Monte Carlo studies of the spontaneous rotational symmetry breaking in dimensionally reduced super Yang-Mills models

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Outline

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IIB Matrix Model: Overview

- A non-perturbative definition of string theory in the large N limit
- A theory with only one scale, possibility to dynamically choose a unique vacuum
- Dynamical emergence of space–time and matter content
- Dynamical compactification of extra dimensions
- Tackle cosmological questions, like expansion of $3+1$ dimensional space–time, resolution of cosmic singularity

The IKKT or IIB Matrix Model [Ishibashi,Kawai,Kitazawa,Tsuchiya hep-th/9612115]

$$Z = \int dA\, d\Psi\, e^{iS}$$

$$S = \underbrace{-\frac{1}{4g^2}\mathrm{tr}\left([A_\mu, A_\nu][A^\mu, A^\nu]\right)}_{=S_B} \underbrace{- \frac{1}{2g^2}\mathrm{tr}\left(\Psi_\alpha (\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right)}_{=S_F}.$$

$A_\mu(\mu = 0, \ldots, 9)$,
$\Psi_\alpha(\alpha = 1, \ldots, 16)$ (10D Majorana-Weyl spinor),
$(A_\mu)_{ij}$, $(\Psi_\alpha)_{ij}$, $i, j = 1, \ldots, N$ hermitian matrices.

- manifest SO(9,1) symmetry and SU(N) gauge invariance
- $\mathcal{N} = 2$ Supersymmetry

Relation to String Theory

- Matrix regularization of IIB string action in the large N limit:

$$S_{\text{Schild}} = -\int d^2\sigma\,\sqrt{g}\,\left(\frac{1}{4}\left\{X_\mu(\sigma), X_\nu(\sigma)\right\}^2 + \frac{1}{2}\Psi(\sigma)\mathcal{C}\Gamma^\mu\left\{X_\mu(\sigma), \Psi(\sigma)\right\}\right)$$

$$X_\mu(\sigma) \to (A_\mu)_{ij} \qquad \Psi_\alpha(\sigma) \to (\Psi_\alpha)_{ij}$$

$$\{\cdot,\cdot\} \to -\imath\,[\cdot,\cdot] \qquad \int d^2\sigma\,\sqrt{g} \to \mathrm{tr}$$

- non-commutative world sheet
- block structure in matrices $\to$ second quantized string theory
- reproduce interaction between D-branes at one loop level
- loop equation for Wilson loops $\to$ light cone IIB string field theory: $w(C) = \mathrm{tr}P\exp\left[\imath\int_C k^\mu A_\mu\right] \to \Psi\left[k(\cdot)\right]$

[Fukuma, Kawai, Kitazawa, Tsuchiya ('97)]

$\mathcal{N} = 2$ Supersymmetry

$$\begin{cases} \delta^{(1)} A_\mu &= i\epsilon_1 \mathcal{C}\Gamma_\mu \Psi \\ \delta^{(1)}\Psi &= \frac{i}{2}\Gamma^{\mu\nu}[A_\mu, A_\nu]\epsilon_1 \end{cases} \quad \begin{cases} \delta^{(2)} A_\mu &= 0 \\ \delta^{(2)}\Psi &= \epsilon_2 \mathbf{1} \end{cases}$$

and bosonic symmetry

$$\begin{cases} \delta^{(1)} A_\mu &= c_\mu \mathbf{1} \\ \delta^{(1)}\Psi &= 0 \end{cases}$$

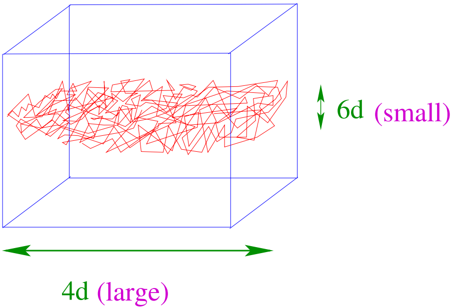
Generators: $Q^{(1)}$, $Q^{(2)}$, P_μ resp.

