Phase Structure of Strongly Coupled Gauge Theories

Nick Evans University of Southampton

Keun-Young Kim

Maria Magou

Astrid Gebauer

James French

Kristan Jensen
(Seattle)

Ingo Kirsch,
Tigran Kalaydzhyan
(DESY)

Crete, May 2010

Introduction – equlibrium computations

Probe brane duals of 3+1d (& 2+1d) gauge theories

Chiral symmetry breaking dynamics

T & μ phase diagrams – chiral + meson melting transitions

First order transitions

Second order transitions

BKT transitions

Non-mean field second order transitions

Motive:

QCD

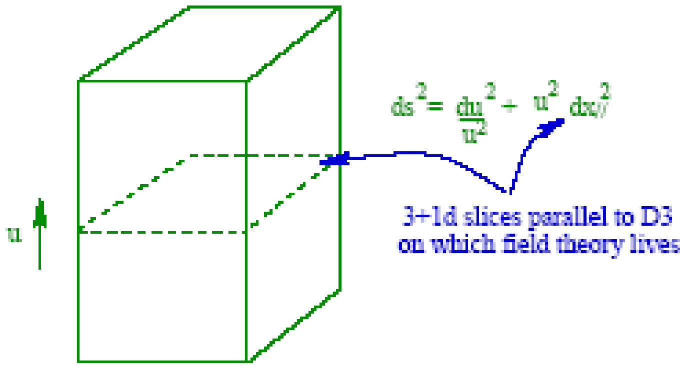
(CM)

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$

Pretty well established by this point!



u corresponds to energy (RG) scale in field theory

The SUGRA fields act as sources

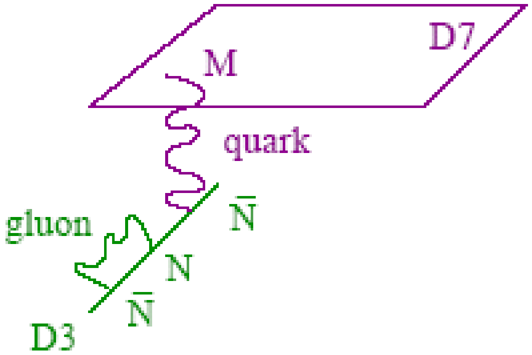
$$\int d^4x \; \Phi_{SUGRA}(u_0)\lambda\lambda$$

eg asymptotic solution ($u \to \infty$) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle\lambda\lambda\rangle}{u^3}$$

Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...



Quarks can be introduced via D7 branes in AdS

The brane set up is

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	•	•	•	•	•	•
D7	—	—	—	—	—	—	—	—	•	•

fills radial dirn of AdS S^3 embedded in S^5 (columns 4–7); $U(1)_A$ (columns 8–9)

We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

Quarks In AdS

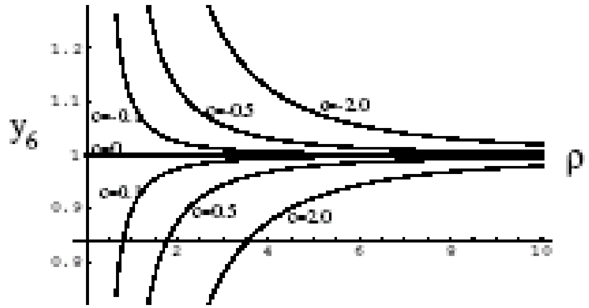
Myers et al

$$S_{D7} = -T_7 \int d^8\xi \; \epsilon_3 \; \rho^3 \sqrt{1 + \frac{g^{ab}}{\rho^2 + u_5^2 + u_6^2}(\partial_a u_5 \partial_b u_5 + \partial_a u_6 \partial_b u_6)}$$

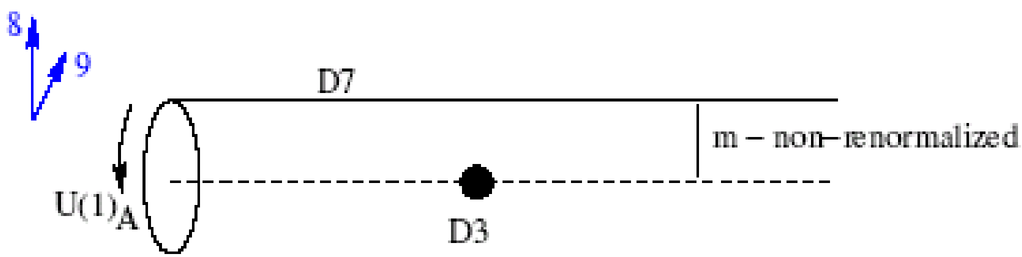
EoM is: $$\frac{d}{d\rho}\left[\frac{\rho^3}{\sqrt{1+\left(\frac{du_6}{d\rho}\right)^2}}\frac{du_6}{d\rho}\right] = 0$$

UV asymptotic solution is
$$u_6 = m + \frac{c}{\rho^2} + \ldots$$

m is the quark mass, c the $\langle \bar{q} q \rangle$ condensate



In AdS regular D7 solution is flat brane



Mesons in AdS_5

Kruczenski,Myers, Mateos, Winters

The D7 lie flat in AdS. We can consider fluctuations that describe R-chargeless mesons

$$w_6 + iw_5 = d + \delta(\rho) e^{ik.x}$$

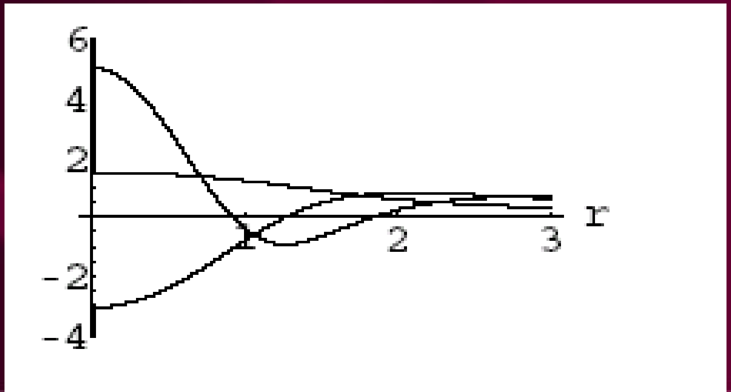
δ satisfies a linearized EoM

$$\partial_\rho^2 \delta + \frac{3}{\rho} \partial_\rho \delta + \frac{M^2}{(\rho^2+1)^2} \delta = 0$$

and the mass spectrum is

$$M = \frac{2d}{R^2} \sqrt{(n+1)(n+2)} \sim \frac{2m}{\sqrt{\lambda_{YM}}}$$

Tightly bound - meson masses suppressed relative to quark mass



Orthonormal wave functions

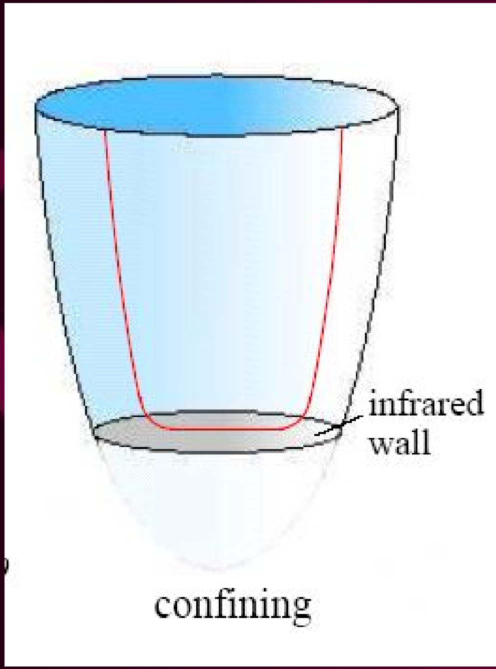
Add Confinement and Chiral Symmetry Breaking

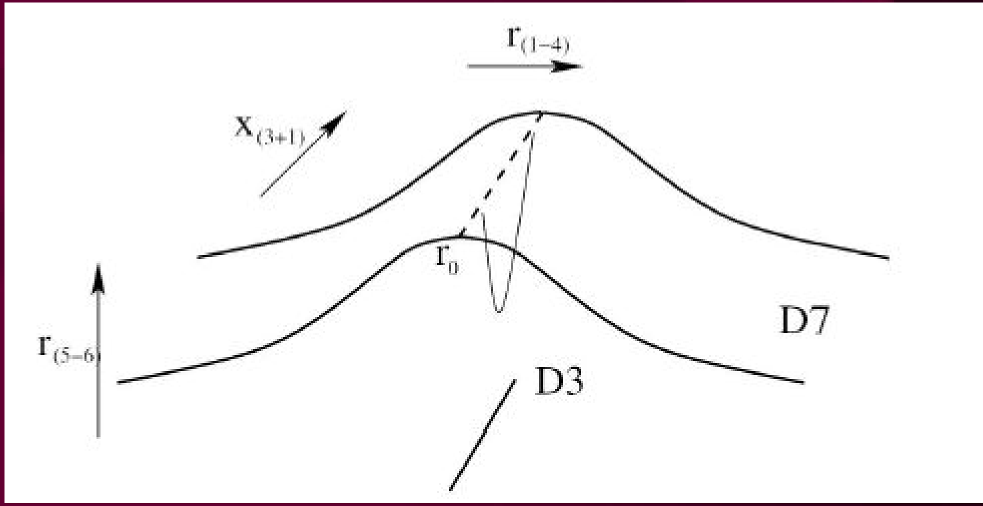
$$ds^2 = \frac{r^2}{R^2} A^2(r) dx_{3+1}^2 + \frac{R^2}{r^2} dr_6^2,$$

$$A(r) = \left(1 - (\frac{r_w}{r})^8\right)^{1/4}, \qquad e^{\phi} = \left(\frac{1+(r_w/r)^4}{1-(r_w/r)^4}\right)^{\sqrt{3/2}}$$

Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall….





