Free Gravitons Break de Sitter Invariance (arXiv:0907.4930, 1002.4037)

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Spacetime Exp. Strengthens QFT

- Why?
  - Loops → classical physics of virtuals
  - Expansion → holds virtuals apart longer
- Maximum Effect for
  - Inflation
  - M=0
  - No conformal invariance (classically)
- Two Particles
  - MMC scalars
  - gravitons

Primordial Inflation was nearly de Sitter with small GH^2

- $ds^2 = -dt^2 + a^2(t)\, dx.dx$
  - $H(t) = \dot{a}/a$ & $\varepsilon(t) = -\dot{H}/H^2$
- For single-scalar inflation with $k = H(t_k)a(t_k)$
  - $\Delta^2_{\mathcal{R}}(k) \approx GH^2(t_k)/\pi\varepsilon(t_k)$ & $\Delta^2_h(k) \approx 16GH^2(t_k)/\pi$
- WMAP data for k = .002/Mpc
  - $\Delta^2_{\mathcal{R}} = 2.441 \times 10^{-9}$ & $r = \Delta^2_h/\Delta^2_{\mathcal{R}} < 0.22$
- Hence
  - $\varepsilon \approx r/16 < 0.014$ (even smaller before t_k!)
  - $GH^2 \approx \pi/16 \times r \times \Delta^2_{\mathcal{R}} < 10^{-10}$

MMC Scalar Models

1. $\lambda\varphi^4$ (Brunier, Kahya, Onemli)
   - $M^2(x;x') \rightarrow \Delta u(t,k)$ & $\langle T_{\mu\nu} \rangle$
   - Growing scalar mass & pos. vac. Energy
2. SQED (Kahya, Prokopec, Tornkvist, Tsamis)
   - $M^2(x;x') \rightarrow \Delta u$ & $[{}^{\mu}\Pi^{\nu}](x;x') \rightarrow \Delta\varepsilon_{\mu}$
   - $\langle \varphi^* \varphi \rangle$, $\langle (D_\mu \varphi)^* D_\nu \varphi \rangle$, $\langle F_{\mu\nu} F_{\rho\sigma} \rangle$ & $\langle T_{\mu\nu} \rangle$
   - Growing photon mass & neg. vac. Energy
3. Yukawa (Duffy, Prokopec, Miao)
   - $M^2(x;x') \rightarrow \Delta u$, $\Sigma(x;x') \rightarrow \Delta u$ & $\langle \varphi \psi^\dagger \gamma^0 \psi \rangle$
   - Growing fermion mass & neg. vac. Energy

Quantum Gravity Models

1. QG + Dirac (Miao)
   - $[{}_i\Sigma_j](x;x') \rightarrow \Delta u(t,k)$
   - Growing fermion field strength
2. QG + MMC Scalar (Kahya,Park)
   - $M^2(x;x') \rightarrow \Delta u(t,k)$
   - $[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x') \rightarrow \Delta\varepsilon_{\mu\nu}$ & force of gravity
   - Possible tilt in Power Spectrum
3. QG (Tsamis,Mora)
   - $[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ & $\langle h_{\mu\nu} \rangle$
   - Consistent with relaxation of Λ

Enhanced QFT as IR Logs

- What? ➔ factors of ln(a) = Ht
  - Eg $\rho = \lambda(H/2\pi)^4 \times \frac{1}{8} \ln^2(a) + O(\lambda^2)$
- Through propagators
  - $i\Delta(x;x') = (\text{dS inv}) + H^2/8\pi^2 \ln(aa')$
  - $i[{}_{ij}\Delta_{k\ell}](x;x') = [2\delta_{i(k}\delta_{\ell)j} - 2\delta_{ij}\delta_{k\ell}] \times \text{same}$
- Also from vertex integrations
  - $\int^t dt' \, 1 = t = \ln(a)/H$
  - NB occur even if no dS breaking in iΔ!

Math Guys ***Hate*** IR logs

- Reluctantly accept in $i\Delta(x;x')$
  - But struggle to avoid consequences
- But deny in $i[{}_{ij}\Delta_{k\ell}](x;x')$
- NB vertex integrations still break dS

  $\int d^4x' \sqrt{-g(x')}\ \theta(x^0 - x'^0)\ \theta[-\ell^2(x;x')]$

  $= \int^t dt'\ a^3(t')\ x\ 4\pi/3H^3\ (1/a' - 1/a)^3$

  $= 4\pi/3H^4\ [\ln(a) + O(1)]$
- But simplest IR logs come from props

dS Inv Eqns Don't Always Have Invariant Solutions

- MMC φ: $\square i\Delta(x;x') = i\delta^4(x-x')/\sqrt{-g}$
  - Allen & Folacci, PRD35 (1987) 3371.
- $ds^2 = -dt^2 + a^2(t)\, dx\cdot dx \qquad a(t) = e^{Ht}$
  - $i\Delta(x;x') = \int d^3k/(2\pi)^3\, e^{k\cdot(x-x')}$

    $\times[\theta(t-t')u(t,k)u^*(t',k) + \theta(t'-t)u^*(t,k)u(t',k)]$
  - $u(t,k) = H/(2\,k^3)^{1/2}\,[1 - ik/Ha]\,\mathrm{Exp}[ik/Ha]$
  - IR problem: $uu^* \sim H^2/2k^3$

What about $i[_{\mu\nu}\Delta_{\rho\sigma}](x;x')$? Cosmologists: not invariant

- Grishchuk (Sov. Phys. JETP 40 (1975) 409)
  - Gravitons have same u(t,k) as MMC φ
- This IS observable!