$$\tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)}\,, \qquad \tilde{Q}^{(1)} = i(Q^{(1)} - Q^{(2)})$$

$$[\epsilon_1 \mathcal{C}\tilde{Q}^{(i)}, \epsilon_2 \mathcal{C}\tilde{Q}^{(j)}] = -2\delta^{ij}\epsilon_1 \mathcal{C}\Gamma^\mu \epsilon_2 P_\mu$$

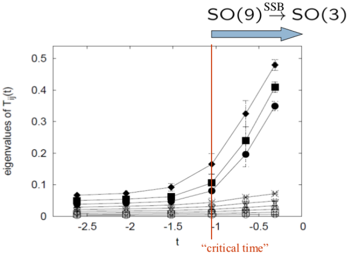
Identify as $D = 10$, $\mathcal{N} = 2$ SUSY and P_μ as translations $\Rightarrow$ eigenvalues of A_μ $D = 10$ space-time coordinates [Aoki et al hep-th/9802985]

Space–Time Interpretation



- non-commutative space–time [Iso,Kawai 99, Ambjørn,KNA,Bietenholz,Hotta,Nishimura et al 00]
- possibility of dynamical compactification of extra dimensions
- possibility of built-in mechanism that generates $(3+1)$-dim space-time

Emergence of $(3+1)$ dimensional spacetime



- Simulations of Lorentzian model: no sign problem, introduce large scale cutoffs in $\mathrm{tr}(A_0^2)$ and $\mathrm{tr}(A_i^2)$ [Kim,Nishimura,Tsuchiya 1108.1540]
  - dynamical time from A^0 (SUSY crucial)
  - expanding 3+1 universe after a critical time
- classical, expanding solutions at late times [Kim,Nishimura,Tsuchiya 1110.4803,1208.0711]
- local field theory as fluctuations around classical solns representing commutative space-time [1208.4910]
- constructively realize chiral fermions at finite-N by imposing conditions on extra dims [1305.5547]

Euclidean Model

$$A_0 \to iA_{10} \qquad \Gamma^0 \to -i\Gamma^{10}$$

- SO(10) rotational symmetry
- Finite: quantum effects, despite flat directions

  [Krauth,Nicolai,Staudacher 98, Austing, Wheater 01]
- Gaussian Expansion Method Calculations [Nishimura,Sugino 02, Kawai et.al. 03,06] show that: [Nishimura,Okubo,Sugino 1108.1293]
  - $d = 3$ configurations have lowest free energy
  - extent of the shrunken dimensions r is independent of d
  - the extent of the large dimensions R depends on d so that the 10 dimensional volume is a finite constant and independent of d: $R^d r^{10-d} = l^{10}$
  - the ratio R/r remains finite in the large N limit

Dynamical compactification by SSB of SO(10)→SO(3)

6 dimensional Euclidean model

- Need a simpler model to study the above results using Monte Carlo simulations
- $D = 4$ studied before, no SSB of SO(4) [Ambjørn,KNA,Bietenholz,Hotta,Nishimura et al 00]
- $D = 6$ the simplest model with SSB of SO(6):

$$Z = \int dA\, d\psi\, d\bar{\psi}\, \mathrm{e}^{-S_b - S_f}$$

$$S_b = -\frac{1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2 \qquad S_f = -\frac{1}{2g^2}\mathrm{tr}\left(\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right)$$

  - A_μ are $N \times N$, hermitian, traceless, vectors w.r.t. SO(6)
  - $\psi_\alpha, \bar{\psi}_\alpha$ are $N \times N$, grassmannian entries, Weyl spinors w.r.t. SO(6)

Similar to $D = 10$:
SO(6) rotational symmetry, $\mathcal{N} = 2$ SUSY, SU(N) symmetry

Dynamical Compactification

Order Parameter

$$T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$$

- Eigenvalues of $T_{\mu\nu}$: λ_n, $n = 1, \ldots, 6$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{10}$$

- Extended d–dimensions if e.g. in the *large N limit*

$$\langle\lambda_1\rangle = \ldots = \langle\lambda_d\rangle \equiv R^2$$

Shrunk $(6-d)$–dimensions if e.g.