BEEGK, Ghoroku..

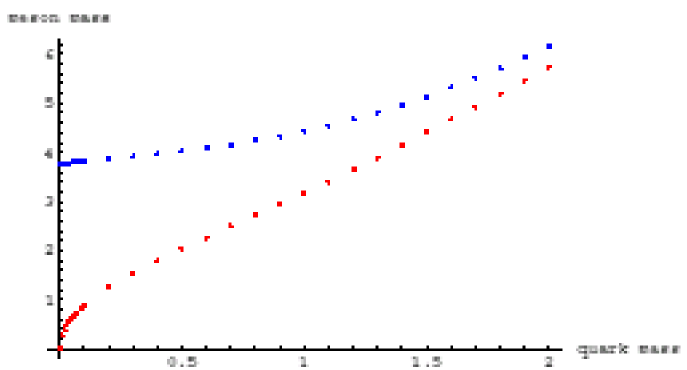
Pion Physics

Seek pion solutions of the form

$$\pi(x,r) = f(\rho)e^{ikx}, \qquad k^2 = -M^2$$

$f(\rho)$ must be smooth - normalizable - at all ρ

The pion and sigma masses can thus be computed as a function of quark mass



There is a Goldstone in the massless limit.

Expected $\sqrt{m}$ behaviour

Magnetic Field Induced Chiral Symmetry Breaking

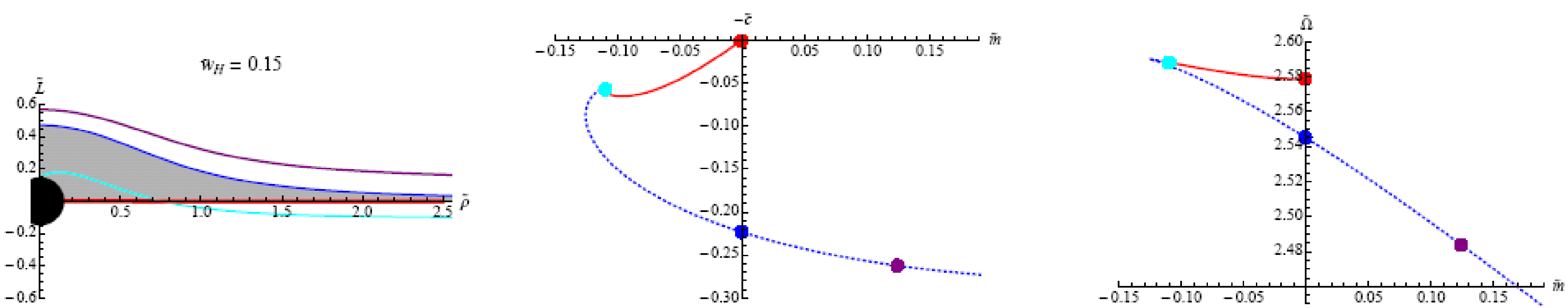
Johnson, Filev, Kundu….

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & & & & \\ & \frac{r^2}{R^2} & B^z & & & & & \\ & -B^z & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2 .. & \\ & & & & & & & \frac{R^2}{r^2} .. \end{pmatrix}$$

$$\mathcal{L} = \rho^3 \sqrt{1+(\partial_\rho w_6)^2}\sqrt{1+\frac{B^z R^4}{r^4}}$$

Put in B though susy partner of mesons..

B “just” breaks conformal symmetry…



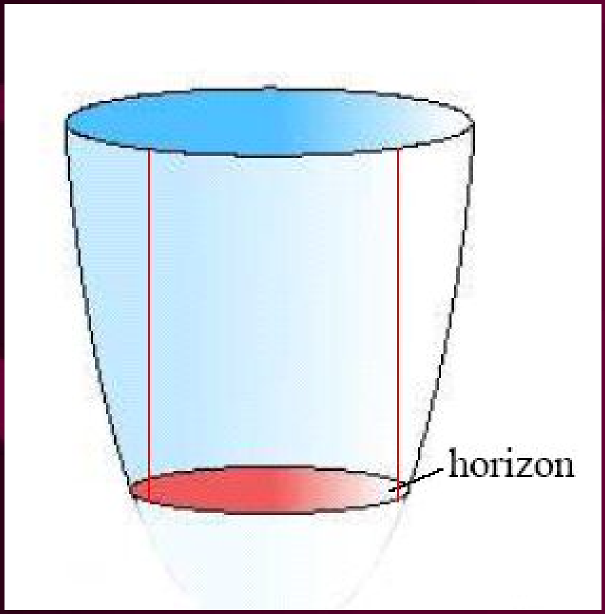
(a)Low temperature - $\widetilde{w}_H = 0.15$. Here we see chiral symmetry breaking with the blue embedding thermodynamically preferred over the red at $\widetilde{m} = 0$.

Finite T - AdS-Schwarzschild

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f} dr^2 + R^2 d\Omega_5^2$$

where $R^4 = 4\pi g_s N \alpha'^2$ and

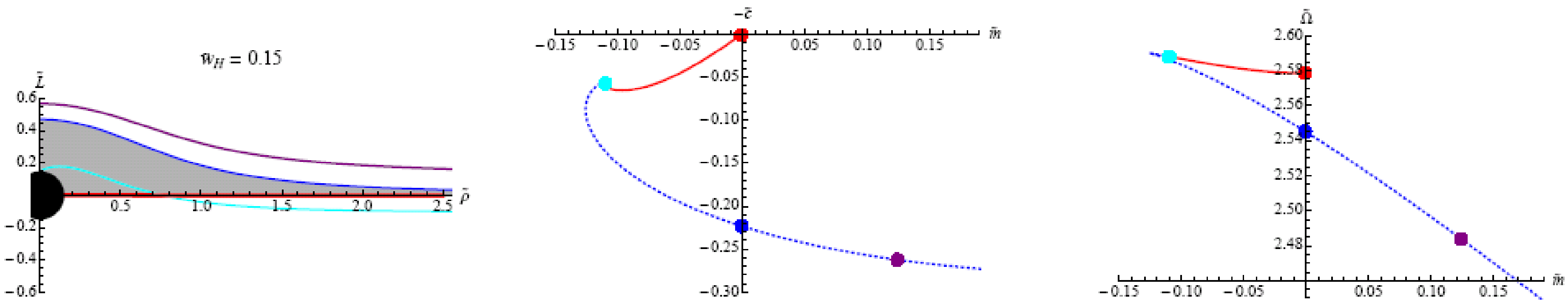
$$f := 1 - \frac{r_H^4}{r^4}\,, \qquad r_H := \pi R^2 T\,.$$



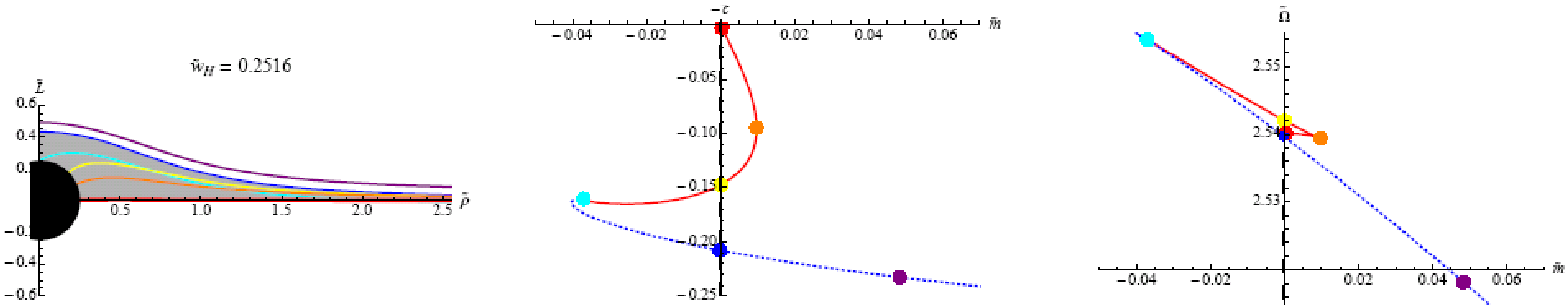
Quarks are screened by plasma

Asymptotically AdS, SO(6) invariant at all scales… horizon swallows information at rH …. Witten interpreted as finite temperature… black hole… has right thermodynamic properties…

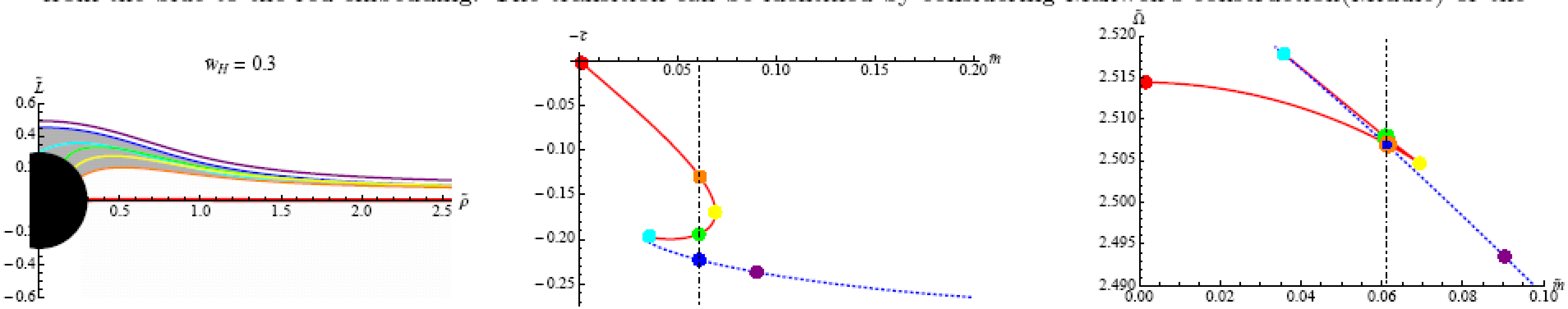
First Order Chiral Symmetry Restoration – B + T



(a)Low temperature - $\widetilde{\omega}_H = 0.15$. Here we see chiral symmetry breaking with the blue embedding thermodynamically preferred over the red at $\widetilde{m} = 0$.



