Non-Abelian T-duality in supergravity and the AdS/CFT correspondence

Daniel C. Thompson

V.U.B. Brussels

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Based on:

- A. Barranco, J. Gaillard, N. Macpherson, C. Núñez, DCT forthcoming
General settings and context – Motivation

Generating solutions in the context of AdS/CFT

- There exist a wide variety of solution generating techniques in supergravity.
- These techniques provide a powerful tool:
  - Matching $\beta$-deformations in gauge theory to the TST transformation in gravity.
  - U-duality transformations.
  - Fermionic T-duality & regular T-duality to explain dual superconformal symmetry at strong coupling.
- Some are well understood as exact string symmetry ($O(d,d;\mathbb{Z})$ dualities), others, like Fermionic T-duality, are probably not - but evidently can be very useful.
Strings and T-dualities in curved backgrounds

- **Abelian T-duality** is well understood and explored.
- **Non-Abelian T-duality?** [Fridling-Jevicki 84, Fradkin-Tseytlin-85], [de la Ossa-Quevedo 93, KS-94, Alvarez–Alvarez-Gaume–Lozano 94...] Not as well understood and **not** likely to be an **exact symmetry**.
- Major advances in the last two years:
  - Understanding how it acts on RR fluxes.

Generating solutions in type-II supergravities

- **Reducing** symmetry/supersymmetry (generically) controllably.
- New examples and features in AdS/CFT
  - It captures generic features of $\mathcal{N} = 2$ superconformal field theories corresponding to quiver (moose-type) diagrams.
  - New (non-singular) flows with $\mathcal{N} = 1$ supersymmetry.
Outline

► Constructing the backgrounds: General strategy
  ▶ The transformation of the NS fields.
  ▶ The induced Lorentz transformation.
  ▶ The transformation of the RR fluxes.

► Type-II backgrounds with $SO(4)$ isometry:
  ▶ In Principal Chiral Models (intermediate step).
  ▶ The general transformation rules.
  ▶ Supersymmetry.

► Non-Abelian T-duality and AdS/CFT: The road map.
  ▶ $\mathcal{N} = 2$ backgrounds, from D3-brane near horizon, M-theory lift and interpretation.
  ▶ $\mathcal{N} = 1$ backgrounds from conifolds, Conformal and non-conformal cases, fate of charges (central and brane).

► Concluding remarks.
Constructing the backgrounds: General strategy

Consider a type-II background with an isometry group $G$.

The transformation of NS fields

Write the 2-dim $\sigma$ model action for the NS fields

$$S(X) = \int d^2\sigma (G_{\mu\nu} + B_{\mu\nu}) \partial_+ X^\mu \partial_- X^\nu, \quad \text{also a } \Phi.$$ 

- **Gauge** $H \subset G$: gauge fields $A_\pm \in \mathcal{L}(H)$ and $\partial \to \nabla$,

  $$S_{\text{T-dual}}(X, \nu, A_\pm) = S_g(X, A_\pm) - i \int d^2\sigma \text{Tr}(\nu F_{+-}), \quad S_g(X, 0) = S(X).$$

  Invariant under the transformation of the $X^\mu$'s and

  $$A_\pm \to \Lambda^{-1}(A_\pm - \partial_\pm) \Lambda, \quad \nu \to \Lambda^{-1} \nu \Lambda.$$

- **Gauge fix** $\text{dim}(H)$ parameters in $X^\mu$ and in the $\nu$'s.
The gauge fields $A_{\pm}$ appear quadratically and non-dynamically. Integrating them out (in a [Buscher 87]–like procedure) gives the T-dual $\sigma$-model.

