

- It would be very interesting to systematically measure $\rho_{DC}\left(T\right)/s\left(T\right)$ as a function of doping and to look for more experimental signatures of hydrodynamics in metals.

Summary

- To get sensible transport properties in holographic theories, we need to introduce a mechanism that dissipates momentum.
- This can be done in a simple way that keeps the calculations tractable. The resulting states can be metallic or insulating.
- The key effect of momentum dissipation is that it generates an effective mass term for the graviton, which controls the resistivity. This extends to more realistic cases: lattices and random disorder.
- Inspired by holography, identified a simple physical mechanism by which metals can acquire a linear resistivity.