(b)Transition temperature - $\widetilde{\omega}_H = 0.2516$. This shows the point where the first order chiral symmetry phase transition occurs from the blue to the red embedding. The transition can be identified by considering Maxwell's construction(Middle) or the



(c)Above the transition - $\widetilde{\omega}_H = 0.3$. This is the chiral restored phase with the $\widetilde{m} = 0$ curve lying along the $\widetilde{\rho}$ axis (red).

Quasi-normal modes & meson melting

BEEGK… Sonnenschein… Hoyos…. Myers, Mateos…

Linearized fluctuations in eg the scalars on the D7 brane must now enter the black hole horizon…

Quasi-normal modes are those modes that near the horizon have only in-falling pieces…

The mass of the bound states become complex – they decay into the thermal bath…

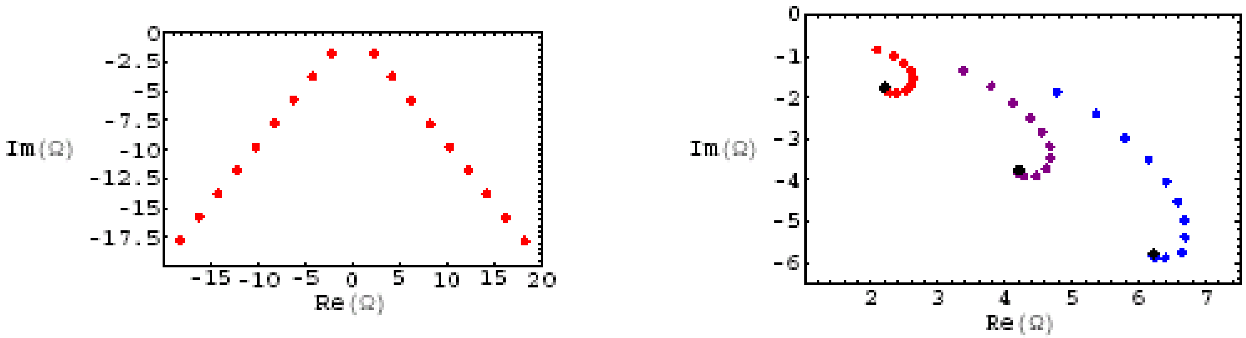
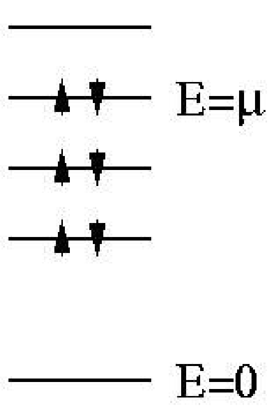


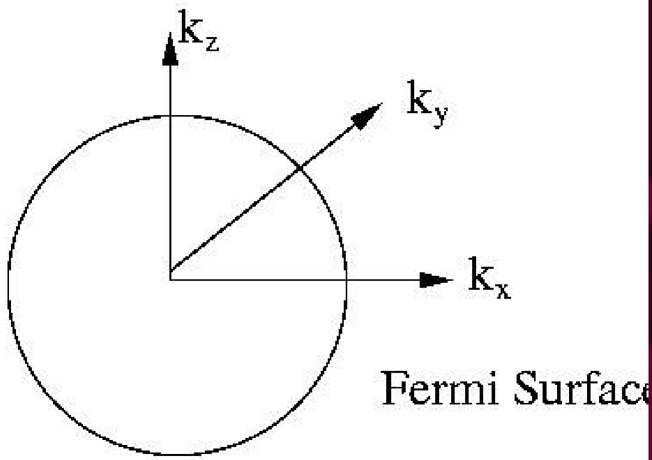
Figure 7.4: The lowest quasi-normal modes for $m_q = 0$ on the left and the three lowest quasi-normal modes for increasing m_q on the right. The black points on the right show the limiting values for $m_q = 0$.

Chemical Potential

At finite density the Fermi-sea of quarks fills up to an energy called the chemical potential



$$\bar{\psi}_i (i \not{D} + \mu \gamma_0) \psi_i$$



We can think of μ as a background vev for the temporal component of the photon…

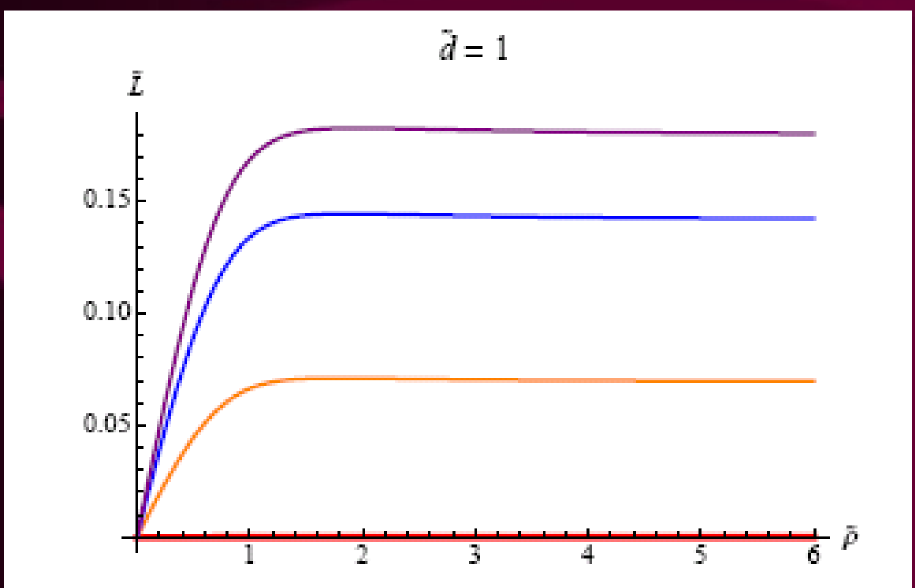
$$\bar{\psi} i (-i A^t \gamma_0) \psi \quad \rightarrow \quad \bar{\psi} \mu \gamma_0 \psi$$

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & \partial_\rho A_0 & & & \\ & \frac{r^2}{R^2} & & & & & & \\ & & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ -\partial_\rho A_0 & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & & & \frac{R^2}{r^2}.. \end{pmatrix}$$

$$\mathcal{L} = \rho^3\sqrt{1+(\partial_\rho w_6)^2-(\partial_\rho A_0)^2}$$

$$A^0 \simeq \mu + \frac{d}{\rho^3} + \ldots$$

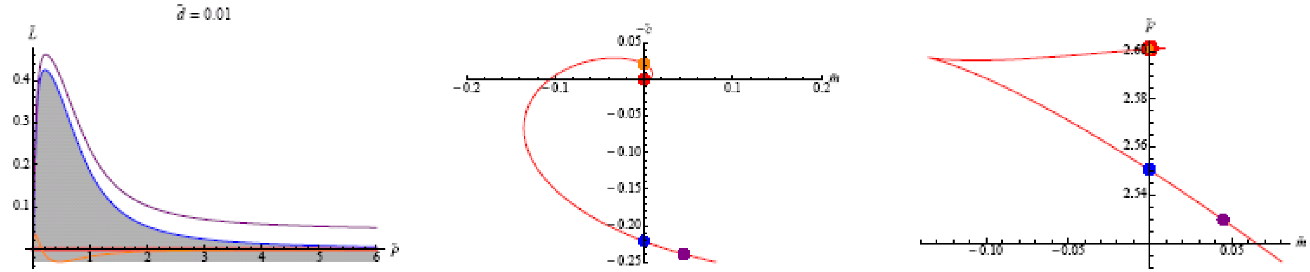
Myers, Mateos



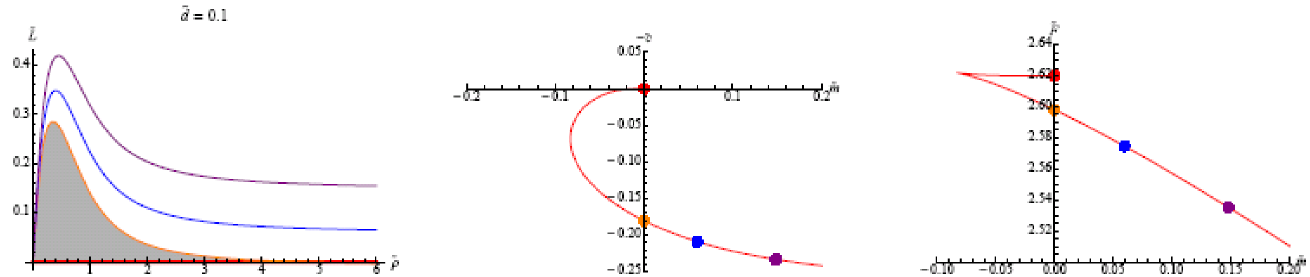
μ induces quarks to fill the vacuum....

ie a spike of strings grows between the D7 and the D3...

