Holographic Charge Density Waves

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Plan of the talk

- Density Waves
- Holographic Superconductors
- Holographic Charge Density Waves
- Conclusions-Discussion

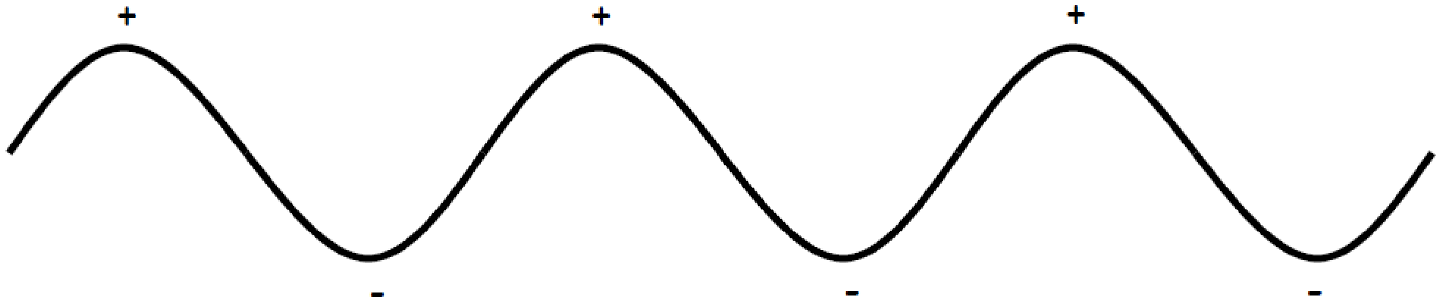
Density Waves

Fröhlich “principle” for superconductivity:

The crystalic systems shape their spacing (lattice) to serve the needs of conductivity electrons

The simpler perhaps example: Peierls Phase Transition

- In a 1-D lattice, the metal system of ions – electrons in the fundamental level is unstable in low temperatures
- It returns to a new state of lower energy via a phase transition
- In the new phase the ions are shifted to new places and the density of electronic charge is shaped periodically in the space



A **Density Wave**, is any possible kind of ordered state that is characterized by a **modulated** macroscopic physical quantity

Consider correlations of the form:

$$\mathcal{V}_{int} = \iint \mathrm{dxdy}\ \hat{\psi}^\dagger_\alpha(\boldsymbol{x})\hat{\psi}^\dagger_\beta(\boldsymbol{y})V_{\alpha,\beta,\gamma,\delta}(\boldsymbol{x},\boldsymbol{y})\hat{\psi}_\gamma(\boldsymbol{y})\hat{\psi}_\delta(\boldsymbol{x})$$

generate two general types of density waves

I. Density waves in the particle-hole channel

Bound state: $\left\langle \hat{\psi}^\dagger_\alpha(\boldsymbol{x})\hat{\psi}_\beta(\boldsymbol{y})\right\rangle \sim \left\langle f(\boldsymbol{k})\ \psi^\dagger_{\boldsymbol{k},\alpha}\psi_{\boldsymbol{k}+\boldsymbol{q},\beta}\right\rangle$

II. Density waves in the particle-particle channel

Bound state: $\left\langle \hat{\psi}^\dagger_\alpha(\boldsymbol{x})\hat{\psi}^\dagger_\beta(\boldsymbol{y})\right\rangle \sim \left\langle f(\boldsymbol{k})\ \psi^\dagger_{\boldsymbol{k},\alpha}\psi^\dagger_{-\boldsymbol{k}+\boldsymbol{q},\beta}\right\rangle$

q: momentum of the pair **k**: relative momentum
f(**k**) denotes the irreducible representation
a,b,..-> Spin, Isospin, Flavour, etc

- **Density waves (p-h) → Neutral particle-hole pair electromagnetic U(1) symmetry is preserved**

- **Pair Density waves (p-p) → Charged 2e particle-particle pair electromagnetic U(1) symmetry is spontaneously broken**

Both kinds of Density waves are distinguished in:

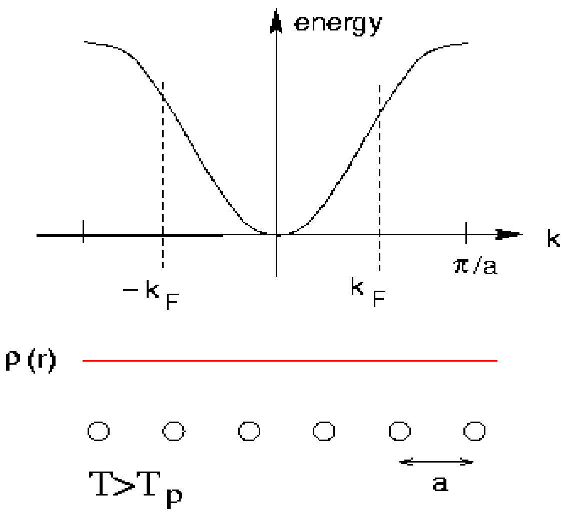
Commensurate: when the ordering wave-vector can be embedded to the underlying lattice