- Some of the Langrange multipliers become $\sigma$-model variables.
- The dilaton transformation is a 1-loop quantum effect.
- This procedure is a canonical transformation in phase space.
**Induced Lorentz transformation**

Using the frame of the original background construct

\[ e^a_{\pm} = e^a_{\mu} \partial_{\pm} X^\mu , \]

- Left and right world-sheet derivatives transform differently and give rise to different T-dual frames \( \tilde{e}^a_{\mu \pm} \).
- A Lorentz transformation relates the frames in the vector rep

\[ \tilde{e}^a_{\mu +} = \Lambda^a_{\; b} \tilde{e}^b_{\mu -} . \]

- Then compute the Lorentz transform in the spinor rep using

\[ \Omega^{-1} \Gamma^a \Omega = \Lambda^a_{\; b} \Gamma^b , \]

which preserves the Clifford algebra \( \{ \Gamma^a, \Gamma^b \} = 2\eta^{ab} \).
Transformation of Ramond fluxes

Use the democratic formulation of type-II [Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen 01]; All $F_p$'s $p = 0, 1, \ldots, 9$ appear. For Minkowski signature

Reduction conditions: $F_p = (-1)^{\lfloor \frac{p}{2} \rfloor} \ast F_{10-p}$,

reduce to the right degrees of freedom.

- We combine the forms RR-forms into a bi-spinor as

**IIB:** $P = \frac{e^\Phi}{2} \sum_{n=0}^{4} \frac{\mathcal{F}_{2n+1}}{(2n+1)!}$,  

**IIA:** $\hat{P} = \frac{e^\Phi}{2} \sum_{n=0}^{5} \frac{\hat{\mathcal{F}}_{2n}}{(2n)!}$,

where $\mathcal{F}_p = \Gamma_{\mu_1 \ldots \mu_p} F^\mu_1 \mu_2 \ldots \mu_p$.

- The above fluxes transform according to

$\hat{P} = P \Omega^{-1}$,

using the pure spinor superstring formalism of [Berkovits 00].
General comments on the T-dual background

Abelian T-duality:

- For $d$ successive ones

$$\Lambda = \text{diag}(\begin{pmatrix} -1_d & 1_{10-d} \end{pmatrix}) .$$

The spinorial representation is [Hassan 99]

$$\Omega = \prod_{i=1}^{d} (\Gamma^i \Gamma_{11}) ,$$

where $\Gamma_{11} = \Gamma^0 \Gamma^1 \cdots \Gamma^9$, obeying $\Gamma_{11}^2 = 1$.

- A single Abelian factor changes the chirality form type-IIA to type IIB and vice versa.
Non-Abelian T-duality:

- \( \dim(G) = \text{odd} \): \( \Omega \) starts with \( \Gamma_{11} \) followed by a linear combination of products of an odd number of \( \Gamma \)-matrices.
- \( \dim(G) = \text{even} \): Then \( \Gamma_{11} \) is omitted and the linear combination has products of an even number of \( \Gamma \)-matrices.
- Hence, chirality might change or stay the same.
- Massive IIA solutions, when \( \dim(G) \) equals the rank of a form.
- Equations of motion should be automatically satisfied.
- Supersymmetry:
  - This may be reduced to a fraction of the original.
  - Later we develop a criterion based on the action of the Lie-Lorrentz or Kosmann derivative on the Killing spinors of the original background.
Type-II backgrounds with $SO(4)$ isometry

Original background with $SO(4)$ isometry

We consider (for concreteness) type-IIB backgrounds:

▶ The NS fields are

\[
\begin{align*}
    ds^2 &= ds^2(M_7) + e^{2A} ds^2(S^3) , \quad B , \quad \Phi .
\end{align*}
\]

▶ The factor $A$, the 2-form $B$ and $\Phi$ may depend on $M_7$.

▶ The RR fluxes respecting the symmetry of the round $S^3$ are

\[
\begin{align*}
    F_5 &= G_2 \wedge \text{Vol}(S^3) - e^{-3A} \star_7 G_2 , \\
    F_3 &= G_3 - m \text{Vol}(S^3) , \\
    F_1 &= G_1 .
\end{align*}
\]

The $G_i$'s are lying entirely on $M_7$.