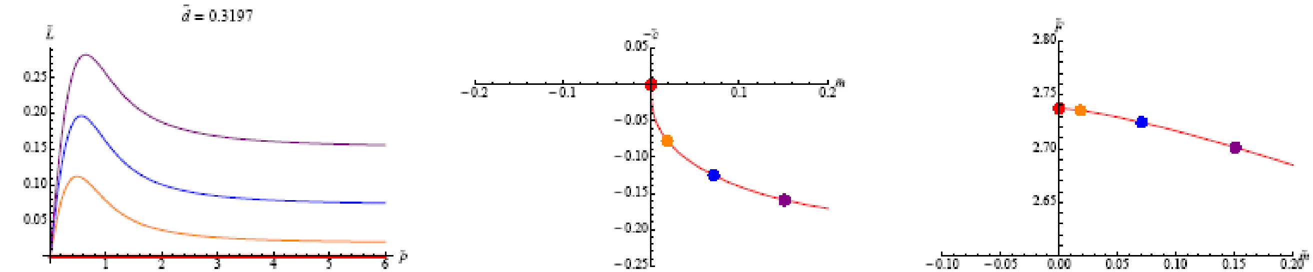
Second Order Chiral Symmetry Restoration – B + density



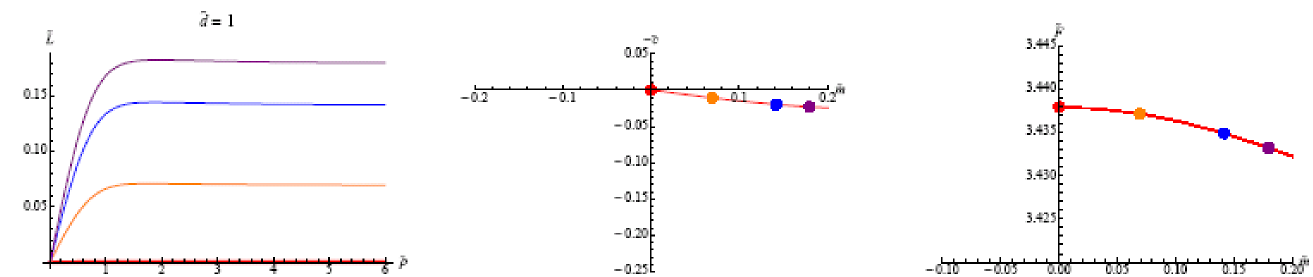
(a)Low density - $\tilde{d} = 0.01$. Here we see chiral symmetry breaking (the blue embedding is preferred over the red embedding) and a spiral structure in the $\tilde{m}$ vs $\tilde{c}$ plane.



(b)Increasing density below the transition - $\tilde{d} = 0.1$. There is still chiral symmetry breaking here with the orange embedding

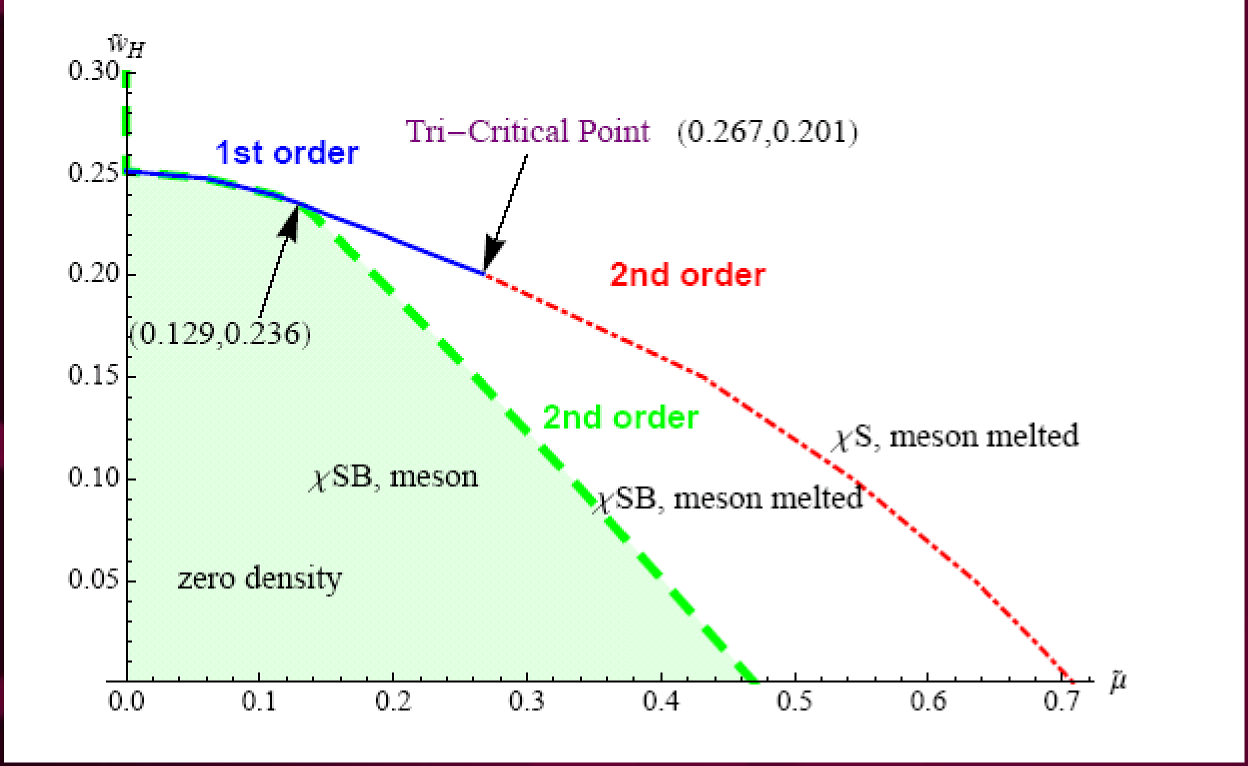


(c)Transition point - $\tilde{d} = 0.3197$. This shows the point where the second order chiral symmetry phase transition occurs.

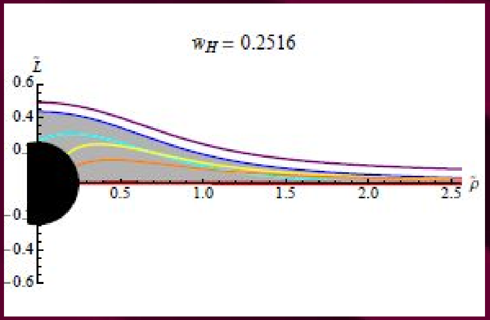


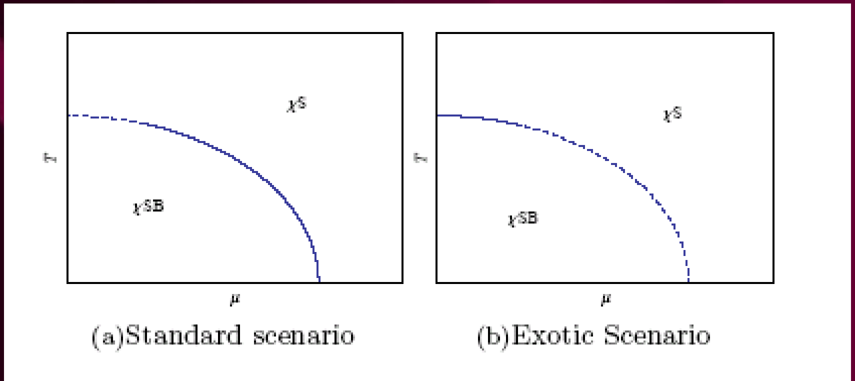
(d)High density $\tilde{d} = 1$. This is the chiral restored phase with the $\tilde{m} = 0$ curve lying along the $\tilde{\rho}$ axis. For larger $\tilde{m}$ the usual spike like embedding can be seen.

Phase Diagram for B Field Theory, m=0



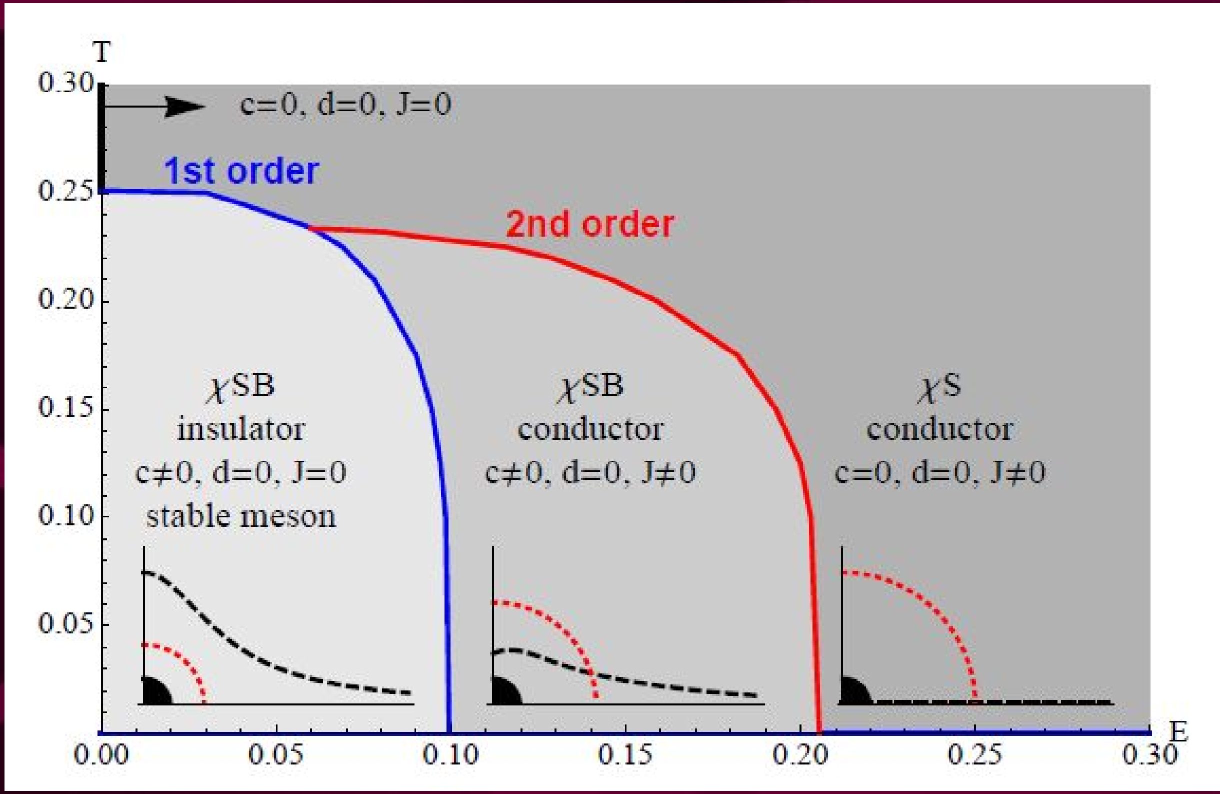
JHEP 1003:132,2010.
e-Print:
arXiv:1002.1885
[hep-th]