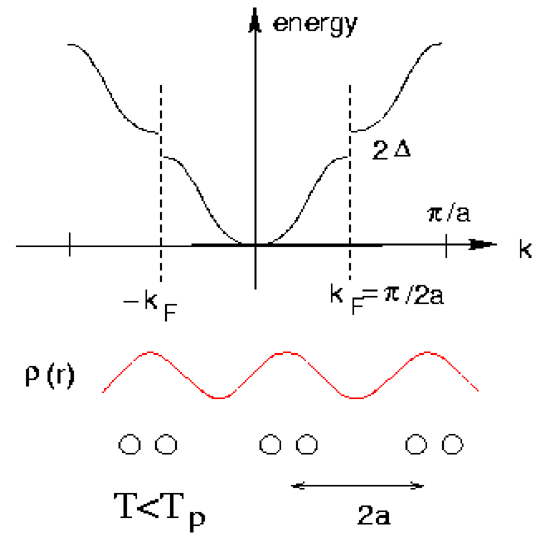
→ Translational symmetry is downgraded

Incommensurate: when the ordering wave-vector cannot be related to any wave-vector of the reciprocal lattice

→ U(1) Translational symmetry is spontaneously broken

1D - Charge Density Waves





Minimization of the energy:

→Opening of a gap at kf, -kf.

Origin of the interaction:

→ Electron-phonon Peierls transition with a lattice distortion.

→ Electron-electron effective interaction not coupled to the lattice

G. Gruener
Rev. Mod. Phys. 60, 1129 (1988)
Rev. Mod. Phys. 66, 1 (1994)

Collective phenomena in density waves

In incommensurate density waves U(1) translational symmetry is broken

→Appearance of the Nambu-Goldstone mode of the U(1) symmetry.
→The 'phason' interacts with the electromagnetic field due to chiral anomaly in 1+1D.
→Ideally the sliding of the phason leads to the Fröhlich supercurrent.

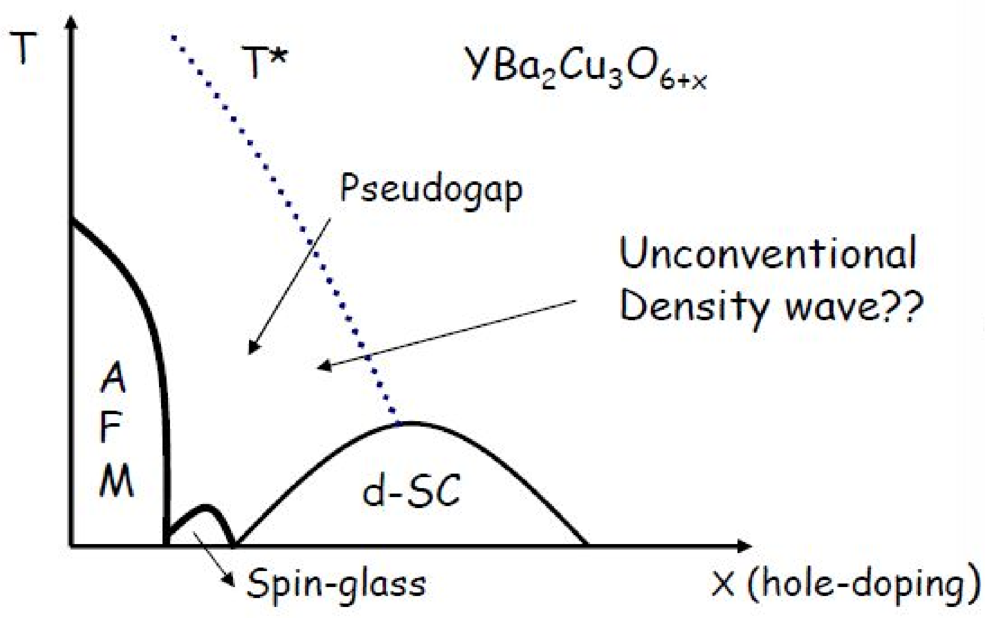
In commensurate or 'pinned' density waves translational symmetry is only downgraded