$$\langle\lambda_{d+1}\rangle = \ldots = \langle\lambda_{10}\rangle \equiv r^2$$

- SSB of SO(6) invariance

$$\mathrm{SO}(6) \to \mathrm{SO}(d)$$

Gaussian Expansion Method (GEM) - Improved Mean Field Approximation

[P.M.Stevenson 81], [Kabat,Lifschytz,Lowe 00-02], [Nishimura,Sugino 01],[Kawai et.al. 02]

- a systematic expansion method to study non perturbative effects
- introduce Gaussian $S_0[M_\mu, \mathcal{A}_{\alpha\beta}]$ where $M_\mu, \mathcal{A}_{\alpha\beta}$ parameters

$$S = (S + S_0) - S_0 = (S_b + S_f + S_0) - S_0$$

$$S_0[M_\mu, \mathcal{A}_{\alpha\beta}] = M_\mu \mathrm{tr}(A_\mu^2) + \mathcal{A}_{\alpha\beta} \mathrm{tr}(\bar{\psi}_\alpha \psi_\beta)$$

- expand $\tilde{S} = S_0 + \epsilon\, S_b + \sqrt{\epsilon}\, S_f$ w.r.t ϵ
- replace $M \to (1-\epsilon)M$, $\mathcal{A} \to (1-\epsilon)\mathcal{A}$
- reorganize series, truncate, set $\epsilon = 1$
- look for "plateaux" in parameter space $(M_\mu, \mathcal{A}_{\alpha\beta})$, in practice by solving

$$\frac{\partial F}{\partial M_\mu} = 0 \qquad \frac{\partial F}{\mathcal{A}_{\alpha\beta}} = 0$$

GEM results [Aoyama, Nishimura, Okubo (arXiv:1007.0883)]

- parameter space is very large: simplify by considering SO(d) invariant ansätze, $2 \le d \le 5$

$$\langle\lambda_1\rangle_{SO(d)} = \ldots = \langle\lambda_d\rangle_{SO(d)} = (R_d)^2$$

- compute free energy and observables at solutions in the large N limit
- compare free energy of ansätze, minimum free energy for the $d = 3$ ansatz, i.e. conclude SO(6)$\to$ SO(3)
- The extent of the shrunken dimensions r (“compactification scale”) is independent of d
- The extent of the large dimensions R depends on d so that the 6 dimensional volume is a finite constant and independent of d:

$$R^d r^{6-d} = \ell^6 \qquad r^2 \approx 0.223 \qquad \ell^2 \approx 0.627$$

- The ratio R_d/r is finite

(in units where $g\sqrt{N} = 1$)

Outline

Monte Carlo Simulation [KNA, Azuma, Nishimura (unpublished)]

Integrate out fermions first:

$$Z = \int dA d\bar{\psi} d\psi \, \mathrm{e}^{-S_b - S_f} = \int dA \, \mathrm{e}^{-S_b} \, Z_f[A]$$

$$Z_f[A] = \int d\bar{\psi} d\psi \, \mathrm{e}^{-S_f} = \det \mathcal{M}$$

Monte Carlo simulations hard due to the strong complex action problem

$$\det \mathcal{M} = |\det \mathcal{M}| \, \mathrm{e}^{\imath \Gamma} \qquad \text{is Complex}$$

The Algorithm

Phase Quenched Model: ignore the phase $e^{\imath\Gamma}$

$$Z_0 = \int dA\, e^{-S_0} \qquad S_0 = S_b - \log|\det \mathcal{M}|$$

Simulate using Rational Hybrid Monte Carlo: use rational approximation

$$x^{-1/2} \simeq a_0 + \sum_{k=1}^{Q} \frac{a_k}{x + b_k}$$

- increased accuracy and range of x requires higher Q
- coefficients a_k and b_k computed using Remez algorithm

[E. Remez 34, Clark and Kennedy 05 `github.com/mikeaclark/AlgRemez`]