  $\Delta^2_h = k^3/2\pi^2 \int d^3x\, e^{ik\cdot x} \langle h_{ij}(t,x)\, h_{ij}(t,0)\rangle$

  $= k^3/2\pi^2 \times 32\pi G \times 2 \times |u(t,k)|^2$

  $= 16/\pi\, GH^2$ (a.k.a. SCALE INVARIANCE)
- Kleepe (PLB 317B (1993) 305)
  - Comp. trans. does not restore invariance

What about $i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x')$? Math Physicists: Yes it is!

- Add $\alpha(D^{\nu}h_{\nu\mu} + \beta D_{\mu}h^{\nu}{}_{\nu})^2$
- Solve in Euclidean space & continue
- Ok except few "singular" choices of α and β

Burden of my Talk: Math Physicists are wrong

- Obstacle to adding gauge fixing term
- Obstacle to analytic continuation
- Origin of "singular" gauges

“Exact” vs “Average” Gauges

- Illustrate with EM in flat space
  - Exact: $\partial_i A_i = 0$ (Coulomb)
  - Average: $\mathcal{L} \rightarrow \mathcal{L} - \frac{1}{2} (\partial_\mu A^\mu)^2$ (Feynman)
- Derive Average from Exact
  - Start in canonical functional formalism

    $\int [dE^T]\,[dA^T]\, e^{iS_{fixed}}$
  - S. Coleman, Erice 1973

Coleman's Seven Steps

1. Integrate out E^T
2. Use $\int[dA^T] = \int [dA]\, \delta[\partial_i A_i] \sqrt{\det[\partial_i \partial_i]}$
3. Undo A_0 constraint
4. Write integrand as invariant
5. $\delta[\partial_\mu A^\mu]$ w field dependent gauge trans
6. $\delta[\partial_\mu A^\mu - f(x)]$ w C-number gauge trans
7. Multiply by $\int[df]\, \mathrm{Exp}[-\tfrac{1}{2} i \int f^2]$

Obstacle on $T^3 \times R$

- Invariant: $\partial_i F^{i0} = J^0 \rightarrow Q = 0$
- Feynman: $[-\partial_t^2 + \partial_i \partial_i] A^0 = J^0 \rightarrow Q \neq 0$ ok
- Problems at Coleman's steps 2 & 3
  - No 0-modes for $\delta[\partial_i A_i]$ and A_0
  - Hence no 0-mode for gauge fixing term
- Same Obstacle on de Sitter
  - IR ∞ of $\varphi\varphi^*$ self-energy (gr-qc/0508015)

Analytic Continuation Sees Only Logarithmic IR Divergences

- $[\square - M^2] i\Delta(x;x') = i\delta^4(x-x')/\sqrt{-g}$

  $\rightarrow i\Delta(x;x') = \int d^3k/(2\pi)^3 \, e^{ik\cdot(x-x')}$

  $\times[\theta(t-t')u(t,k)u^*(t',k)+\theta(t'-t)u^*(t,k)u(t',k)]$
- $u(t,k) = [\pi/(4Ha^3)]^{1/2} \, H_\nu^{(1)}[k/Ha]$
  - $\nu = [9/4 - M^2/H^2]^{1/2}$
  - $uu^* \sim k^{-2\nu} \, [1 + O(k^2)]$
- IR ∞'s for $2\nu \geq 3 \rightarrow M^2 \leq 0$
  - But only logarithmic for $M^2 = -N(3+N) \, H^2$

IR ∞'s Signal Wrong Physics

- DON'T subtract them, fix the physics
  - Exclusive → Inclusive in flat QED, QCD & QG
- Physical Problem:
  - Can't enforce Bunch-Davies for $k < Ha_{initial}$
- Standard Fixes
  - Vilenkin (NPB:226,527,1983)

    → Change Bunch-Davies for $k < Ha_{initial}$
  - NCT and RPW (CQG:11,2969,1994)

    → Keep Bunch-Davies on T^3xR with no $k < Ha_{initial}$

How It Works In Practice

- $i\Delta_{naïve}(x;x') = \int d^3k/(2\pi)^3\, e^{k\cdot(x-x')}$
  $\times[\theta(t-t')u(t,k)u^*(t',k) + \theta(t'-t)u^*(t,k)u(t',k)]$
- Just cut off IR → $\int d^3k/(2\pi)^3\, e^{k\cdot(x-x')}\, \theta(k-k_0)$ x (Same)
- Resolves old problem of Ford & Parker (1977)
  - Scalar-driven FRW
    Iliopoulos,Tomaras,NCT,RPW (NPB:534,419,1998)
  - MMC scalars on FRW with constant ε
    Janssen,SPM,Prokopec,RPW (CQG:25,245013,2008)
  - de Sitter with $M^2_S = -N(N+3)H^2$ and $M^2_V = -(N+1)(N+3)H^2$
    SPM,NCT,PRW (JMP:51,072503,2010)

Why Not Use the Subtracted Solutions for Power Law IR ∞'s?