▶ The T-duality will be performed w.r.t. the $SU(2)_L$ subgroup of $SO(4) \sim SU(2)_L \times SU(2)_R$. 
Non-Abelian T-duality in Principal Chiral Models (PCM)

- The 2-dim $\sigma$-model action for a PCM is
  \[
  S(g) = - \int d^2\sigma \text{Tr}(g^{-1} \partial_- gg^{-1} \partial_+ g), \quad g \in G.
  \]

- It has an $G_L \times G_R$ global symmetry.
- Gauge the symmetry $G_L$ by introducing gauge fields $A_\pm$.
- The corresponding action is
  \[
  S_{\text{nonab}} = - \int d^2\sigma \text{Tr}(g^{-1} D_- gg^{-1} D_+ g) + i \text{Tr}(vF_{+-}),
  \]
  with $D_{\pm} g = \partial_{\pm} g - A_{\pm} g$ and $F_{+-}$ the field strength.
- Invariance under $G_L$ (local) $\times$ $G_R$ (global).
Gauge fix $g = \mathbb{1}$ and integrate out the $A_\pm$'s

$$S_{\text{T-dual}}(v) = \int d^2\sigma \, \partial_+ v_a (M^{-1})^{ab} \partial_- v_b ,$$

where

$$M_{ab} = \delta_{ab} + f_{ab} , \quad f_{ab} \equiv f_{ab}^c v_c .$$

Contribution to the Dilaton: $\Phi = -\frac{1}{2} \ln \det(M)$.

Original isometry $G_L \times G_R$ is broken to $G_R$.

Introduce coordinates $X^\mu \in g$ and the left-invariant forms $L^a$.

Then, the relation of world-sheet derivatives is

$$\partial_+ v_a = M_{ba} L^b_\mu \partial_+ X^\mu , \quad \partial_- v_a = -M_{ab} L^b_\mu \partial_- X^\mu .$$

These define two frames related by the Lorentz transformation

$$\Lambda_{ab} = -(MM^{-1}T)_{ab} .$$

Then

$$\Omega = \exp \left( \frac{1}{2} f_{ab} \Gamma^{ab} \right) \prod_{i=1}^{\dim(G)} (\Gamma_{11} \Gamma_i) .$$
T-dual background w.r.t. SU(2); NS-sector

- We specialize to the case of a PCM for SU(2) and
  - use spherical coordinates and
  - take into account $e^{2A}$.
- The NS-sector fields are given by

$$d\hat{s}^2 = ds^2(M_7) + e^{-2A}dr^2 + \frac{r^2e^{2A}}{r^2 + e^{4A}}ds^2(S^2),$$

$$\hat{B} = B + \frac{r^3}{r^2 + e^{4A}}\text{Vol}(S^2),$$

$$e^{-2\hat{\Phi}} = e^{-2\Phi} e^{2A(r^2 + e^{4A})},$$

- The round $S^2$ sphere appears; Manifest SU(2) symmetry.
\(T\)-dual background w.r.t. \(SU(2);\) RR-sector

- The Lorentz transformation matrix \(\Omega\) acting on the spinors is

\[
\Omega = \Gamma_{11} \frac{e^{2A} \Gamma_{789} + \mathbf{v} \cdot \Gamma}{\sqrt{r^2 + e^{4A}}} \quad \Rightarrow \quad \Omega^{-1} = \Gamma_{11} \frac{e^{2A} \Gamma_{789} - \mathbf{v} \cdot \Gamma}{\sqrt{r^2 + e^{4A}}}.
\]

- The isometry group is 3-dim and has “legs” along \(F_3\).
Hence we expect a massive IIA solution.

- Indeed, the massive IIA fluxes are

\[
\hat{F}_0 = m , \\
\hat{F}_2 = \frac{mr^3}{r^2 + e^{4A}} \text{Vol}(S^2) + rdr \wedge G_1 - G_2 , \\
\hat{F}_4 = \frac{r^2 e^{4A}}{r^2 + e^{4A}} G_1 \wedge dr \wedge \text{Vol}(S^2) - \frac{r^3}{r^2 + e^{4A}} G_2 \wedge \text{Vol}(S^2) \\
+ rdr \wedge G_3 + e^{3A} \star_7 G_3 .
\]
Supersymmetry

- Is any/how much Supersymmetry preserved? what conditions?
  *Preserved susy when Lie-Lorentz or Kosmann derivative vanishes*

\[
\mathcal{L}_\xi \epsilon = \xi^\mu D_\mu \epsilon + \frac{1}{4} D_\mu \xi_v \Gamma^\mu_v \epsilon,
\]