The Algorithm

Define $\mathcal{D} = \mathcal{M}^\dagger \mathcal{M} \Rightarrow \det \mathcal{D}^{1/2} = |\det \mathcal{M}|$, then we can approximate

$$\det \mathcal{D}^{1/2} \simeq \int dF\, dF^*\, \mathrm{e}^{-S_{PF}[F,F^*,A]} \qquad (F_\alpha)_{ij} \text{ pseudofermions}$$

where

$$S_{PF}[F,F^*,A] = \mathrm{tr}\left\{ a_0 F^\dagger F + \sum_{k=1}^{Q} a_k F^\dagger (\mathcal{D} + b_k)^{-1} F \right\}$$

- spectrum of $\mathcal{D}$ determines Q
- rescale A, F to adjust spectrum to desired range

The Algorithm

$$H = \frac{1}{2}\mathrm{tr}\Pi^2 + \mathrm{tr}\tilde{\Pi}^\dagger\tilde{\Pi} + S_{\mathrm{eff}}[F, F^*, A]$$

where

$$S_{\mathrm{eff}}[F, F^*, A] = S_0[A] + S_{PF}[F, F^*, A]$$

- $\Pi_{ij}^{\mu} = (\Pi^*)_{ji}^{\mu}$, $\tilde{\Pi}_{ij}^{\alpha}$ canonical momenta of $((A_\mu)_{ij}, (F_\alpha)_{ij})$
- $\int d\tilde{\Pi} d\tilde{\Pi}^* d\Pi dF dF^* dA\, \mathrm{e}^{-H} = \int dF dF^* dA \mathrm{e}^{-S_{\mathrm{eff}}}$
- τ-evolution according to eom preserve H:

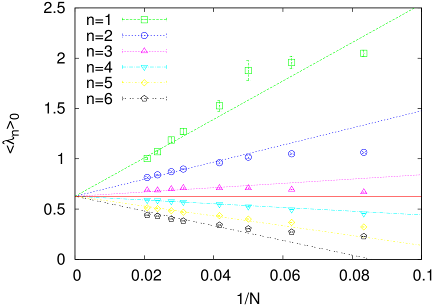
$$\frac{dA_\mu}{d\tau} = \frac{\partial H}{\partial \Pi^\mu} = \Pi_\mu^* \,, \qquad \frac{dF_\beta}{d\tau} = \frac{\partial H}{\partial \tilde{\Pi}^\beta} = \tilde{\Pi}_\beta^* \,,$$

$$\frac{d\Pi^\mu}{d\tau} = -\frac{\partial H}{\partial A_\mu} = -\frac{\partial S_{\mathrm{eff}}}{\partial A_\mu} \,, \qquad \frac{d\tilde{\Pi}^\beta}{d\tau} = -\frac{\partial H}{\partial F_\beta} = -\frac{\partial S_{\mathrm{eff}}}{\partial F_\beta}$$

The Algorithm

- Discretize eom: $\tau_f = N_\tau \Delta\tau$
- Discretization errors: $\Delta H \sim \mathcal{O}(\Delta\tau^2)$. To maintain detailed balance condition use a Metropolis accept/reject decision. Acceptance rate depends on ΔH, tune parameters in order to maximize acc. ratio and minimize autocorrelation times.
- Main part of computational effort: terms $(\mathcal{D} + b_k)^{-1}F$. Replace by solutions χ of $(\mathcal{D} + b_k)\chi_k = F$
- Use conjugate gradient method for the smallest of b_k's. ($\mathcal{O}(N^3)$ ops if cleverly done)
- Use multimass Krylov solvers for other b_k ($\mathcal{O}(Q)$ gain).
- Conjugate gradient method needs $\mathcal{O}(N^2)$ iterations to converge. (instead of $\mathcal{O}(1)$ in typical LQCD)

Results



- In the large–N limit $\langle \lambda_1 \rangle_0 = \ldots = \langle \lambda_6 \rangle_0 = \ell^2 \approx 0.627$ consistent with GEM result
- No SO(6) SSB $\Rightarrow$ phase fluctuations are important in inducing SSB as expected