QCD scenarios for comparison… Wilczek vs Philipsen (Lattice)

Phase Diagram for B=1,T,E Theory, m=0

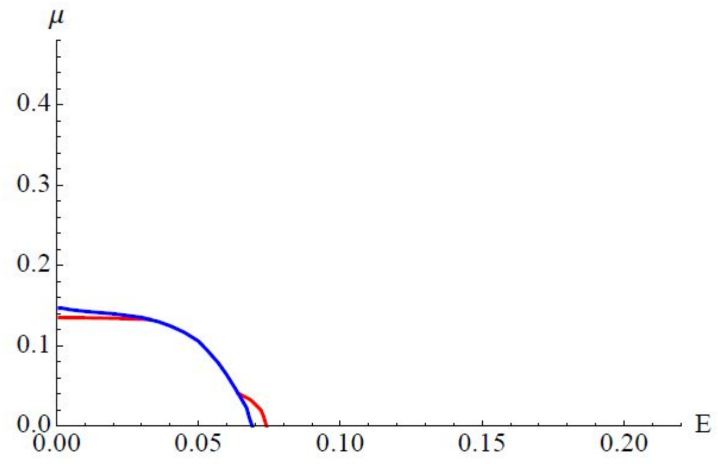


JHEP to appear
e-Print:
arXiv:1103.5627
[hep-th]

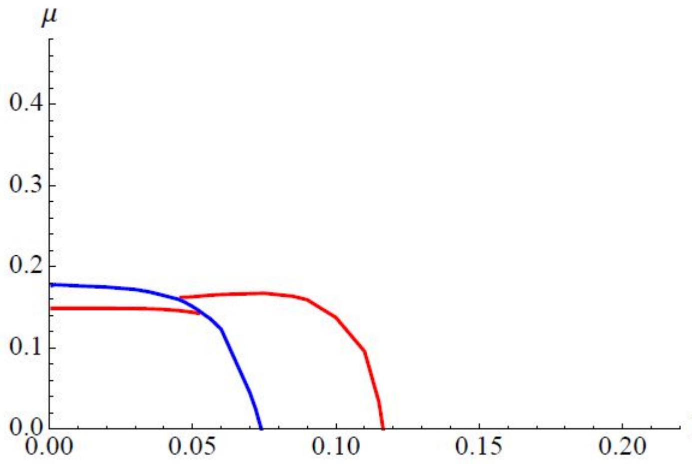
An E field tends to dissociate the mesons – when it does a current must be introduced (Karch, O'Bannon)

It opposes chiral symmetry breaking like mu.

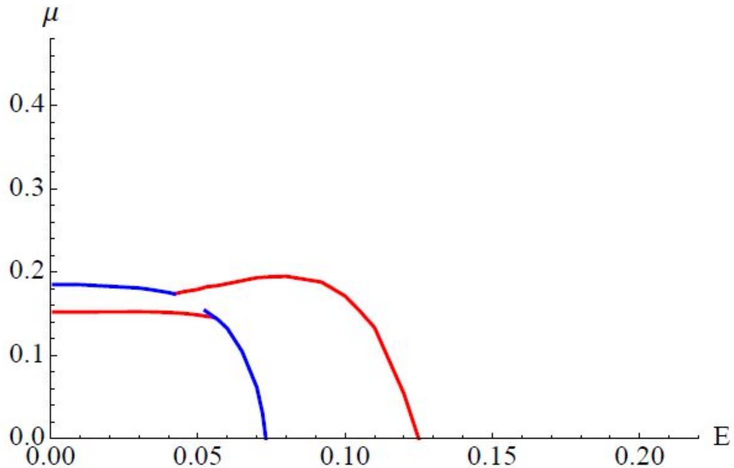
Phase Diagram for B=1,T,E,μ Theory, m=0



(a) $T = 0.232$



(b) $T = 0.224$



(c) $T = 0.222$

Thanks to Astrid Gebauer!

Second Order Mean Field Behaviour

A mean field second order transition is just an effective Landau -Ginsberg (Higgs) Model

$$V_{\text{eff}} = V_0 + k(B - B_c)\phi^2 + \lambda\phi^4 + \ldots$$

$$\phi \sim \sqrt{B - B_c}$$

A Phenomenological Operator

with Kristan Jensen

$$\tilde{S}_7 = -\mathcal{N}_7 \int d\rho \sqrt{1+L'^2} \sqrt{\tilde{d}^2 + \rho^6 \left(1 + \frac{B^2}{w^4} + \frac{O^2}{w^{2\Delta}}\right)}.$$

$$\Delta = 3$$

holographic Berezinskii-Kosterlitz-Thouless transition

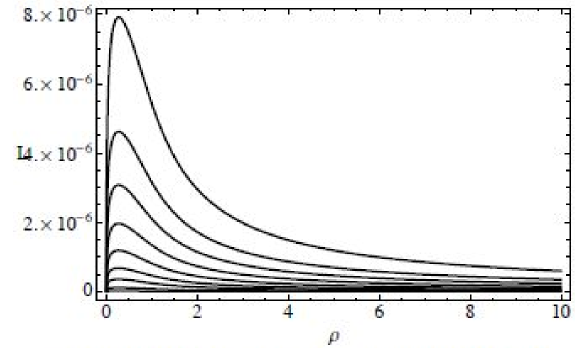
T=0 transition changes…

K. Jensen, A. Karch, D. T. Son, and E. G. Thompson, Phys. Rev. Lett. **105**, 041601 (2010), 1002.3159.

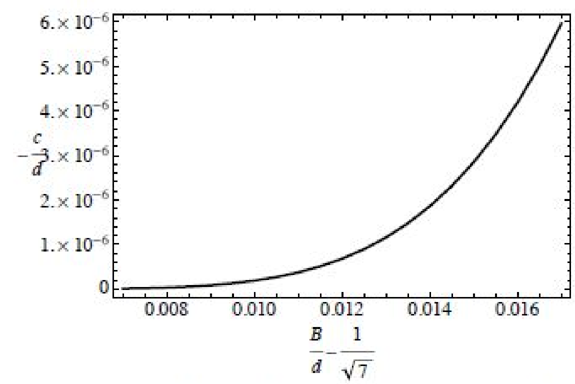
Exponential scaling of order parameter away from the transition…

D. B. Kaplan, J.-W. Lee, D. T. Son, and M. A. Stephanov, Phys. Rev. **D80**, 125005 (2009), 0905.4752.

First identified in D3/D5 system where d and B have same dimension… AdS/CM link

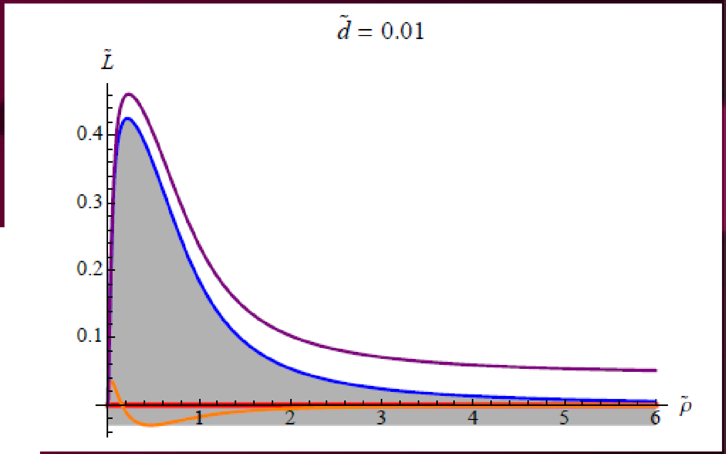


(a)The embedding L of a D5 brane in the D3 geometry for various $B/\tilde{d}$ showing the BKT transition.



(b)A plot of the quark condensate c versus B across the D3/D5 BKT transition.

Instability of flat embedding



$$\mathcal{L}_7 = \sqrt{d^2 + O^2 + \rho^6}\, L'^2 + \frac{3O^2 L^2}{\rho^6 \sqrt{d^2 + O^2 + \rho^6}}$$

Small rho limit solutions:

$$\frac{L}{\rho} \sim \left(\frac{1}{\rho}\right)^{\Delta}$$

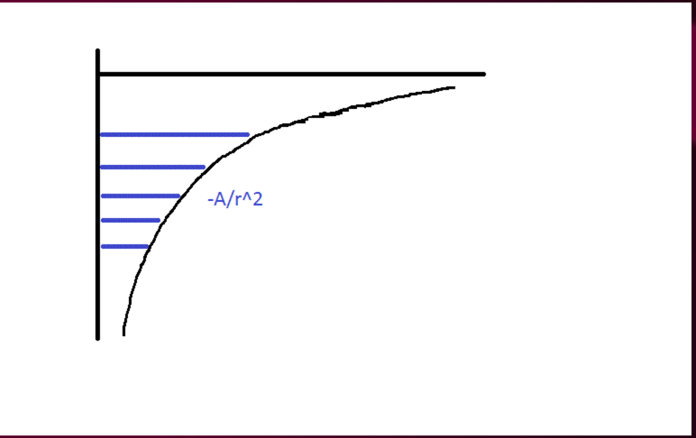
$$\Delta_{\pm} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 + m^2}$$

$$m^2 = -3O^2/(\tilde{d}^2 + O^2)$$

$$\Delta_{\mathrm{IR}} = \frac{1 + \sqrt{\frac{\tilde{d}^2 - 11O^2}{\tilde{d}^2 + O^2}}}{2}.$$

O and d enter on same footing because same dimension (not true in B case!) …. For fixed d raising O triggers complex □□□□ an instability that correctly predicts the transition point…

The Schroedinger well becomes unstable (A > 1/4) with an infinite number of negative energy states growing from zero… leading to exponential behaviour…



Breitenlohner-Freedman (BF)

In our analysis we use the results for a scalar in AdS_{p+1}: The solution of the equation of motion is

$$\frac{L}{\rho} \sim \left(\frac{1}{\rho}\right)^{\Delta} \qquad (11)$$

$$\Delta_{\pm} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 + m^2}\,. \qquad (12)$$

and the Breitenlohner-Freedman (BF) bound [65] is given by $-p^2/4$

$$AdS_2,\ m^2_{BF} = -1/4$$

0+1d theory rules IR?