- Is there a mapping of the Killing spinor eqs/spinors?

\[
\hat{\epsilon}_1 = \Omega \epsilon_1, \quad \hat{\epsilon}_2 = \epsilon_2
\]

- The fraction of supersymmetry preserved is determined by the independent extra conditions needed to make the Kosmann derivative vanish.
Non-Abelian T-duality and AdS/CFT: The Road Map

$AdS_5 \times S^5; AdS_5 \times \frac{S^5}{\mathbb{Z}_2}$

Non-Abel T-dual

Type IIA & M theory geometry
(Caitto-Maldacena)

Mass deformation

$AdS_5 \times T^{1,1}$

Mass deformation

Type IIA & M-theory geometry
(Bah-Beem-Bobev-Wecht)

Fractional D3

New Massive IIA backgrounds

KT/KS

Baryonic Branch

D5 on $S^2$
\( \mathcal{N} = 2 \) backgrounds

**D3 near horizon**

- We write the \( S^5 \) of the original type-II\( B \) background as
  \[
  ds^2(S^5) = 4(d\theta^2 + \sin^2 \theta \, d\phi^2) + \cos^2 \theta \, ds^2(S^3) .
  \]

- The NS-part of the T-dual background has
  \[
  ds^2 = ds^2(\text{AdS}_5) + 4(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{dr^2}{\cos^2 \theta} + \frac{r^2 \cos^2 \theta}{\cos^4 \theta + r^2} d\Omega_2^2 ,
  \]
  a dilaton and an NS 2-form, as well as
  \[
  F_2 = -8 \cos^3 \theta \sin \theta \, d\theta \wedge d\phi ,
  \]
  \[
  F_4 = -8 \frac{r^3 \cos^3 \theta \sin \theta}{\cos^4 \theta + r^2} \, d\theta \wedge d\phi \wedge \text{Vol}(S^2) ,
  \]
- A type-II\( A \) solution with \( SO(4,2) \times SU(2) \times U(1) \) symmetry.
- Solution is singular at \( \theta = \pi/2 \) and 1/2 supersymmetric.
M-theory lift and gauge theory interpretation

$\mathcal{N} = 2$ superconf. quiver theories using D4,D6 and NS5-branes.

\[ \begin{array}{c}
\k_1 \quad \k_2 \quad \ldots \quad \k_N \\
d_1 \quad d_2 \quad \ldots \quad d_N 
\end{array} \]

► Notation:
Circular: $SU(k_n)$ gauge group. Square: $SU(d_n)$ global ($d_n$ fundamentals), Horizontal lines: bi-fundamental ($k_{n-1}, \bar{k}_n$).

► They admit 11-dim dual geometries containing $AdS_5$ factors and possessing $SU(2) \times U(1)$ isometry [Gaiotto-Maldacena 09],

► The details of the solution are fed up using solutions of

\[ (\partial_x^2 + \partial_y^2)\Psi + \partial_z^2 e^\Psi = 0 , \]

the continual Toda eq. [Boyer-Finley 82, Saveliev 89].
The relation to the quiver theories is very clear in the subclass having an additional $U(1)$ symmetry:

- Corresponds, via dim reduction, to a type-IIA solution.
- There is a mapping of the continual Toda to the Laplace eq.
- **Electrostatic problem**: Find the potential $V(\rho, \eta)$ of a semi-infinite charged line with density $\lambda(\eta)$.

![Figure: A charged line perpendicular to an infinite conducting plane.](image)

- $\lambda(\eta)$ is composed of linear segments with integer slopes.
- The **rank** of the gauge group $k_n = \lambda(n)$, with $n = 1, 2, \ldots$. Changes in slope correspond to extra fundamentals $d_n$. 
The non-Abelian T-dual captures **generic features**:

- Our solution can be cast into that form with
  \[
  \lambda(\eta) = \eta, \quad V(\rho, \eta) = \eta \ln \rho + \eta \left( \frac{\eta^2}{3} - \frac{\rho^2}{2} \right),
  \]

- Describes the **general geometry near the origin** (small \( \eta \)).