Complex Action Problem

$$Z = \int dA\, e^{-S_0}\, e^{\imath\Gamma} \qquad Z_f[A] = |\det \mathcal{M}|\, e^{\imath\Gamma}$$

- no ordinary Monte Carlo importance sampling possible: not a positive definite probability measure
- A serious and important *technical* problem
  - Lattice QCD at high T/finite μ [1302.3028]
  - Lattice QCD with θ-vacua [0803.1593]
  - Real time QFT [hep-lat/0609058]
  - Electron structure calculation [PRL 71(93)1148, J.Chem.Phys 102,4495+109,6219]
  - Repulsive Hubbard model [PRB 41(90) 9301]
  - Nuclear shell model [Phys.Repts. 278(97)1]
  - Polymer theory [Phys.Repts. 336(00)167]

Complex Action Problem

Possible approach: use the phase quenched model $Z_0 = \int dA\, e^{-S_0}$:

$$\langle \lambda_n \rangle = \frac{\langle \lambda_n\, e^{\imath\Gamma} \rangle_0}{\langle e^{\imath\Gamma} \rangle_0}$$

- $\langle e^{\imath\Gamma} \rangle_0$ decreases as $e^{-N^2 \Delta f} \sim Z/Z_0$, $\Delta f > 0$.
  Need $\mathcal{O}(e^{cN^2})$ statistics for given accuracy goal.
- Overlap problem: distribution of sampled configs in Z_0 has exponentially small overlap with Z

Dominant configurations determined by competition of entropy, action and phase fluctuations.

Factorization Method [KNA, Nishimura 01]

$$\tilde{\lambda}_n = \frac{\lambda_n}{\langle \lambda_n \rangle_0}$$

- $\langle \tilde{\lambda}_n \rangle_0 \equiv 1$, deviation from 1 is the effect of the phase
- Consider the distribution functions

$$\rho(x_1, \ldots, x_6) = \left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\rangle \qquad \rho^{(0)}(x_1, \ldots, x_6) = \left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\rangle_0$$

- Consider the ensemble

$$Z_{x_1,\ldots,x_6} = \int dA \, \mathrm{e}^{-S_0[A]} \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k)$$

then $\rho(x_1, \ldots, x_6) = \frac{1}{C} \rho^{(0)}(x_1, \ldots, x_6) \, w(x_1, \ldots, x_6)$
where $w(x_1, \ldots, x_6) = \langle \mathrm{e}^{i\Gamma} \rangle_{x_1,\ldots,x_6}$
$C = \langle \mathrm{e}^{i\Gamma} \rangle_0$ not needed in the calculation.

Factorization Method

$$\langle \tilde{\lambda}_n \rangle = \int \prod_{k=1}^{6} dx_k \, x_n \, \rho(x_1, \ldots, x_6)$$

- In the large-N limit, dominating configs determined by minimum of the "free energy":

$$\begin{aligned} \mathcal{F}(x_1, \ldots, x_6) &= -\frac{1}{N^2} \log \rho(x_1, \ldots, x_6) \\ &= -\frac{1}{N^2} \log \rho^{(0)}(x_1, \ldots, x_6) - \frac{1}{N^2} \log w(x_1, \ldots, x_6) + \frac{1}{N^2} \log C \end{aligned}$$

- The minimum is determined by solutions of

$$\frac{1}{N^2} \frac{\partial}{\partial x_n} \log \rho^{(0)}(x_1, \ldots, x_6) = -\frac{\partial}{\partial x_n} \frac{1}{N^2} \log w(x_1, \ldots, x_6) \quad \text{for} \quad n = 1, \ldots, 6$$

Factorization Method

$$\frac{1}{N^2}\frac{\partial}{\partial x_n}\log\rho^{(0)}(x_1,\ldots,x_6) = -\frac{\partial}{\partial x_n}\frac{1}{N^2}\log w(x_1,\ldots,x_6) \quad \text{for} \quad n=1,\ldots,6$$