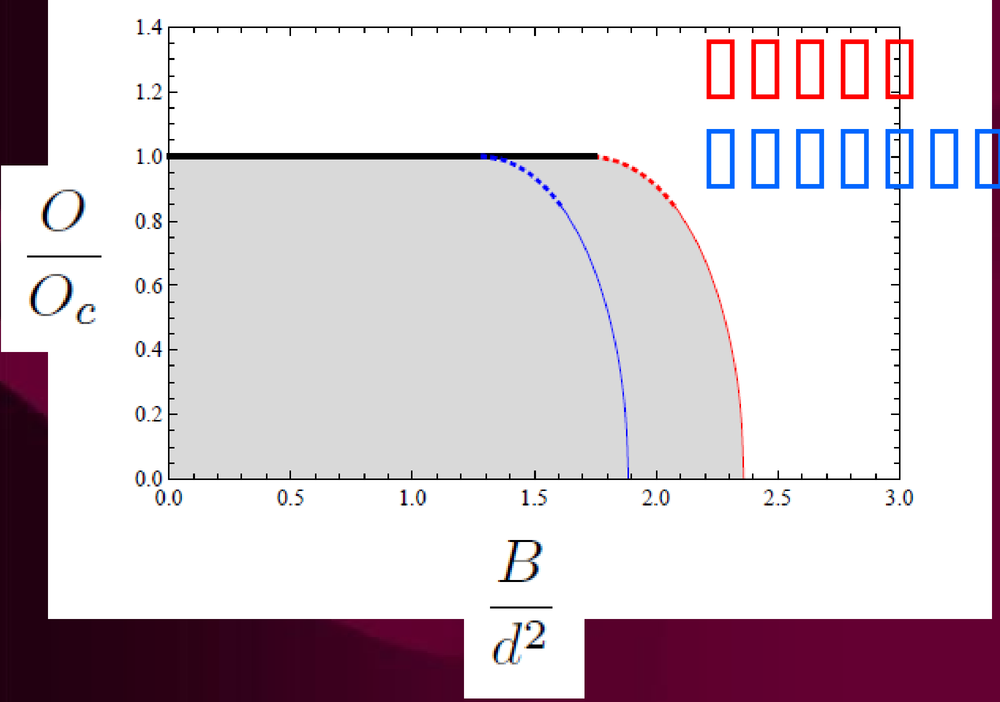
Small rho limit mass

$$m^2 = -3O^2/(\tilde{d}^2 + O^2)$$

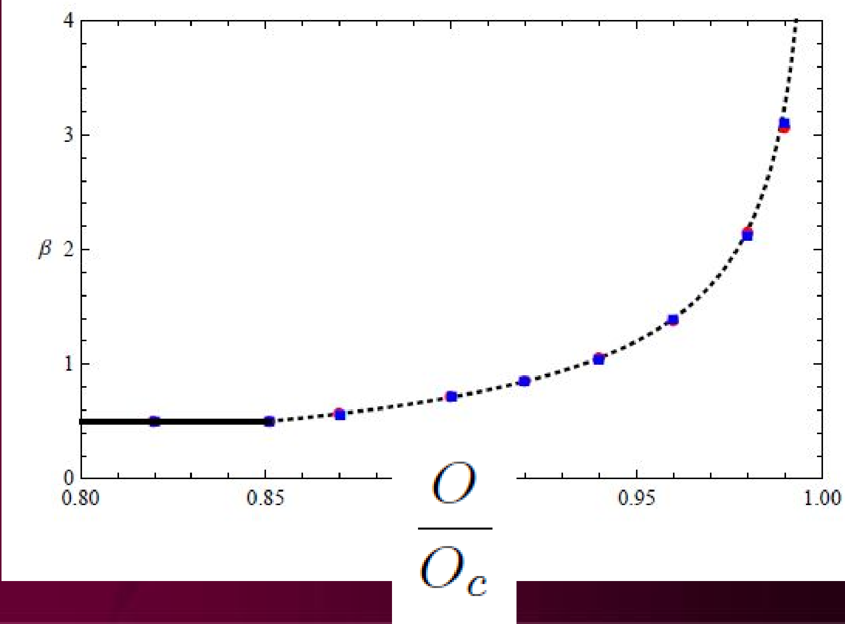
From Mean-Field 2nd Order to BKT

$$S_7 = \int d\rho \sqrt{1 + L'^2} \sqrt{d^2 + \rho^6 \left(1 + \frac{B^2}{w^4} + \frac{O^2}{w^6}\right)}$$

O+d triggers BKT…. B+d is second order mean-field… what about O+B+d:



$$c \sim (B - B_c)^{\beta}$$



Faulkner, Horowitz,Roberts - **arXiv:1008.1581** [hep-th]

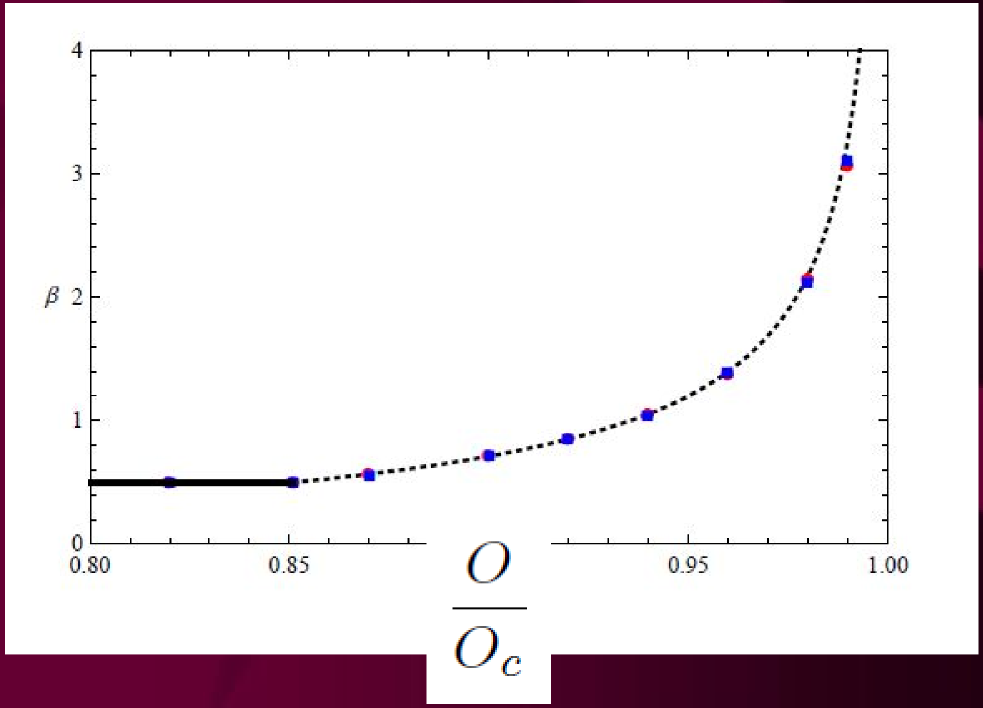
Effective Theory

$$V_{\text{eff}} = V_0 + k(B - B_c)\phi^2 + \lambda\phi^4 + \alpha_{\text{IR}}\phi^{p/(p-\Delta_{\text{IR}})}$$

$$\phi \sim (B - B_c)^{\beta} \equiv (B - B_c)^{\frac{p-\Delta_{IR}}{2\Delta_{IR}-2}}$$

$$\Delta_{\text{IR}} < \Delta_c = 3/4p$$

$$\beta = \frac{1}{2}\left(\sqrt{\frac{d^2 + O^2}{d^2 - 11O^2}} - 1\right)$$



Comments

Lots of complex phase structure with B,E mu, T, O!

Non mean-field quantum critical points…

A full effective theory showing role of 0+1d IR… (pretty cool)

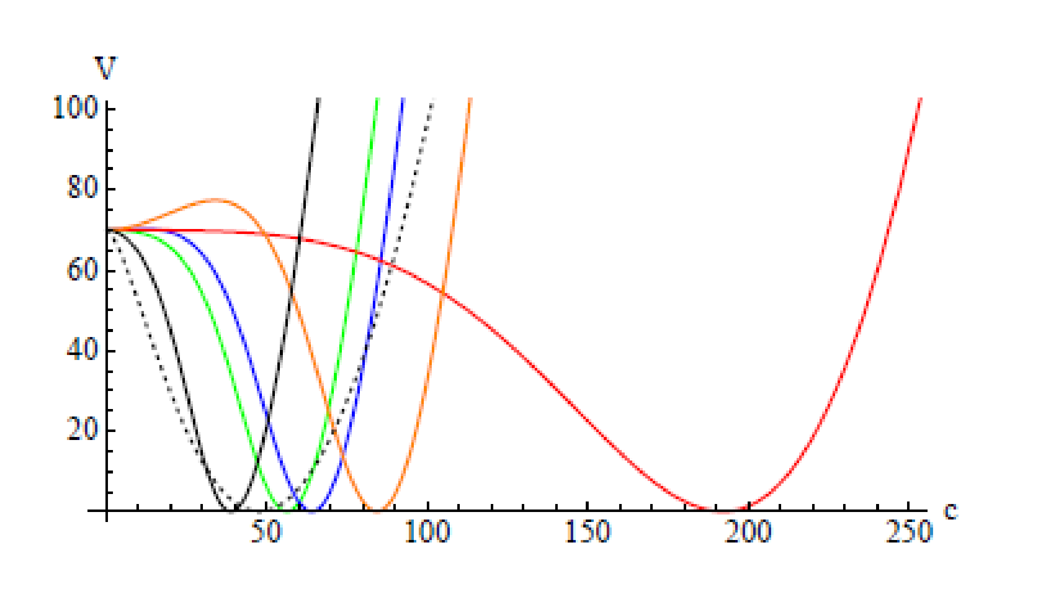
What’s it good for?

