AdS/Condensed Matter

Supergravity helps Condensed Matter Physics. A first example: Holographic Superconductors

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Supergravity helps Condensed Matter Physics.
A first example: Holographic Superconductors

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Summary

1 Motivations

2 AdS/CFT and Hol. Superconductors: minimal structure

3 $\mathcal{N} = 2$ sugra coupled to $SU(2, 1)$ hypermultiplet

4 Probing the IR geometry: EE

5 Dual Field Theory and Marginal Deformations
Motivations

Non Fermi Liquid/ Non BCS
some unconventional features

Strange Metal phase, examples:
- $\rho \approx T^n$ with $n$ function of the doping and varying in $[1, 2]$.
- Drude tail falls off with non integer power law, example $w^{2/3}$.

High $T_c$ SuperC phase, examples:
- d-wave order parameter
- $T_c$ around $100K$
- $C_e \approx T^n$ for $T < T_c$. 

ReCe(CuO)
The Spin Fermion Model

\[ \mathcal{L}_{sf} = g \, c_{w,k}^{\dagger} \sigma \sigma_{w, k, q} + \vec{S}_{\Omega, q} \]

\[ \mathcal{L}_{f} = c_{w,k}^{\dagger} G_{0}^{-1}(w, k) c_{w,k} \]

\[ \mathcal{L}_s = \chi_{0}^{-1}(q, \Omega) \, S_{\Omega, q} S_{\Omega, q} \]

\[ G_{0}(w, k)^{-1} = w - \epsilon(k) \]

\[ \chi_{0}(\Omega, q)^{-1} = \left( \xi^{-2} + (q - Q)^2 - (\Omega/v_s)^2 \right) \]
\[ \Delta_{BCS} = \langle c_{k \uparrow}^+ c_{-k \downarrow} \rangle \]

\[ \epsilon(k) = v_x k_x + v_y k_y \]
\[ \epsilon(k + Q) = v_y k_y - v_x k_x \]
\[ v_F^2 = v_x^2 + v_y^2 \]

Hot spot: point \( \vec{x} \in \text{FS} \) such that \( \exists \vec{y} \) with the property: \( \vec{x} - \vec{y} = Q \)

Loop contr. organized in powers of \( \lambda \sim \frac{g^2}{v_f \xi^{-1}} \)

Results
- d-wave from BCS at H.S: \( \Delta_k = -\Delta_{k+Q} \).
- NFL from Loop corrections:
  \[ G_{H.S.}^{-1} \sim \text{sign} w \sqrt{|i|w| - \vec{v} \vec{k}} \]

Limitations
- strong coupling
- No quasiparticle at H.S.
- Large N exp. not under control
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**AdS/CFT Minimal Structure**

Large $N$ gauge theory

- $d$ dimensions

Classical gravity

- $d + 1$ dimensions

---

**aAdS$_{d+1}$ spacetime**

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2}dr^2$$

Scalars $m^2 = \Delta(\Delta - d)$

Warp Factor

$$r = 0 \quad r \to \infty$$

Conformal Field Theory

$$\phi(r, x_\mu) = \begin{cases} 
\phi_0/r^{\Delta-} & \text{source} \\
O/r^{\Delta+} & \text{vev}
\end{cases}$$

$$\langle \exp \int \phi_0 O \rangle_{CFT} = Z_{\text{Grav}}(\phi_0)$$

AdS

$$\{t, \vec{x}\} \to \lambda \{t, \vec{x}\} \quad r \to r/\lambda$$

CFT

$$\{t, \vec{x}\} \to \{t, \vec{x}\}/s \quad E \to s \ E$$
**AdS/CFT minimal structure**

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- $d + 1$ dimensions

AdS$_{d+1}$ spacetime

$$ds^2 = \frac{r^2}{L^2} (-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} dr^2$$