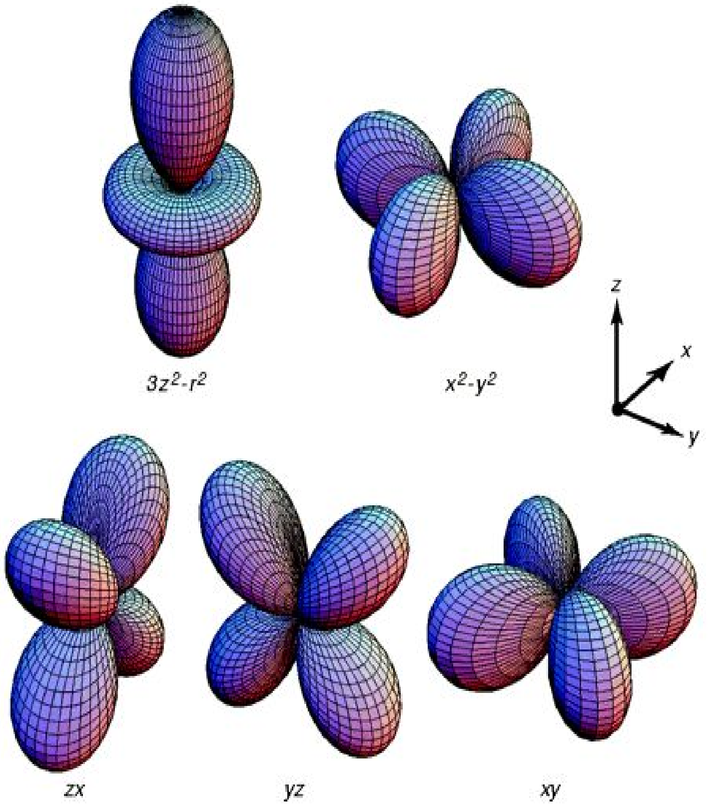
→The U(1) Nambu-Goldstone mode is gapped.
→However, the remnant Z2 symmetry allows the formation of solitons, corresponding to inhomogeneous phase configurations 'connecting' domains.
→Solitons can propagate giving rise to a charge current.

2D-Unconventional Density Waves

For ‘higher-dimensional’ materials, the emergence of a complex extended Fermi surface can lead to the formation of density waves belonging to non-trivial irreducible representations.

The case of high Tc cuprates





Holographic Superconductivity

According to AdS/CFT correspondence:

Bulk:	Gravity Theory	Boundary:	Superconductor
	Black hole		Temperature
	Charged scalar field		Condensate

We need "Hairy" Black Holes in the gravity sector

Consider the Lagrangian

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

For an electrically charged black hole the effective mass of Ψ is

S. Gubser

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

the last term is negative and if q is large enough (in the probe limit) pairs of charged particles are trapped outside the horizon

Probe limit

S. Hartnoll, C. Herzog, G. Horowitz

Rescale A ->A/q and Ψ-> Ψ /q, then the matter action has a $1/q^2$n front, so that large q suppresses the backreaction on the metric

Consider the planar neutral black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

where

$$f = \frac{r^2}{L^2} - \frac{M}{r}$$

with Hawking temperature

$$T = \frac{3M^{1/3}}{4\pi L^{4/3}}$$

Assume that the fields are depending only on the radial coordinate

$$A = \phi(r)dr \ \ , \Psi = \psi(r)$$

Then the field equations become

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\psi' + \frac{\phi^2}{f^2}\psi + \frac{2}{L^2 f}\psi = 0\,,$$

$$\phi'' + \frac{2}{r}\phi' - \frac{2\psi^2}{f}\phi = 0$$

There are a two parameter family of solutions with regular horizons

Asymptotically:

$$\Phi = \mu - \frac{\rho}{r} + \cdots \qquad \Psi = \frac{\Psi^{(1)}}{r} + \frac{\Psi^{(2)}}{r^2} + \cdots\,.$$

For ψ, either falloff is normalizable. After imposing the condition that either ψ(1) or ψ(2) vanish we have a one parameter family of solutions