- **Change of the gauge group**:
  The original \( SU(N) \) has changed to \( \prod_{i=2} SU(i) \).
**N = 1 Backgrounds**

Remarks/Questions:

- Can the same amount of supersymmetry be preserved?
- Are there non-singular (non)-Abelian T-duals?

The answer is yes!

- D3-brane on a conifold singularity [Klebanov-Witten 98],
  
  N=1 SCFT with $SU(N) \times SU(N)$ gauge group.

  - The gravity dual is $AdS_5 \times T_{1,1}$, with
    
    $$T_{1,1} = \frac{SU(2) \times SU(2)}{U(1)}.$$  

  The symmetry group is $SO(4, 2) \times SU(2) \times SU(2) \times U(1)$. 
Up lift of the non-Abelian dual of the KW
The non-Abelian T-dual is a type-IIA with $g$, $B$, $\Phi$, $F_2$ and $F_4$.

Uplift to 11-dims:

- The metric

\[ ds^2 = \Delta^{1/3} \left( ds_{AdS_5}^2 + \lambda_1^2 (\sigma_1^2 + \sigma_2^2) \right) + \Delta^{-2/3} \left[ (x_1^2 + \lambda^2 \lambda_1^2) dx_1^2 + (x_2^2 + \lambda_1^4) dx_2^2 + 2x_1 x_2 dx_1 dx_2 + \lambda^2 \lambda_1^2 x_1^2 \sigma_3^2 + (dx_\# + \frac{\sigma_3}{27})^2 \right], \]

where $\Delta = \lambda_2^2 x_1^2 + \lambda^2 (x_2^2 + \lambda_2^4)$ and $\lambda = \frac{1}{3}$, $\lambda_1 = \lambda_2 = \frac{1}{\sqrt{6}}$.

- There is also an $F_4$ flux.

- Preserves $\mathcal{N} = 1$ superconformal. In the class of [Gauntlett-Martelli-Sparks-Waldram 04] and [Bah-Beem-Bobev-Wecht 12].
Susy preserved and non singular

- The Kosman derivative of the original $T^{1,1}$ Killing spinors vanishes
  \[ \mathcal{L}_{k_1} \epsilon \sim \mathcal{L}_{k_2} \epsilon \propto (\Gamma^{12} + \Gamma^{34}) \epsilon \sim 0, \quad \mathcal{L}_{k_3} \epsilon = 0, \]

- Singularities associated to fixed points of the isometry (c.f. polar $U(1)$ duality in $R^2$), points where the norms of the Killing vectors vanish. But
  \[ |k_i|^2 > 0 \]

- Subtlety: Removable bolt singularity dictates a halving of the range of angular coordinate $\psi$: possible sign of an orientifold?

- Preserves $\mathcal{N} = 1$ superconformal.
Can we brake conformal invariance but retain supersymmetry?

- Include $M$ fractional D3-branes by turning $H_3$, $F_3$ and a $\Phi$ [Klebanov-Nekrasov 99, Klebanov-Tseytlin 00, Klebanov-Strassler 00].

- Theory becomes non-conformal $\mathcal{N} = 1$ with $SU(N + M) \times SU(N)$ gauge group.

- An RG flow which is non-singular at all scales.

- For simplicity concentrate on the UV regime in which

\[
ds^2 = e^{-5q} ds_5^2 + \frac{1}{6} e^{2f+3q} \sum_{i=1}^{2} (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) + \frac{1}{9} e^{3q-8f} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 ,
\]

where

\[
ds_5^2 = du^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu .
\]

Also, there is a self-dual $F_5$, $\Phi = \text{const.}$ and

\[
B = P T(u) [\text{Vol}(S_1^2) - \text{Vol}(S_2^2)] , \quad F_3 \sim P [\text{Vol}(S_1^2) - \text{Vol}(S_2^2)] \wedge (d\psi + \cdots).
\]

The constant $P \sim M/N$. 
Important characteristics:

- $f, q, T$ and $A$ satisfy a non-linear system of eqs.; Solvable.
- The $R$-symmetry corresponds to shifts of $\psi$.
- The Killing spinors do not depend on $\psi$.