Scalars $m^2 = \Delta(\Delta - d)$

Warp Factor

Conformal Field Theory

$r = 0 \quad r \rightarrow \infty$

AdS $\{t, \vec{x}\} \rightarrow \lambda \{t, \vec{x}\}$

CFT $\{t, \vec{x}\} \rightarrow \{t, \vec{x}\}/s$ $E \rightarrow s \ E$

$$\langle \exp \int \phi_0 \mathcal{O} \rangle_{CFT} = e^{-N^2 S_{\text{class}}(\phi)}$$

$$\phi(r, x_\mu) = \begin{cases} 
\phi_0/r^{\Delta-} & \text{source} \\
\mathcal{O}/r^{\Delta+} & \text{vev}
\end{cases}$$
**AdS/CFT minimal structure**

Large $N$ gauge theory  
$d$ dimensions

Classical gravity  
$d + 1$ dimensions

AdS$_{d+1}$ spacetime

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2}dr^2$$

gauge boson $A = A_\mu dx^\mu$

Warp Factor  
$r = 0$  
$r \to \infty$

Conformal Field Theory

AdS  
$\{t, \vec{x}\} \to \lambda \{t, \vec{x}\}$  
$r \to r/\lambda$

CFT  
$\{t, \vec{x}\} \to \{t, \vec{x}\}/s$  
$E \to s\ E$

$$\left\langle \exp \int (S_0^\mu J_\mu) \right\rangle_{CFT} = e^{-N^2 S_{\text{class}}(A)}$$
Basic Lagrangian density

\[ \mathcal{L}[g_{\mu\nu}, A_\mu, \zeta] = \frac{1}{2\kappa^2} \left[ \mathcal{R} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \zeta \overline{D^\mu \zeta} - V(|\zeta|) \right] \]

The ansatz

\[ ds^2 = -g(r)e^{-\chi(r)} dt^2 + \frac{dr^2}{g(r)} + r^2 d\vec{x}_d^2, \]

\[ A_\mu dx^\mu = \Phi(r) dt \]

\[ |\zeta| = \eta(r) \]

\[ g(r_h) \quad \text{Eq. I order for } g \]
\[ \chi(r_h) \quad \text{Eq. I order for } \chi \]
\[ \Phi(r_h), \Phi'(r_h) \quad \text{Eq. II order for } \Phi \]
\[ \eta(r_h), \eta'(r_h) \quad \text{Eq. II order for } \eta \]

\[ \rightarrow 6 + 1 \text{ parameters} \]
Basic Lagrangian density

\[ \mathcal{L}[g_{\mu\nu}, A_\mu, \zeta] = \frac{1}{2\kappa^2} \left[ \mathcal{R} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \zeta \overline{D^\mu \zeta} - V(|\zeta|) \right] \]

The ansatz

\[ ds^2 = -g(r) e^{-\chi(r)} dt^2 + \frac{dr^2}{g(r)} + r^2 \, d\vec{x}_d^2, \quad A_\mu \, dx^\mu = \Phi(r) \, dt \]

\[ |\zeta| = \eta(r) \]

---

Eq. I order for \( g \) \hfill 7 - 2 = 5
Eq. I order for \( \chi \)
Eq. II order for \( \Phi \) \hfill 2 scalings
Eq. II order for \( \eta \)

\( \rightarrow \) 6 + 1 parameters

- fix the value of \( r_h \)
- \( \chi(r_h) = \chi_0 \)
The ansatz

\[ ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2 d\vec{x}^2, \quad A_\mu dx^\mu = \Phi(r)dt \]

\[ |\zeta| = \eta(r) \]

---

| Eq. I order for g | 5 − 2 = 3 |
| Eq. I order for \( \chi \) | |
| Eq. II order for \( \Phi \) | because \( g(r) \sim (r - r_h) \) |
| Eq. II order for \( \eta \) | \( \bullet \quad \Phi(r) \sim \Phi_0(r - r_h) \) |
| \( \rightarrow \text{6 + 1 parameters} \) | \( \bullet \quad \eta'(r_h) \propto \eta(r_h) \equiv \eta_0 \) |
**Basic Lagrangian density**

\[
\mathcal{L}[g_{\mu\nu}, A_\mu, \zeta] = \frac{1}{2\kappa^2} \left[ R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \zeta \overline{D^\mu \zeta} - V(|\zeta|) \right]
\]