- each function has a well defined large-N limit
- dominating solution can be used as an *estimator* of $\langle\tilde{\lambda}_n\rangle$
- no need to know $\rho(x_1,\ldots,x_6)$ everywhere to compute $\langle\tilde{\lambda}_n\rangle$
- RHS has complex action problem but scales fast with increasing $N \Rightarrow$ extrapolation to larger N
- errors do not propagate exponentially with N as with a naive large N extrapolation

Factorization Method

- key in using the method: find the right observables to constrain
- determine the ones that are strongly correlated with the phase expectation values of all others computed at the saddle point solution: no sign problem! [KNA,Azuma,Nishimura 1009.4504,1108.1534]
- d-dimensional configs:
  $d = 6 \Rightarrow \det \mathcal{M} \in \mathbb{C}$, $d = 5 \Rightarrow \det \mathcal{M} \in \mathbb{R}$, ($\mathbb{R}_+$ dominates at large N)
  $d = 4, 3 \Rightarrow \det \mathcal{M} \in \mathbb{R}_+$, $d \leq 2 \Rightarrow \det \mathcal{M} = 0$
- phase is stationary w.r.t. perturbations around $d < 6$ configs
  [Nishimura, Vernizzi 00]
- strong evidence that $\lambda_1, \ldots, \lambda_6$ found to be the only ones strongly correlated with the phase: our choice for studying their distribution functions [1009.4504]

Strong complex phase fluctuations play central role in the SSB mechanism

[Nishimura, Vernizzi 00, KNA, Nishimura 01]

Simplifications

- hard to solve the saddle point equations in full 6D parameter space
- we study SO(d) symmetric vacua $2 \leq d \leq 5$, compare to GEM
  $x_1 = \ldots = x_d > 1 > x_{d+1} = \ldots = x_6$
- we find that large evs, when sufficiently large, decorrelate from the phase
  $\Rightarrow$ omit large evs from $\rho(x_1, \ldots, x_6)$
- we find that small evs to acquire the same value in the large-N limit
  $\Rightarrow$ omit smallest evs from $\rho(x_1, \ldots, x_6)$

Therefore, in order to study the SO(d) vacuum, consider only $\rho(x_{d+1})$

Observables

We take $n = d + 1$ for the SO(d) vacuum

- Define $w_n(x) = \langle e^{i\Gamma} \rangle_{n,x}$ w.r.t $Z_{n,x} = \int dA\, e^{-S_0[A]}\, \delta(x - \tilde{\lambda}_n)$
- Define $\rho_n^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_n) \rangle_0$
- Let $\bar{x}_n$ be the solution to the saddle point equation

$$\frac{1}{N^2} f_n^{(0)}(x) \equiv \frac{1}{N^2} \frac{d}{dx} \log \rho_n^{(0)}(x) = -\frac{d}{dx} \frac{1}{N^2} \log w_n(x)$$

in the $x < 1$ region. Then we define the estimator

$$\langle \tilde{\lambda}_n \rangle_{\mathrm{SO}(d)} = \bar{x}_n \,, \qquad n = d + 1$$

Observables

Given $\bar{x}_n$ we also use the estimators

- $\langle\tilde{\lambda}_k\rangle_{\mathrm{SO}(d)} = \langle\tilde{\lambda}_k\rangle_{n,\bar{x}_n}$
- Compute free energy

$$\mathcal{F}_{\mathrm{SO}(d)} = \int_{\bar{x}_n}^{1} \frac{1}{N^2} f_n^{(0)}(x) dx - \frac{1}{N^2} \log w_n(\bar{x}_n) , \quad \text{where } n = d+1$$

By computing $\mathcal{F}_{\mathrm{SO}(d)}$ for different d we can in principle determine the true vacuum

Simulations

We simulate the system

$$Z_{n,V} = \int dA \, e^{-S_0[A] - V(\lambda_n[A])} \, , \quad V(z) = \frac{1}{2}\gamma \, (z-\xi)^2$$

- γ large enough $e^{-V} \to \delta(x - \tilde{\lambda}_n)$
- in practice, we make sure that results are independent of γ
- study the distribution function

$$\rho_{n,V}(x) = \left\langle \delta(x - \tilde{\lambda}_n) \right\rangle_{n,V} \propto \rho_n^{(0)}(x) \exp\left\{-V\left(x \langle \lambda_n \rangle_0\right)\right\}$$