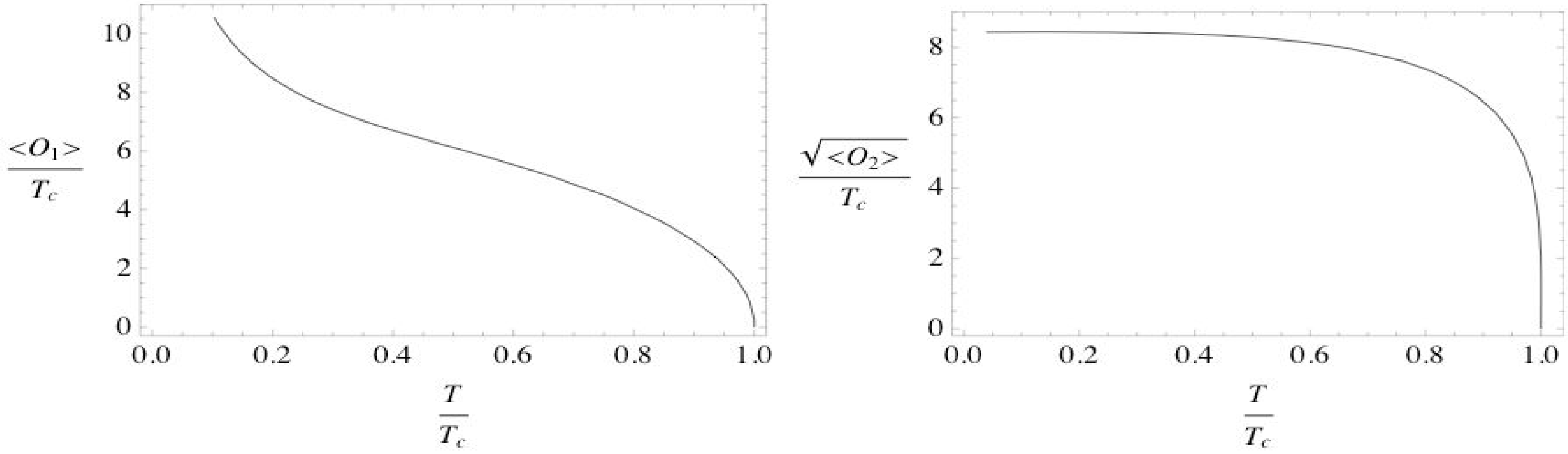
Dual Field Theory

Properties of the dual field theory are read off from the asymptotic behaviour of the solution:

μ = chemical potential, ρ = charge density

If *O* is the operator dual to ψ, then

$$\langle O_i \rangle = \sqrt{2}\Psi^{(i)}\,, \quad i = 1, 2$$



Condensate as a function of T

From
S. Hartnoll, C. Herzog, G. Horowitz
Phys. Rev. Lett. 101, 031601 (2008)

Conductivity

Consider A_x fluctuations in the bulk with time dependence of the form $e^{-i\omega t}$

$$A_x'' + \frac{f'}{f} A_x' + \left(\frac{\omega^2}{f^2} - \frac{2\Psi^2}{f} \right) A_x = 0$$

Solve this with ingoing wave boundary conditions at the horizon

The asymptotic behaviour is

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots$$

From the AdS/CFT correspondence we have

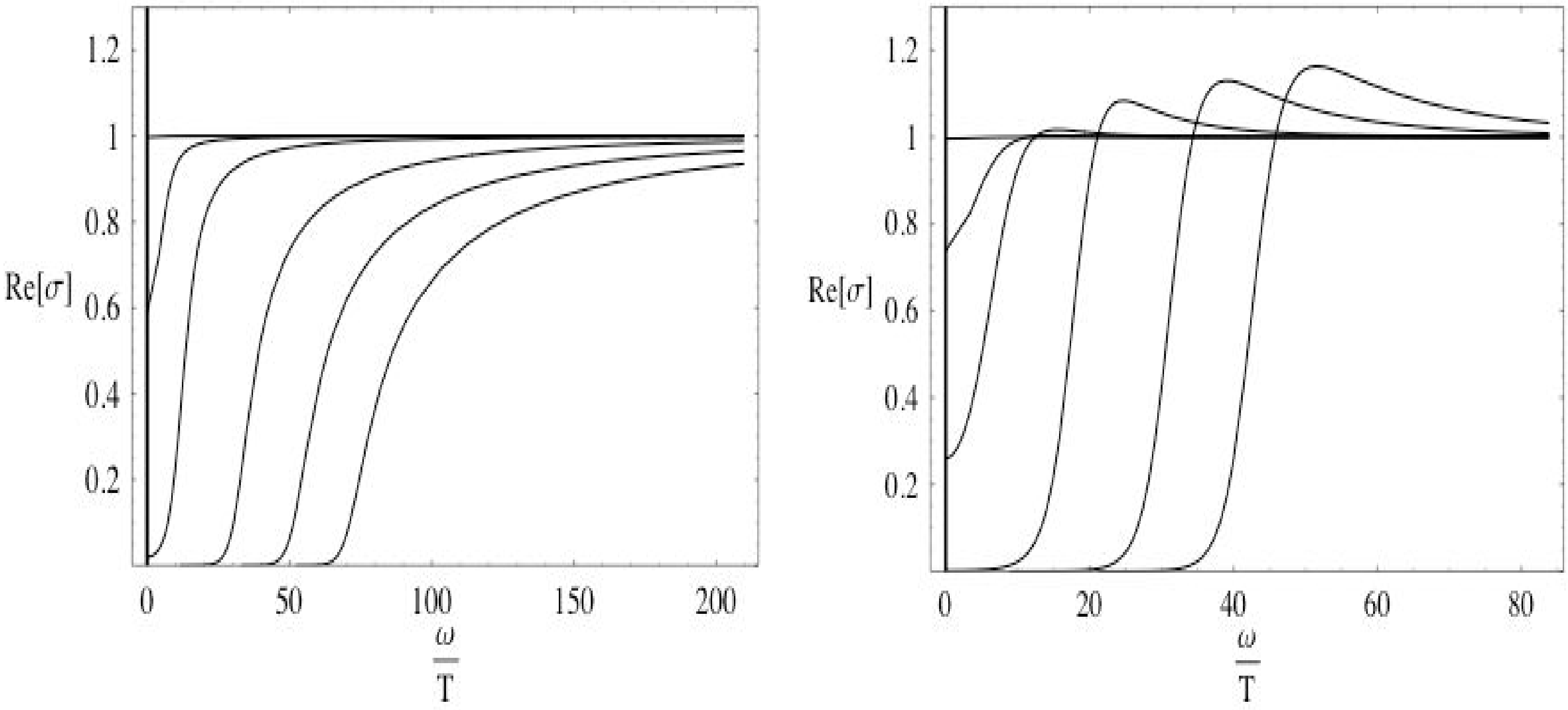
$$A_x = A_x^{(0)} \,, \qquad \langle J_x \rangle = A_x^{(1)}$$