What happens when T-duality acts?

- Abelian T-duality: Will be a solution of type-IIA.
  - w.r.t. $\psi$ will break supersymmetry and
  - w.r.t. to any $\phi_i$ will lead to a singular background.
- Non-Abelian T-duality: Will be a solution of massive type-IIA.
  - Will have $F_0, F_2$ and $F_4$ turned on. In particular,
    \[ F_0 \sim M, \]
    the Romans’ mass is quantised.
  - The global symmetry is $SU(2) \times U(1)_\psi$.
  - Non-singular with unbroken Supersymmetry.
Fate of the central charge

- The central charge: For the metric

\[ ds^2 = \alpha(u) dx^2_{1,3} + du^2 + g_{ij}(x, u) dx^i dx^j , \]

it is of the form

\[ c \sim \frac{H^{7/2}}{\alpha^{3/2}(H')^3} , \quad H = \alpha^3 V_{\text{int}}^2 , \quad V_{\text{int}} = \int d^5 x e^{-2\Phi} \sqrt{\det g} . \]

Non-increasing towards the IR [Girardello et. al. 98].

- Invariant of T-duality up to a single RG scale background independent coefficient.

- Invariance of the ratio

\[ \frac{c_{\text{IR}}}{c_{\text{UV}}} = \frac{27}{32} , \]

for the RG flow from \( AdS_5 \times S^5 / \mathbb{Z}_2 \) to \( AdS_5 \times T_{1,1} \) as well as for their T-duals.
Fate of the brane charges

There are three type of charges (for a review, see [Marolf 00]).

<table>
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<td>No</td>
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<td>No</td>
<td>Yes</td>
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Maxwell:

Before : \[ Q_{D3} \sim \int_{\theta_i, \phi_i, \psi} F_5 \sim N \ln r , \quad Q_{D5} \sim \int_{\theta_1, \phi_1, \psi} F_3 = M , \]

After : \[ Q_{D6} \sim \int_{\theta_1, \phi_1} F_2 \sim N \ln r , \quad Q_{D8} = M . \]

Page:

Before : \[ Q_{D3} \sim \int_{\theta_i, \phi_i, \psi} (F_5 - B \wedge F_3) = N_0 , \quad Q_{D5} \sim \int_{\theta_1, \phi_1, \psi} (F_3 - B \wedge F_1) = M \]

After : \[ Q_{D6} \sim \int_{\theta_1, \phi_1} (F_2 - F_0 B) = N_0 , \quad Q_{D8} = M . \]

Non-singular number of “mobile” D3 \( N_0 = 0 \)
Seiberg duality

To compute the change in the gauge group along the RG flow one computes the effective brane charges. Following a procedure as in [Benini-Canoura-Cremonesi-Nunez-Ramallo 07]:

- This can be done by realizing that a shift in the NS charge as 
  \[ b_0 \rightarrow b_0 + n, \quad b_0 = \frac{1}{4\pi^2} \int_{\Sigma^2} B, \]
  leaves the string theory invariant.

- Can be compensated by a shift in the holographic coordinate which changes the Maxwell charges.

- At a fixed energy scale perform a large gauge transformation by changing \( B \rightarrow B + \Delta B \). This affects the Page charges.

- Both procedures give the same result. It turns out that a change of \( \Delta Q_{D3} = M \) units in the KT case, induces a change of \( \Delta Q_{D6} = 2M \) after the transformation.
**Other probes**

We can consider further observables and also probe IR physics (using dual of full KS or KS+bb or Wrapped D5)

- Domain walls in IR: matching of effective tension \( \text{wrapped } D5 \rightarrow \text{unwrapped } D2 \)
- Wilson loop/\( q\bar{q} \) potential preserved
- ’t Hooftline/monopole potential not preserved but still becoming effective tension goes to zero in IR (wrapped \( D3 \rightarrow \text{wrapped } D4 \) on \( \Sigma_2 \))
- Gauge coupling (instanton action of euclidean \( D1 \) brane on \( \Sigma_2 \) \( \rightarrow \) euclidean \( D3 \) brane on \( \Sigma_3 \)) \( 1/g^2 \rightarrow \rho \) in IR
G-structures