Simulations

- position of the peak of $\rho_{n,V}(x)$ solution of

$$0 = \frac{d}{dx}\log \rho_{n,V}(x) = f_n^{(0)}(x) - \langle\lambda_n\rangle_0\, V'\left(x\,\langle\lambda_n\rangle_0\right)$$

- we take the peak sharp and use

$$x_p = \langle\tilde{\lambda}_n\rangle_{n,V}$$

- we define the estimators

$$\begin{aligned} w_n(x_\mathrm{p}) &= \langle\cos\Gamma\rangle_{n,V} \;, \\ f_n^{(0)}(x_\mathrm{p}) &= \langle\lambda_n\rangle_0\, V'\left(\langle\lambda_n\rangle_{n,V}\right) = \gamma\langle\lambda_n\rangle_0\left(\langle\lambda_n\rangle_{n,V} - \xi\right) \;. \end{aligned}$$

- γ too small, distribution of $\tilde{\lambda}_n$ wide, large error in $\langle\tilde{\lambda}_n\rangle_{n,V}$
  γ too large, small error in $\langle\tilde{\lambda}_n\rangle_{n,V}$ propagates by factor of γ to $f_n^{(0)}(x_p)$
  $(\langle\tilde{\lambda}_n\rangle_{n,V} - \xi \sim 1/\gamma$

Simulations

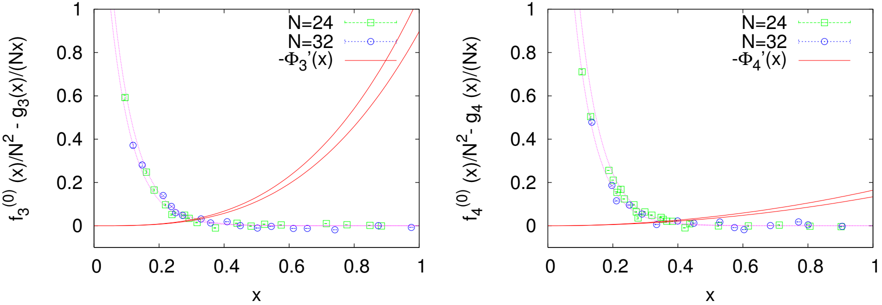
- It is possible to compute $f_n^{(0)}(x)$, $w_n(x)$ for x suppressed by many orders of magnitude in Z_0
- $w_n(x)$ hard due to the complex action problem, but

$$\Phi_n(x) = \lim_{N\to\infty} \frac{1}{N^2} \log w_n(x)$$

  scales for small enough N
- $f_n^{(0)}(x)$, $w_n(x)$ computed by interpolation or fits. Fitting functions determined by simple scaling arguments for small x
- We find that the function $f_n^{(0)}(x)$ scales as $\frac{1}{N} f_n^{(0)}(x)$ for $x \gtrsim 0.4$, but as $\frac{1}{N^2} f_n^{(0)}(x)$ for smaller x. Need to subtract the $\mathcal{O}(1/N)$ finite size effects in the calculations.

Outline

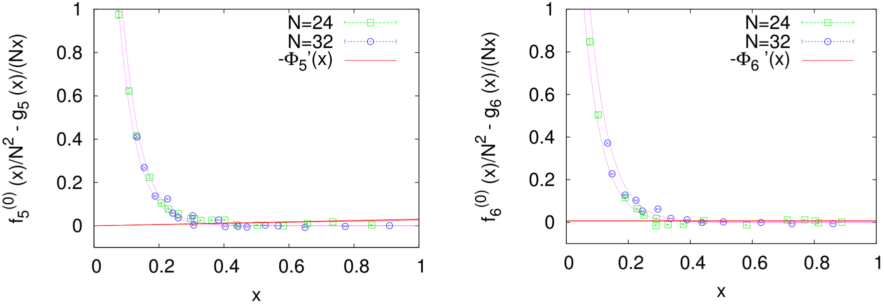
$\langle \tilde{\lambda}_n \rangle$



Compute the solution to $\frac{1}{N^2} f_n^{(0)}(x) = -\Phi'(x)$ (after subtracting finite size effects): Compare to the GEM result $r^2/\ell^2 \approx 0.223/0.627 = 0.355$

$$\langle \tilde{\lambda}_3 \rangle_{\mathrm{SO}(2)} = \bar{x}_3 = 0.31(1) \qquad \langle \tilde{\lambda}_4 \rangle_{\mathrm{SO}(3)} = \bar{x}_4 = 0.35(1)$$