From Ohm's law we obtain the conductivity

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\dot{A}_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -\frac{i A_x^{(1)}}{\omega A_x^{(0)}}$$

Then we get

From
S. Hartnoll, C. Herzog, G. Horowitz
Phys. Rev. Lett. 101, 031601 (2008)



Curves represent successively lower temperatures. Gap opens up for T < Tc.

Holographic Charge Density Waves

Can we construct a holographic charge density wave?

Problems which should be solved:

- The condensation is an electron-hole pair therefore it should be charge neutral
- The current on the boundary should be modulated
- The translational symmetry must be broken (completely or partially)
- The U(1) Maxwell gauge symmetry must be unbroken

The Lagrangian that meets these requirements is

$$\mathcal{L} = \frac{R+6/L^2}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m^2\Phi^2 - \frac{1}{2}{\lambda_1}^2\Phi^2\partial_\mu\vartheta\partial^\mu\vartheta$$
$$- \lambda_2^2\Phi^2\epsilon^{\mu\nu\rho\sigma}\big(B_{\mu\nu} - \partial_\mu\omega_\nu^{(2)} + \partial_\nu\omega_\mu^{(2)}\big)\big(A_\rho - \partial_\rho\omega^{(1)}\big)\partial_\sigma\vartheta$$

Where:

A_μ is a Maxwell gauge field of strength F=dA,

$B_{\mu\nu}$ is an antisymmetric field of strength H=dB

and $\omega^{(1)}, \omega_\mu^{(2)}$ are auxiliary Stueckelberg fields.

The last term

$$-\lambda_2^2 \Phi^2 (B - d\omega^{(2)}) \wedge (A - d\omega^{(1)}) \wedge d\vartheta$$

Is a topological term (independent of the metric)

The Lagrangian is gauge invariant under the following gauge transformations

$$A \to A + d\chi^{(1)},\ \omega^{(1)} \to \omega^{(1)} + \chi^{(1)}\ ,$$
$$B \to B + d\chi^{(2)},\ \omega^{(2)} \to \omega^{(2)} + \chi^{(2)}.$$

We shall fix the gauge by choosing

$$\omega^{(1)} = 0,\ \omega^{(2)} = 0.$$

Apart from the above gauge symmetries, the model is characterized by an additional global U(1) symmetry

$$\vartheta \longrightarrow \vartheta + \alpha$$

that corresponds to the translational symmetry.

This global U(1) symmetry will be spontaneously broken for $T<T_c$ in the bulk and it will give rise to the related Nambu-Goldstone mode, the phason as it is called in condensed matter physics.

One may alternatively understand this global symmetry, by unifying the fields as

$$\Psi = \Phi e^{i\lambda_1 \vartheta}$$

where λ_1 corresponds to the charge of the U(1) translational invariance.

Remarks

- In a usual condensed matter CDW, when translational symmetry is completely broken, the phason is massless and the CDW is called incommensurate or sliding
- The sliding originates from the freely propagating phason that gives rise to the Froelich supercurrent
- In spite of the dissipationless electric charge conduction, U(1) gauge invariance is intact and no Meissner effect arises.

However

if the phason is gapped then there is a remnant discrete translation symmetry that prevents sliding and suppresses the Froelich conduction. In this case, the CDW is termed commensurate or pinned.

Field Equations

By varying the metric we obtain the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R + \frac{6}{L^2} \right) = 8\pi G (T^{EM}_{\mu\nu} + T^{B}_{\mu\nu} + T^{\Phi,\vartheta}_{\mu\nu} + T^{\text{int}}_{\mu\nu})$$

By varying A_μ we obtain the Maxwell equations

$$\nabla^\mu F_{\mu\nu} = J_\nu \, , \qquad J^\mu = \lambda_2^2 \Phi^2 \epsilon^{\mu\nu\rho\sigma} B_{\nu\rho} \partial_\sigma \vartheta$$

By varying $B_{\mu\nu}$ we obtain

$$\nabla^\mu H_{\mu\nu\rho} = \lambda_2^2 \Phi^2 \epsilon_{\mu\nu\rho\sigma} A^\mu \partial^\sigma \vartheta$$

By varying $\Phi \text{ and } \vartheta$ we obtain two more field equations.