The ansatz

\[
\begin{align*}
    ds^2 &= -g(r) e^{-\chi(r)} dt^2 + \frac{dr^2}{g(r)} + r^2 \, d\vec{x}_d^2, \\
    A_\mu \, dx^\mu &= \Phi(r) \, dt \\
    |\zeta| &= \eta(r)
\end{align*}
\]

---

| Eq. I order for g | $3 - 1 = 2$ |
| Eq. I order for $\chi$ | $g'(r_h)$ gives the temperature and depends on $(\eta_0, \Phi_0)$ |
| Eq. II order for $\Phi$ | |
| Eq. II order for $\eta$ | $(\eta_0, \Phi_0)$ to set the source $= 0$. |

\[\rightarrow 6 + 1 \text{ parameters}\]
**Two Simple Potentials**

\[ V = -6 - m^2 |\zeta|^2 \]

\[ V = -6 - m^2 |\zeta|^2 + \lambda |\zeta|^4 \]

AdS$_4$ $\rightarrow$ Domain Wall  
(Modulo Lifschitz solutions)

\[ T \approx 0.18 \rho^{1/2} \]
Scalar Manifold from $\mathcal{N} = 8 \ d = 4$

\[ \mathcal{L}_{\text{scalar}} = \frac{1}{2} \partial_{\mu} \eta \partial_{\mu} \eta - J(\eta)A_{\mu}A^{\mu} - V(\eta) \]

$U(1)^4 \rightarrow U(1)$
STU $\oplus$ inc. hypers

$J_1 = \sinh^2 \left( \frac{\eta}{2} \right)$
$V_1 = -2(2 + \cosh \eta)$

Unexpected condensation

$J_2 = \frac{1}{4} \sinh^2(\eta)$
$V_2 = \frac{1}{2} \sinh^4 \left( \frac{\eta}{2} \right) + V_1$

Similar to the Abelian Higgs model
Scalar Manifold from $\mathcal{N} = 8 \ d = 4$

\[ \mathcal{L}_{\text{scalar}} = \frac{1}{2} \partial_\mu \eta \partial_\mu \eta - J(\eta) A_\mu A^\mu - V(\eta) \]

$U(1)^4 \rightarrow U(1)$

STU $\oplus$ inc. hypers

$\langle O \rangle$

Retrograde Condensate

$J_2 = \frac{1}{4} \sinh^2(\eta)$

$V_2 = \frac{1}{2} \sinh^4 \left( \frac{\eta}{2} \right) + V_1$

Similar to the Abelian Higgs model
Summary

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The $\mathcal{N} = 2$ supergravity setup: Bosonic Sector

- graviton multiplet = \{ gravity $\oplus$ graviphoton \}.
- Hyperscalars $q^u$ parametrize a $Q$ Kahler manifold: $(J^x)_u^v$, $H_{uv}$
- The $SU(2)$ group of R-symmetry acts on the hyperscalars.
- The gauge group is introduced by gauging compact isometries (Killing vectors $K_u^\lambda$) of $Q$.

As a consequence of the gauging procedure, the Lagrangian gets a unique scalar potential.

$$V = 2g^2 \left( 4H^{uv} \partial_u \mathcal{W} \partial_v \mathcal{W} - 3\mathcal{W}^2 \right), \quad \mathcal{W} = \sqrt{\frac{2}{3} P^x P^x}$$

$$2n_H P^x_\lambda = (J^x)_u^v \nabla_v K^u_\lambda = (J^x)_u^v \left( \partial_v K^u_\lambda - \Gamma^u_{vp} K^p_\lambda \right)$$
The Universal Hypermultiplet $SU(2, 1)/U(2)$

$\dim_{\mathbb{R}} SU(2, 1)/U(2) = 4$. The coset space is top. a ball in $\mathbb{C}^2$

$$
\zeta_1 = \tau \cos \frac{\theta}{2} e^{i(\varphi + \psi)/2} \quad \zeta_2 = \tau \sin \frac{\theta}{2} e^{-i(\varphi - \psi)/2}
$$

$$
H_{uv} dq^u dq^v = \frac{d\tau^2}{(1-\tau^2)^2} + \frac{\tau^2}{4(1-\tau^2)} (\sigma_1^2 + \sigma_2^2) + \frac{\tau^2}{4(1-\tau^2)^2} \sigma_3^2
$$