KW, KT etc. $\mathcal{N} = 1$ backgrounds: $\Omega_3$ and $J_2$ or $SU(3)$ structure.

$$\Phi_1 = \Omega_3, \quad \Phi_2 = e^{-ij}, \quad (d + H)\Phi_1 = 0, \quad (d + H)\Phi_2 = F_{RR}$$

These pure spinors transform like RR fluxes:

$$e^\phi \Phi_i = e^{\hat{\phi}} \hat{\Phi} \cdot \Omega^{-1}$$

Shows the duals produced above are $SU(2)$ structure and $\mathcal{N} = 1$ SUSY

$$\hat{\Phi}_1 = e^{-v^w} \wedge \omega_2, \quad \hat{\Phi}_2 = e^{-ij} \wedge (v + iw)$$
Flavours

- Add D7 flavour branes to KW (susy callibration conditions [Areán et al., Martucci and Smyth])
- Beyond probe/quenched limit $N_f \sim N_c$ by smearing encoded by a smearing form

$$\Xi_2 = -N_f \left( \sin \theta d\theta \wedge d\phi + \sin \tilde{\theta} d\tilde{\theta} \wedge d\tilde{\phi} \right)$$

- Modified Bianchi identities $dF_1 = \Xi_2$ and accommodate back reaction with this ansatz

$$ds^2 = e^{\Phi_2} \sqrt{h} \left( dr^2 + \lambda_1^2 e^{2g} \left( \sin^2 \theta d\phi^2 + d\theta^2 \right) + \lambda_2^2 e^{2g} \left( \sigma_3^2 + \sigma_4^2 \right) + \lambda_1^2 e^{2f} \left( \sigma_3 + \cos \theta d\phi \right)^2 \right) ,$$

$$F_1 = \frac{N_f}{4\pi} \left( \sigma_3 + \cos \theta d\phi \right) , \quad F_5 = (1 + *) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge Kdr \ .$$

- $1^{st}$ order BPS equations which can be solved!
Flavours: The T-dual

- Can T-dualise as before, complicated geometry but $SU(2)$ structure and SUSY
- Source D7s $\rightarrow$ source D6 and D8.
- T-dual smearing forms transform similar to RR fluxes:
  \[ e^{\Phi\Xi} = e^{\hat{\Phi}} e^{B^\wedge \hat{\Xi}} \]
- T-dual Bianchi Identities
  \[ (d - \hat{H}) \wedge \hat{F} = e^{\hat{B}^\wedge \hat{\Xi}} \]
- Can now add flavours to all the interesting IIA backgrounds described.
- Side remark - change in $B$ redistributes $D4$, $D6$ charges
$Y^{p,q}$ and its T-dual

- $Y^{p,q}$ are an infinite class of Sasaki-Einstein manifolds [Gauntlett et al.]
- $AdS_5 \times Y^{p,q}$ are an infinite class of AdS-CFT dual pairs in type IIB [Martelli and Sparks, Benvenuti et al., Franco et al.]
- Isometry group $SU(2) \times U(1) \times U(1)$ so can dualise w.r.t. $SU(2)$
- Kosmann vanishes and Killing norm non-zero $\Rightarrow$ susy and smooth
- Result: a new infinite class of solutions in type IIA with dynamic $SU(2)$ structure and M-theory lifts!
- Metric retains many of the features of the dual KW and supported by $B, F2, F4$
Concluding remarks

1. Non-Abelian T-duality extended to type II supergravity
2. $AdS_5 \times T_{11}$ and its deformations give new smooth solutions with $\mathcal{N} = 1$ in (massive) IIA & M-theory
3. Probes indicate duality cascade and confining
4. Classified solutions in terms of $SU(2)$ structure
5. Showed how to add flavour branes beyond the quenched approximation
6. A new infinite class of M-theory solutions from dualising $Y^{p,q}$

- Opens Qs: better handle on gauge theory; integrability; global issues; generalised geometry