Non-equilibrium Dynamics At The Transitions

AdS/CFT lets us solve full time dependence of non-equilibrium configurations… some sample ideas…

Seeking Inflating Gauge Theories

Nick Evans, James French, Keun-Young Kim, arXiv:1009.5678 [hep-th]



2nd order transition leaves us at c=0 in T=0 V

Slow roll to V=0 with +ve cosmological constant -> inflation

Scalars are unnatural

An inflating strongly coupled gauge theory?

Watching the roll with a B field

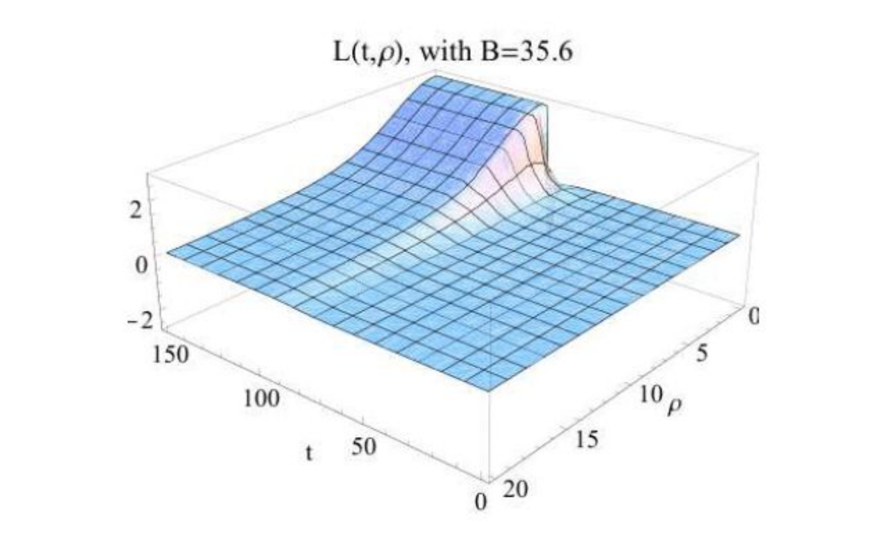
$$\mathcal{L} = -\widetilde{T}_7 R^4 \int d\rho\, \beta\, \rho^3 \sqrt{\frac{g^{00}\dot{L}^2}{(\rho^2+L^2)^2} + 1 + L'^2}$$

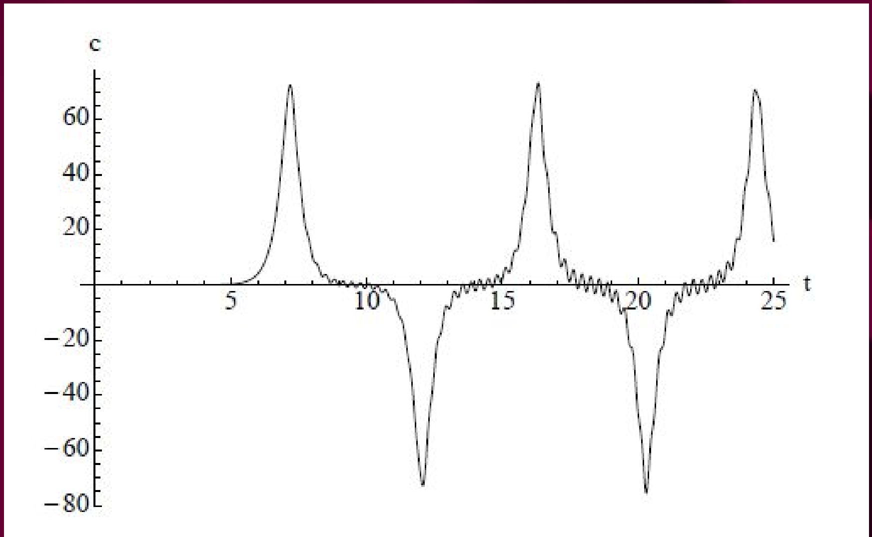
$$\beta = \sqrt{1 + \frac{B^2}{(\rho^2+L^2)^2}}$$

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

Probe limit – inflation slow relative to N=4 glue dynamics.. Or just a model…

$$L(\rho, t=0) = 0, \qquad \dot{L}(\rho, t=0) = v e^{-\rho^2}$$

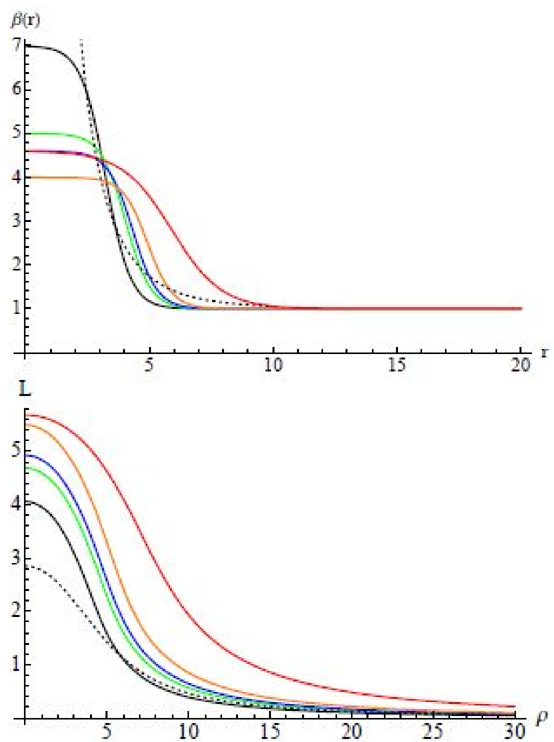




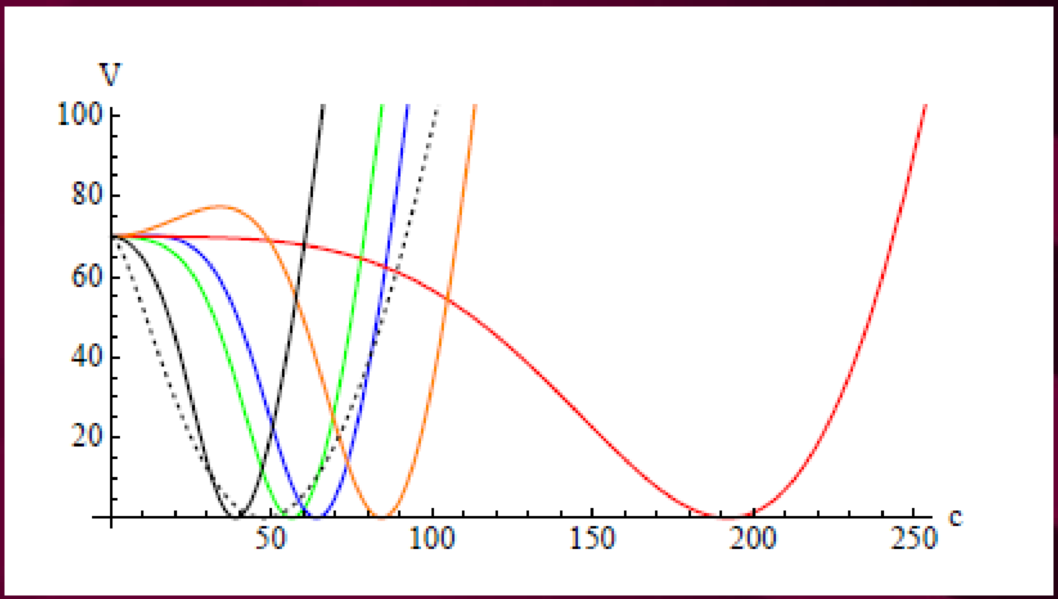
How should the coupling run to favour inflation?

$$e^{\Phi} = g_{\mathrm{YM}}^2(r^2) = g_{\mathrm{UV}}^2\Big(A + 1 - A\tanh\left[\Gamma(r-\lambda)\right]\Big)$$

Cf AdS/QCD

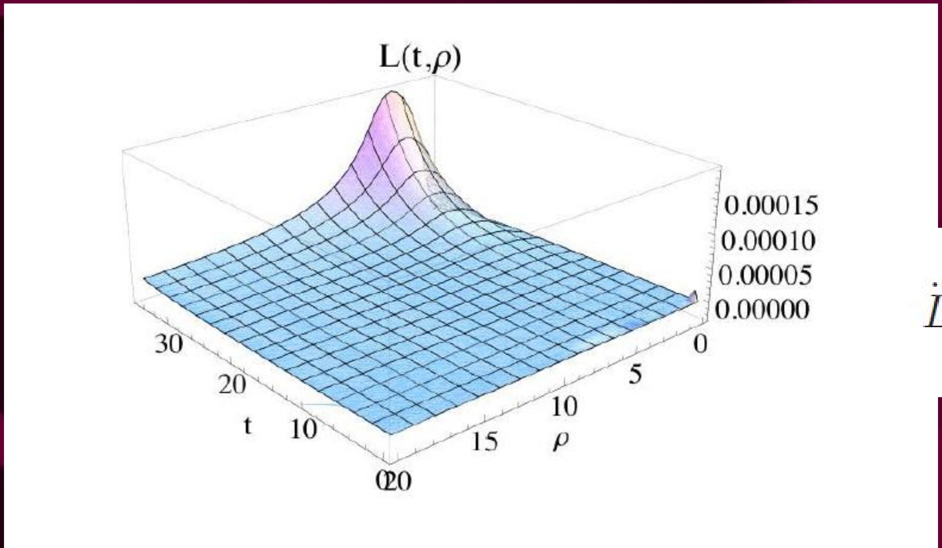


	Γ	A	λ
Black	1	3	3.240
Green	1	2	4.045
Blue	1	1.8	4.325
Orange	1	1.5	4.940
Red	0.5	1.8	5.882
Black(Dotted)	B = 35.6	-	-