Isotropy group is $U(2) = SU_R(2) \times U(1)$ with $U(1)_3 \subset SU_R(2)$

Result: the gauging of $U(1)_3$ builds the bridge

$U(1)^4 \rightarrow U(1)$

$\theta = \pi/2$

$SO(8)$

$SO(3) \times SO(3)$

$\theta = 0$

$R[\zeta_1] = +1$

$R[\zeta_2] = -1$

$\tau = \tanh \frac{\eta}{2}$
The Universal Hypermultiplet $SU(2, 1)/U(2)$

Idea: If $\theta$ can be fixed arbitrarily in the range $[0, \pi/2]$ we find a new family of solutions interpolating between $1 - 2$.

...from this point of view the retrograde condensate can be used as a clue to detect the existence of a bigger family of sol. $\rightarrow d = 5 \mathcal{N} = 8$

UV Asymptotics $\eta: \quad d = 4 \text{ and } m^2L^2 = -2 \rightarrow \Delta_- = 1 \text{ and } \Delta_+ = 2$

$$\eta(r) = \frac{O_1}{r} + \frac{O_2}{r^2} + \ldots, \quad \theta(r) = \theta_\infty + \frac{\xi}{r} + \ldots;$$

$O_2 = 0 \rightarrow \Delta = 1$ Condensate
The red dot is the saddle point $\theta = 0, \eta = 2 \text{arccosh}\sqrt{5}$.

- The red lines are the horizon values of the functions $\theta(r)$ and $\eta(r)$.
- The value of $\theta_{\infty}$ can be read on the vertical axis.

$\theta_{\infty} \neq 0$ the superconductor is driven towards $\theta(r_h) = \pi/2$ as $T \to 0$. 
\[ \Delta = 1 \text{ Condensates: Numerics} \]

- From bottom to top \( \theta_\infty = 0, 0.1, 0.25, 0.5, 0.75, 1 \)
- First order phase transitions for \( \theta_\infty > \theta_{cr} \approx .95 \).

(The curves should be represented in a 3d space according to their value of \( \theta_\infty \))
Extremal solutions for $\theta_\infty \neq 0$: Phase II

$+dz^2 + dr^2 + r^2 d\Omega_2$

$\cos \alpha = C_\eta$

$z/r = \pm \tan \alpha$

$dr^2 + r^2 \cos \alpha^2 d\Omega_2$

cone geometry

Hints:

- $\theta(r_h) \to \pi/2$

- Solitons with large mass converge to extremal Hol.SC.

$ds^2 = r^2(-dt^2 + d\vec{x}^2) + \frac{dr^2}{r^2 + C_\eta^2}$

$\eta(r) = 2 \text{arcsinh} \frac{C_\eta}{r}$

$\theta(r) = \frac{\pi}{2} + \delta_\theta r$

$\Phi(r) = \delta_\Phi r$
Extremal solutions for $\theta_\infty \neq 0$: Phase II

\[ -dz^2 + dr^2 + r^2 dM^2 \]
\[ \cosh \alpha = C_\eta \]
\[ z/r = \pm \tanh \alpha \]
\[ dr^2 + r^2 \cosh \alpha^2 dM^2 \]
cone geometry

2 Hints:
- $\theta(r_h) \rightarrow \pi/2$
- Solitons with large mass converge to extremal Hol.SC.

$\rho_{FT}$

UV AdS$_4$

\[ ds^2 = r^2(-dt^2 + d\vec{x}^2) + \frac{dr^2}{r^2 + C_\eta^2} \]
\[ \eta(r) = 2 \arcsinh \frac{C_\eta}{r} \]

$\theta(r) = \frac{\pi}{2} + \delta r$
$\Phi(r) = \delta r$
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Probing the IR geometry

- Entanglement Entropy

\[ S(\mathcal{E}) = -\text{Tr} \rho_{\mathcal{E}} \log \rho_{\mathcal{E}}, \quad \rho_{\mathcal{E}} = \text{Tr}_{\overline{\mathcal{E}}} |gs\rangle \langle gs| \]