We wish to solve the field equations in the ***probe limit***

Probe limit

Consider the following rescaling

$\lambda_1 \to \lambda_1/\kappa, \lambda_2 \to \lambda_2/\kappa,$
the fields $\Phi \to \kappa\Phi, A_\mu \to \kappa A_\mu, \vartheta \to \kappa\vartheta$
and let $\kappa \to 0$

The equation for the antisymmetric field simplifies to

$$\nabla^\mu H_{\mu\nu\rho} = 0$$

which is solved by

$$H_{\mu\nu\rho} = 0$$

We shall choose the solution

$$B^{ry} = -B^{yr} = 1$$

with all other components vanishing

The Einstein equations then simplify to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R + \frac{6}{L^2} \right) = 0$$

They can be solved by the Schwarzschild black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2) \quad , \qquad f(r) = r^2 - \frac{r_h^3}{r}$$

Then the other field equations come from the Lagrangian density

$$\mathcal{L}/\kappa^2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m^2\Phi^2 - \frac{1}{2}\lambda_1^2\Phi^2\partial_\mu\vartheta\partial^\mu\vartheta - \lambda_2^2\Phi^2\varepsilon_{ry\rho\sigma}A^\rho\partial^\sigma\vartheta$$

It is independent of κ and therefore well-defined
in the probe limit $\kappa \to 0$

B. Sakita, K. Shizuya
Phys. Rev. B 42, 5586 (1990)

The resulting coupling in the probe limit of the scalar fields with the gauge field is of the chiral anomaly type in t-x spacetime

The two scalar field can alternatively be understood as a modulus and a phase of a complex field as

$$\Psi = \Phi e^{i\lambda_1\vartheta}$$

V. Yakovenko, H. Goan
Phys. Rev. B 58, 10648 (1998)

where λ_1 corresponds to the charge of the U(1) translational invariance.

Our aim is to end up with a scalar potential of the form

$$A_t \equiv V(r,x) = v(r)\cos(kx)$$

where k corresponds to the modulation wavevector

According to the AdS/CFT dictionary, the asymptotic expansion of

$$V(r,x) \sim -\mu(x) + \frac{\rho(x)}{r} + \ldots$$

will source a chemical potential

$$\mu(x) = \mu\cos(kx)$$

and a charge density

$$\rho(x) = \rho\cos(kx)$$

The emergence of the latter operator in the dual CFT, signals the formation of a CDW due to strong interactions

Remarks

- We will show that the modulated chemical potential is dynamically generated.

- The metric is sourced by a dual operator, the stress-energy tensor on the conformal boundary. It has a non-vanishing vacuum expectation value determined by the dependence of the metric on the radial coordinate r near the boundary.

- Similar considerations apply to the B field which is dual to an anti-symmetric field. However, unlike the metric, the B field is constant (independent of r), so there is no source term similar to the stress-energy tensor (in the probe limit). The effect of the B field on the boundary theory is to induce a topological interaction which leads to the formation of a CDW.

- The topological term is responsible for the instability in the bulk.

S. Hartnoll and C. Herzog
Phys. Rev. D77, 106009
(2008)

Equations of motion are

$$\Phi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\Phi' - \left\{\frac{m^2}{f} + (\vartheta')^2 + \frac{1}{r^2 f}(\vartheta_x)^2 + \frac{2}{f^2}V_t\vartheta_x\vartheta\right\}\Phi = 0\,,$$

$$\vartheta_k'' + \left[\frac{f'}{f} + 2\left(\frac{1}{r} + \frac{\Phi'}{\Phi}\right)\right]\vartheta' + \frac{1}{r^2 f}\vartheta_x^2\vartheta + \frac{1}{f^2}\vartheta_x V_t = 0\,,$$

$$V_t'' + \frac{2}{r}V_t' - \frac{1}{r^2 f}\vartheta_x^2 V_t + \frac{\Phi^2}{f}\vartheta_x\vartheta = 0$$

While in k-space they become

$$\Phi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\Phi' - \left\{\frac{m^2}{f} + \sum_k\left[\vartheta_k'\vartheta_{-k}' + \frac{k^2\vartheta_k\vartheta_{-k}}{r^2 f} - 2ik\frac{V_k\vartheta_{-k}}{f^2}\right]\right\}\Phi = 0\,,$$

$$\vartheta_k'' + \left[\frac{f'}{f} + 2\left(\frac{1}{r} + \frac{\Phi'}{\Phi}\right)\right]\vartheta_k' - \frac{k^2}{r^2 f}\vartheta_k + \frac{ikV_k}{f^2} = 0\,,$$

$$V_k'' + \frac{2}{r}V_k' - \frac{k^2}{r^2 f}V_k + \frac{ik\vartheta_k\Phi^2}{f} = 0$$

where we have considered the homogeneous solution $\Phi = \Phi(r)$

and $L = 1\,,\lambda_1 = \lambda_2 = 1$

Fourier Transforms

Suppose that the x-direction has length Lx. Then assuming periodic boundary conditions, the minimum wavevector is

$$k = \frac{2\pi}{L_x}$$

Taking Fourier transforms, the form of the field equations suggests that it is consistent to truncate the fields by including (2n+1)k-modes for ϑ and V and 2nk modes for Φ