- $[\Box - M^2]\, i\Delta(x;x') = i\delta^4(x-x')/\sqrt{-g}$
  but $i\Delta(x;x') \neq \langle\psi|T[\varphi(x)\varphi(x')]|\psi\rangle$
- Eg $i\Delta(x;x') \rightarrow i\, G_{ret}(x;x')$
  - Vanishes for $x'=x$ vs $\langle\psi|\varphi^2|\psi\rangle \neq 0$
- SHO: $-m[(d/dt)^2+\omega^2]\, i\Delta(t;t') = i\delta(t-t')$
  $-i \sin[\omega|t-t'|]/2m\omega + \alpha \cos(\omega t)\cos(\omega t')$
  $+ \beta \sin[\omega(t+t')] + \gamma \sin(\omega t)\sin(\omega t')$
  - Solves for any α, β & γ, but QM requires
    $\alpha + \gamma \geq 1/2m\omega$ and $\alpha\gamma \geq \frac{1}{4}\beta^2$
- Math: Reflection Positivity fails

Exact de Donder Gauge

- $\mathrm{mtrc}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$ with $D^{\nu} h_{\nu\mu} = \frac{1}{2} D_{\mu} h^{\nu}{}_{\nu}$
- $\mathcal{D}_{\mu\nu}{}^{\alpha\beta}\, i[{}_{\alpha\beta}\Delta_{\rho\sigma}](x;x') \neq g_{\mu(\rho} g_{\sigma)\nu}\, i\delta^4(x-x')$
  - Not consistent with gauge condition
- $\mathrm{rhs} = [g_{\mu(\rho} g_{\sigma)\nu} - \frac{1}{2}\, g_{\mu\nu} g_{\rho\sigma}]\, i\delta^4(x-x')$
  $+ 2\sqrt{-g}\ \mathrm{Sym}\{D_{\mu} D_{\rho}'\, i[{}_{\nu}\Delta_{\sigma}](x;x')\}$
- $[\square + 3H^2]\, i[{}_{\mu}\Delta_{\nu}](x;x') = g_{\mu\nu}\, i\delta^4(x-x')/\sqrt{-g}$
  - Corresponds to $M_V^2 = -6H^2$
  - IR ∞'s $M_V^2 \leq 0$, Log $M_V^2 = -(N+2)(N+3)H^2$

Scalar Structure Functions in $i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x')$

- Spin 0 Part: $\mathcal{P}_{\mu\nu}(x)\ \mathcal{P}_{\rho\sigma}(x')\ \mathcal{F}_{0(}x;x')$
  - $\mathcal{P}_{\mu\nu} = D_\mu D_\nu + \frac{1}{2}\,[\square+6H^2]\,g_{\mu\nu}$
  - $\frac{3}{4}[\square+4H^2][\square+6H^2]^2\ \mathcal{F}_0(x;x') = i\delta^4(x-x')/\sqrt{-g}$
  - $M^2 = -4H^2$ is Log ∞, $M^2 = -6H^2$ is power ∞
- Spin 2 Part: $\mathcal{P}_{\mu\nu}{}^{\beta\delta}(x)\ \mathcal{P}_{\rho\sigma}{}^{\kappa\theta}(x')\ [\mathcal{T}_{\beta\kappa}\mathcal{T}_{\delta\theta}\mathcal{F}_2]$
  - $C^{\alpha\beta\gamma\delta} = \mathcal{P}_{\mu\nu}{}^{\alpha\beta\gamma\delta}\,h^{\mu\nu} + O(h^2)$
  - $\mathcal{P}_{\mu\nu}{}^{\beta\delta} = -1/2H^2\ \mathcal{P}_{\mu\nu}{}^{\alpha\beta\gamma\delta}\,D_\alpha D_\gamma$
  - $\mathcal{T}_{\beta\kappa} = -1/2H^2\ \partial^2 y/\partial x^\beta \partial x'^\kappa$ $\quad y = aa'H^2(x-x')^2$
  - $\square^3[\square-2H^2]^2\ \mathcal{F}_2 = 64\,H^4\,i\delta^4(x-x')$

Conclusion: Graviton Propagator Is NOT de Sitter Invariant

- Plausible arguments each way
  - Pro: Inv. solns w some gauge fixing terms
  - Con: Dynamically same as MMC scalars + IR divergences in some gauges
- Long controversy resolved
  - Obstacle to adding gauge fixing terms
  - Obstacle to Euclidean continuation
- De Donder projection operator not invariant