- Excitations

vector type
conductivity

fermionic type
Probing the IR geometry

- Entanglement Entropy

\[ ds_{sp}^2 = \frac{L^2}{z^2} (dx^2 + dy^2 + U(z)dz^2) \]

\[ S(\mathcal{E}) = \frac{2\pi L^2}{\kappa^2} \text{Area}(\gamma\mathcal{E}) \]

\[ S(\mathcal{E}) = \frac{4\pi L^2}{\kappa^2} l_y \left(s + \frac{1}{\epsilon}\right) \]

\[ s = 2L^2 l_y \int_{\mathcal{E}}^{z_*} \frac{dz}{z^2} \sqrt{U(z) + x'(z)^2} \]

\[ \frac{l_x}{2} = \int_{\epsilon}^{z_*} \frac{dz}{z^2} \frac{z^2}{z_*^2} \sqrt{\frac{U(z)}{1 - (z/z_*)^4}} \]
Probing the IR geometry: EE at $T = 0$

From top to bottom
$\theta_\infty = 0.5, 1., 1.4.$

$\gamma(\mathcal{E})$

bulk min. surf.

$z_* \gg 1/C_\eta$

$U(z) \approx (C_\eta z)^{-2}$

$l_x/l_y = z_* \int_\epsilon^1 dz \frac{z^2}{\sqrt{1 - z^4}} U(z z_*)^{1/2}$

$l_x/2 \sim const \frac{1}{C_\eta}$
Probing the IR geometry: EE at $T = 0$

From top to bottom

$\theta_{\infty} = 0.5, 1., 1.4.$

$$s = 2L^2 l_y \int_\epsilon^{z_*} \frac{dz}{z^2} \sqrt{U(z)}$$

$$z_* \gg \frac{1}{C_\eta}$$
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\[ \eta(r) = O(r) \frac{r}{r} + \ldots, \quad \theta(r) = \theta_\infty + \frac{\xi}{r} + \ldots \]

\[ \Delta = 1 \text{ Condensate} \]

\[ \zeta_1(r) = \tanh \frac{\eta(r)}{2} \cos \frac{\theta(r)}{2}, \quad \zeta_2(r) = \tanh \frac{\eta(r)}{2} \sin \frac{\theta(r)}{2} \]
\[ \eta(r) = \frac{\mathcal{O}}{r} + \ldots, \quad \theta(r) = \theta_\infty + \frac{\xi}{r} + \ldots \]

\[ \Delta = 1 \text{ Condensate} \]

\[ \zeta_1(r) = \frac{\mathcal{O}_1^{(1)}}{r} + \frac{\mathcal{O}_1^{(2)}}{r^2} + \ldots, \quad \zeta_2(r) = \frac{\mathcal{O}_2^{(1)}}{r} + \frac{\mathcal{O}_2^{(2)}}{r^2} + \ldots \]

\[ \mathcal{O}_1^{(1)} = \frac{1}{2} \mathcal{O} \cos \frac{\theta_\infty}{2} \quad \mathcal{O}_1^{(2)} = -\frac{1}{4} \mathcal{O} \xi \sin \frac{\theta_\infty}{2} \]

\[ \mathcal{O}_2^{(1)} = \frac{1}{2} \mathcal{O} \sin \frac{\theta_\infty}{2} \quad \mathcal{O}_2^{(2)} = \frac{1}{4} \mathcal{O} \xi \cos \frac{\theta_\infty}{2} \]
The dictionary is
\[ \zeta_1 \rightarrow \Delta = 1 \text{ Op.} \]
\[ \zeta_2 \rightarrow \Delta = 2 \text{ Op.} \]
i.e. opposite quant.