Thus

$$\vartheta(r,x) = \sum_{n=0}^{\infty} \vartheta_{(2n+1)k}(r) e^{i(2n+1)kx} + c.c.$$
$$V(r,x) = \sum_{n=0}^{\infty} V_{(2n+1)k}(r) e^{i(2n+1)kx} + c.c.$$
$$\Phi(r,x) = \frac{1}{2}\sum_{n=0}^{\infty} \Phi_{2nk}(r) e^{2inkx} + c.c.$$

Remarks

- We consider only the contributions of the leading n = 0 terms

$$\vartheta_{\pm k} \;\; V_{\pm k} \;\; and \;\; \Phi_0$$

- Higher over tones can be included successively. At each step, we need to add equations for the new overtones which also introduce corrections to the lower Fourier modes already calculated in the previous step

- At each step the complexity of the system of coupled equations for the overtones of the three fields increases.

Boundary conditions

At the horizon:

$$\Phi' = \frac{m^2}{3r_h} \frac{1+\frac{1}{m^2}\sum_k \left[\frac{k^2\vartheta_k\vartheta_{-k}}{r_h^2} - \frac{2ik\left(V_k'\vartheta_{-k}+V_k\vartheta'_{-k}\right)}{3r_h}\right]}{1-\sum_k \frac{2ikV_k\vartheta_{-k}}{(3r_h)^2}}\Phi,$$

$$\sum_k kV_k\vartheta_{-k} = 0\,,\;\; \frac{k^2}{r_h^2}\vartheta_k - \frac{ikV_k'}{3r_h} = 3r_h\vartheta_k'\,,\;\; kV_k = 0\,,$$

and

$$kV_k = i\vartheta_k(r_h\Phi)^2,\;\; r\to r_h$$

The condition $kV_k = 0,\ \forall k$ permits us to decouple the momenta and consider a single wavevector CDW

Asymptotically

$$\Phi(r) = \frac{\langle \mathcal{O}_\Delta \rangle}{r^\Delta}, \quad \Delta = 1, 2$$

If ϑ and V are normalizable

Setting z=1/r and $\Delta = 1$ we get from the equations at infinity

$$\vartheta_k''(z) - k^2 \vartheta_k(z) + ikV_k(z) = 0$$

$$V_k''(z) - k^2 V_k(z) + ik\langle \mathcal{O}_1 \rangle^2 \vartheta_k(z) = 0$$

a system of linear coupled oscillators providing solutions of the form:

$\vartheta \sim \cos(pz) = \cos(\frac{p}{r})$ V $\sim \sin(pz) = \sin(\frac{p}{r})$

Rendering both normalizable and the $\Delta = 1$ is acceptable

We will not discuss the $\Delta = 2$ case

Transforming back to the r coordinate we have

$$\vartheta(r,x) = 2i\left[a + \frac{bu}{r} + \cdots\right]\sin(kx)\,,$$
$$V(r,x) = 2\langle\mathcal{O}_1\rangle\left[a + \frac{bu}{r} + \cdots\right]\cos(kx)\,,$$

where a and b are constants. To leading order and according to the AdS/CFT correspondence, we obtain in the dual boundary theory a single-mode CDW with a dynamically generated charge density of the form

$$\rho(x) \sim \cos(kx)$$

Observe: when the condensate $\langle\mathcal{O}_1\rangle$ is zero, i.e. for temperatures above the critical temperature, the modulated chemical potential and the charge density vanish, and that they become non zero as soon as the condensate becomes nonzero, i.e. when the temperature is lowered below Tc. Therefore, the modulated chemical potential and the charge density are spontaneously generated and do not constitute fixed parameters of controlling Tc, contrary to what happens In holographic superconductors.

Question: Which quantity controls the temperature?