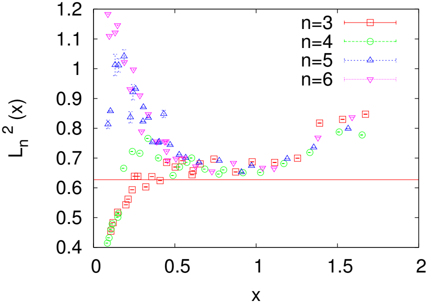
$\langle \tilde{\lambda}_n \rangle$



Compute the solution to $\frac{1}{N^2} f_n^{(0)}(x) = -\Phi'(x)$ (after subtracting finite size effects): Compare to the GEM result $r^2/\ell^2 \approx 0.223/0.627 = 0.355$

$$\langle \tilde{\lambda}_5 \rangle_{SO(4)} = \bar{x}_5 = 0.34(2) \qquad \langle \tilde{\lambda}_6 \rangle_{SO(5)} = \bar{x}_6 = 0.36(3)$$

Constant volume property

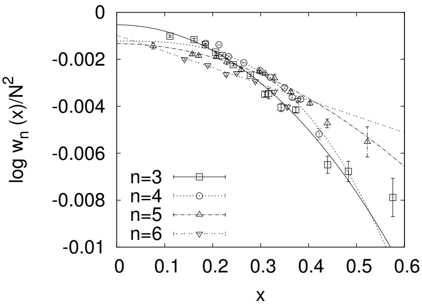


$\langle\lambda_k\rangle_{SO(d)}$, $k \neq n = d+1$, is estimated from $\langle\lambda_k\rangle_{x_p} = \langle\lambda_k\rangle_{n,V}$
In order to minimize the finite size effects, we compute

$$L_n^2(x) = \left(\prod_{k=1}^{6} \langle\lambda_k\rangle_{n,x}\right)^{\frac{1}{6}}$$

and find that $L_n^2(x) \approx \ell^2 \approx 0.627$ for $0.5 < x < 1$

Free Energy



Hard!
After subtracting finite size effects, we fit $\frac{1}{N^2} f_n^{(0)}(x) = p_n \mathrm{e}^{-q_n x}$.
Attempt e.g. to substitute in $\mathcal{F}_{\mathrm{SO}(d)} = \int_{\bar{x}_n}^{1} \frac{1}{N^2} f_n^{(0)}(x) dx - \frac{1}{N^2} \log w_n(\bar{x}_n)$ for $\bar{x}_n \approx 0.355$. Still working!! TBA...

Outline

1. Introduction

2. Monte Carlo Simulations
   - Phase Quenched Model
   - Complex Action Problem
   - The Factorization Method
   - Simulations

3. Results

4. Conclusions

Conclusions

- Simulation from first principles 6D version of IIB matrix model
- Complex action problem very strong, use factorization method successfully
- Computed numerically the maxima of λ_n distributions and estimated $\langle\lambda_n\rangle$ for SO(d) vacua
- Large-N and small-x scaling properties of distribution functions play important role in the calculation
- Short distance, non-perturbative, dynamics of eigenvalues of matrices A play crucial role in determining r
- Results are consistent with GEM prediction $R^d r^{6-d} = \ell^6$, $r^2 \approx 0.223$, $\ell^2 \approx 0.627$
- Consistent with the GEM scenario of dynamical compactification with SSB of SO(6)$\to$SO(3)
- Consistent with (euclidean) spacetime having volume independent of d and R/r finite