- Double trace deformation! \( \mathcal{O}^{(1)}_2 = \lambda \mathcal{O}^{(1)}_1 \) and \( \mathcal{O}^{(2)}_1 = -\lambda \mathcal{O}^{(2)}_2 \)

The marginal coupling is \( \lambda = \tanh \frac{\theta_\infty}{2} \)

\[
\begin{align*}
\mathcal{O}^{(1)}_1 &= \frac{1}{2} \mathcal{O} \cos \frac{\theta_\infty}{2} \\
\mathcal{O}^{(1)}_2 &= \frac{1}{2} \mathcal{O} \sin \frac{\theta_\infty}{2} \\
\mathcal{O}^{(2)}_1 &= -\frac{1}{4} \mathcal{O} \xi \sin \frac{\theta_\infty}{2} \\
\mathcal{O}^{(2)}_2 &= \frac{1}{4} \mathcal{O} \xi \cos \frac{\theta_\infty}{2}
\end{align*}
\]
**Dual Field theory**

The dictionary is

\[
\begin{align*}
\xi_1 & \rightarrow \Delta = 1 \text{ Op.} \\
\xi_\dagger_2 & \rightarrow \Delta = 2 \text{ Op.}
\end{align*}
\]

i.e. opposite quant.

- Double trace deformation! \( O_2^{(1)\dagger} = \lambda O_1^{(1)} \) and \( O_1^{(2)} = -\bar{\lambda} O_2^{(2)\dagger} \)

Taking into account the charges \( \xi_1 \leftrightarrow \xi_\dagger_2 \). Then, \( \lambda = \tanh \frac{\theta_\infty}{2} e^{-i\psi} \)

- \( O_1^{(1)} = \frac{1}{2} O \cos \frac{\theta_\infty}{2} e^{i(\phi+\psi)/2} \), \( O_1^{(2)} = -\frac{1}{4} O \xi \sin \frac{\theta_\infty}{2} e^{i(\phi+\psi)/2} \)

- \( (O_2^{(1)})\dagger = \frac{1}{2} O \sin \frac{\theta_\infty}{2} e^{i(\phi-\psi)/2} \), \( (O_2^{(2)})\dagger = \frac{1}{4} O \xi \cos \frac{\theta_\infty}{2} e^{i(\phi-\psi)/2} \)
Marginal deformation

\[
\int d^3x \left( \lambda \mathcal{O}_1 \mathcal{O}_2 + \overline{\lambda} \mathcal{O}_1^\dagger \mathcal{O}_2^\dagger \right) \mathcal{O}_2^{(2)\dagger} = \frac{1}{2} \xi \mathcal{O}_1^{(1)} e^{-i\psi} .
\]

We interpret \( \theta(r) \) in an RG fashion: 

\[ \theta_\infty \neq 0 \text{ in the UV, drives the theory to } \theta(0) = \pi/2 \text{ in the IR.} \]

This value is associated to the confining background \( \rightarrow \) Issue on \( \lambda ! \)

\( \lambda \) exactly marginal but the confining IR phase has to be associated to a relevant def.

\[ \theta(r) = \theta_\infty + \frac{\xi}{r} + \ldots \]

\[ \xi = 0 \leftrightarrow \theta(r) = \text{const.} \]
Marginal deformation

\[ \sim \int d^3x \ (\xi O_1^\dagger O_1) \]  

\[ O_2^{(2)} = \frac{1}{2} \xi O_1^{(1)} e^{-i\psi}. \]

We interpret \( \theta(r) \) in an RG fashion:

\( \theta_\infty \neq 0 \) in the UV, drives the theory to \( \theta(0) = \pi/2 \) in the IR.

This value is associated to the confining background → Issue on \( \lambda \)!

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\[ \theta(r) = \theta_\infty + \frac{\xi}{r} + \ldots \]

\[ \xi = 0 \leftrightarrow \theta(r) = \text{const.} \]
Conclusions

We have found new holographic superconductor solutions in $\mathcal{N} = 8$ sugra where:

- The topology of the underlying $\mathcal{N} = 2$ model plays an important role.
- The back-reaction changes the IR physics of the solutions and this IR physics is geometrically understood.

It may be that a similar story holds for the 5D $\mathcal{N} = 8$. 

Thanks for the attention!!
We have find new holographic superconductor solutions in $\mathcal{N} = 8$ sugra where:

- The topology of the underlying $\mathcal{N} = 2$ model plays an important role
- The back-reaction changes the IR physics of the solutions and this IR physics is geometrically understood

It may be that a similar story holds for the 5D $\mathcal{N} = 8$.

Thanks for the attention!!