Walking leads to a gap between and V…

Early time rolling is around the dilaton kink…. Acceleration involves holographic behaviour…



$$\ddot{L}(\rho) \sim \left(\lambda^3 \frac{A\,\Gamma}{A+1} L(\lambda) + \lambda^4 L''(\lambda)\right) \delta(\rho - \lambda)$$

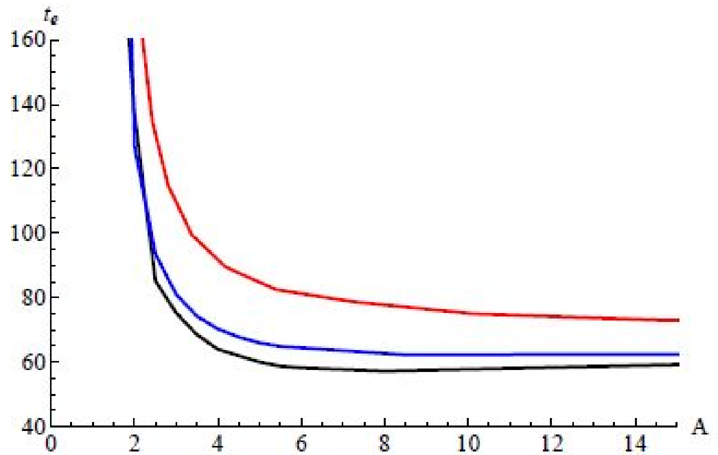


FIG. 11: t_e against A for a sequence of values of Γs: 1.5(Black), 1(Blue), 0.5(Red). $v = 0.00001$. $H = \sqrt{70/3}$.

te is the transition time…

Walking dynamics may lead to inflationnary dynamics…

Chiral Transition in Janik's Cooling Geometry

The black hole grows/shrinks changing the effective potential…

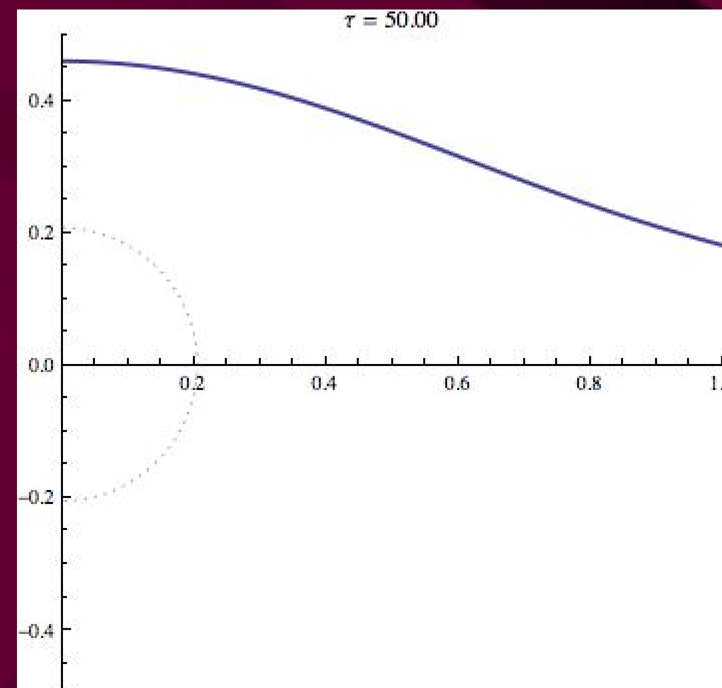
With Ingo Kirsch, Tigran Kalaydzhyan (DESY)

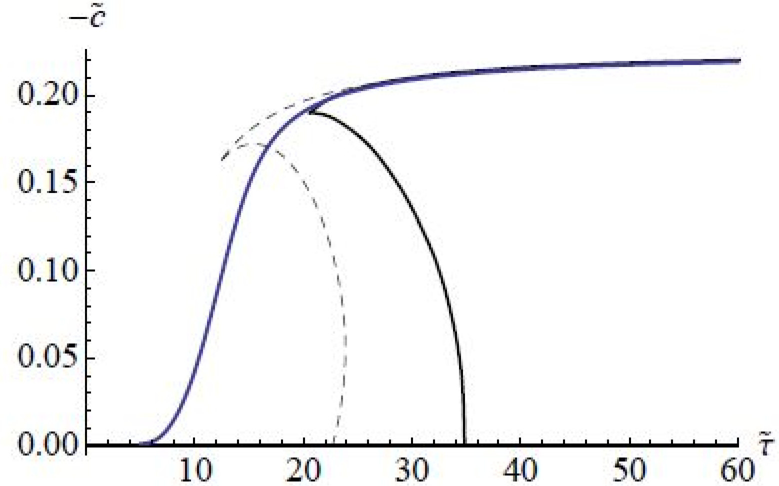
$$ds^2 = \frac{r^2}{R^2}\left(-e^{a(\tau,r)}d\tau^2 + e^{b(\tau,r)}\tau^2 dy^2 + e^{c(\tau,r)}dx_\perp^2\right) + \frac{R^2}{r^2}\left(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2\right)$$

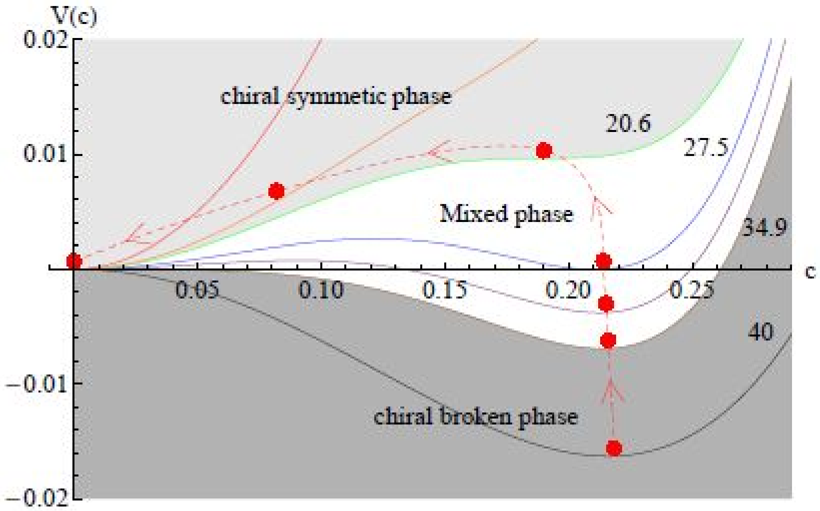
$$a(\tau,z) = \ln\left(\frac{(1-v^4/3)^2}{1+v^4/3}\right) + 2\eta_0\frac{(9+v^4)v^4}{9-v^8}\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau,z) = \ln(1+v^4/3) + \left(-2\eta_0\frac{v^4}{3+v^4} + 2\eta_0\ln\frac{3-v^4}{3+v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau,z) = \ln(1+v^4/3) + \left(-2\eta_0\frac{v^4}{3+v^4} - \eta_0\ln\frac{3-v^4}{3+v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

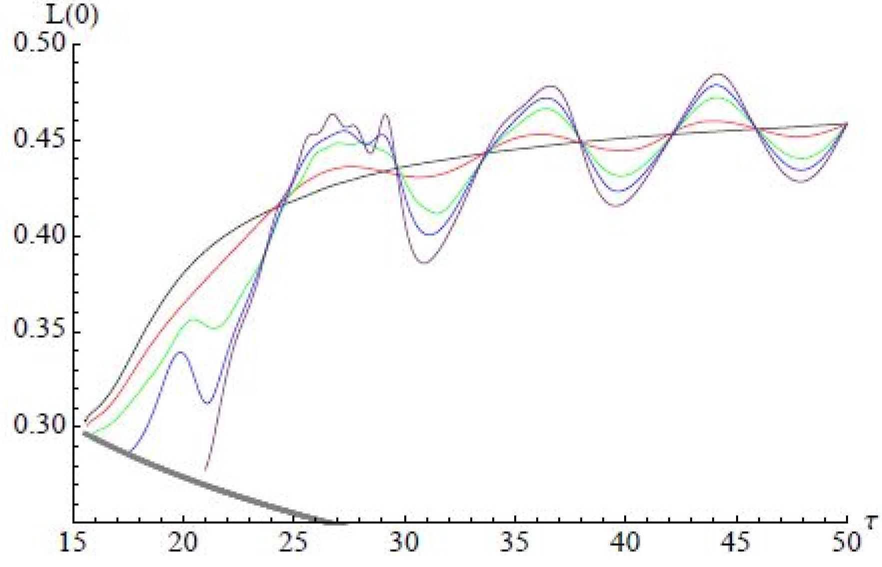






Equilibrium vs PDE solutions…

Bubble formation…



Conclusion

Strongly coupled phase diagrams with rich structure
arXiv:1002.1885 [hep-th], arXiv:1003.2694 [hep-th], arXiv:1103.5627 [hep-th]

Non mean-field quantum critical points… **arXiv:1008.1889** [hep-th]

Out of equilibrium dynamics

Walking dynamics & inflation linked? **arXiv:1009.5678** [hep-th]

Towards strongly coupled quenches, disordered chiral condensates etc… arXiv:1011.2519 [hep-th]

Wilsonian derivation of effective potential, inflaton phenomenology, horizon formation and thermalization…