In the holographic superconductors we have

$$T_c \propto \sqrt{\rho}$$

In our case we have the following scaling symmetry

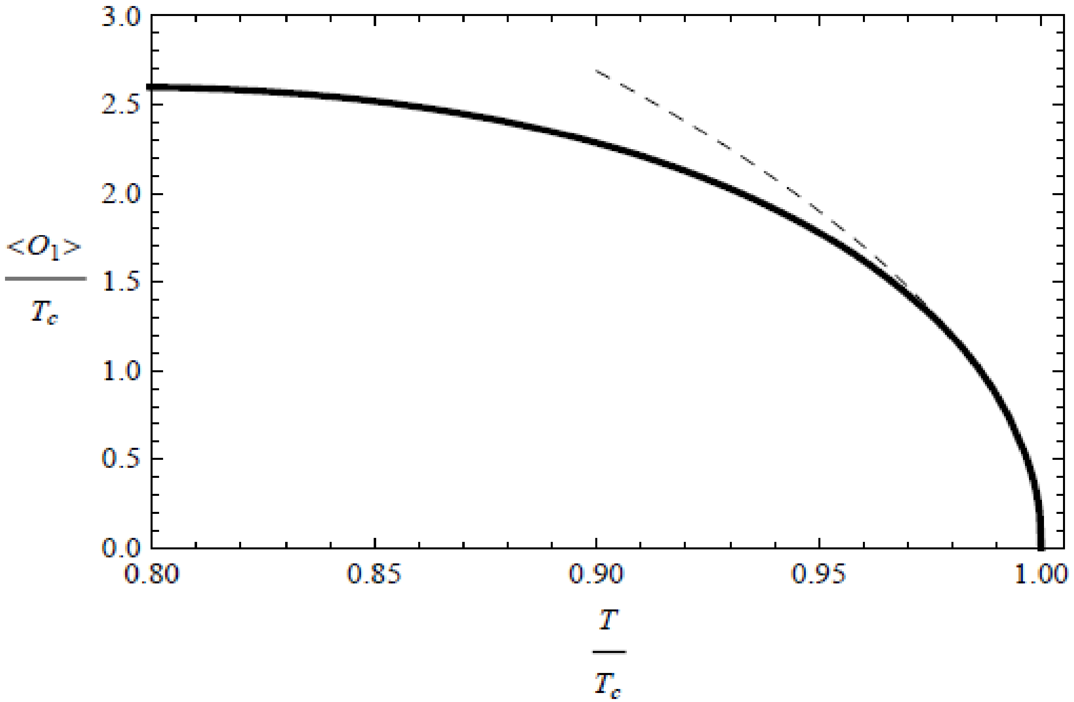
$$x \to \zeta x,\; y \to \zeta y,\; r \to r/\zeta$$

It follows that k/r_h is scale-invariant and therefore

$$T_c \propto k$$

The critical temperature is controlled by the value of the wave-vector k.

Temperature dependence of the condensate



The dashed line is the BCS fit to the numerical values near Tc.
We find

$$\langle \mathcal{O}_1 \rangle \approx 8.5 T_c \left(1 - T/T_c\right)^{1/2} \text{ near } T \to T_c$$

Remarks

- The instability shows up at T/Tc< 0.4 where the fields diverge

- The equations become unstable at low temperature

- We expect that in this regime backreaction effects on the bulk metric become significant

- The instability of numerical solutions may indicate that additional wavevectors become favourable. It is possible that for lower temperatures the system might prefer to condense in a multi-k CDW state, in order to lower its energy.

Collective excitations

Consider fluctuations of the fields

$$A_x(r,x) \to \widetilde{A}_{\omega,q}(r)e^{i(qx-\omega t)},\, V(r,x) \to V(r,x) + \widetilde{V}_{\omega,q}(r)e^{i(qx-\omega t)}$$
$$\vartheta(r,x) \to \vartheta(r,x) + \widetilde{\vartheta}_{\omega,q}(r)e^{i(qx-\omega t)}$$

Then the propagation equations are

$$\widetilde{\vartheta}''_{\omega,q} + \left[\frac{f'}{f} + 2\left(\frac{1}{r} + \frac{\Phi'}{\Phi}\right)\right]\widetilde{\vartheta}'_{\omega,q}$$
$$+ \left(\frac{\omega^2}{f} - \frac{q^2}{r^2}\right)\frac{\widetilde{\vartheta}_{\omega,q}}{f} + \frac{i\left(q\widetilde{V}_{\omega,q} + \omega\widetilde{A}_{\omega,q}\right)}{f^2} = 0\,,$$
$$\widetilde{V}''_{\omega,q} + \frac{2}{r}\widetilde{V}'_{\omega,q} + \left(\frac{\omega^2}{f} - \frac{q^2}{r^2}\right)\frac{\widetilde{V}_{\omega,q}}{f} + \frac{iq\widetilde{\vartheta}_{\omega,q}\Phi^2}{f} = 0\,,$$
$$\widetilde{A}''_{\omega,q} + \frac{f'}{f}\widetilde{A}'_{\omega,q} + \left(\frac{\omega^2}{f} - \frac{q^2}{r^2}\right)\frac{\widetilde{A}_{\omega,q}}{f} - \frac{i\omega(r\Phi)^2\widetilde{\vartheta}_{\omega,q}}{f^2} = 0$$

To study the dynamics of fluctuations in the dual CFT, take the limit $r \to \infty$ and employ the Fourier transformation $z = \frac{1}{r} \to p$

$$(\omega^2 - q^2 - p^2)\,\widetilde{\vartheta}_{\omega,q}(p) + iq\widetilde{V}_{\omega,q}(p) + i\omega\widetilde{A}_{\omega,q}(p) = 0\,,$$
$$+\,iq < \mathcal{O}_1 >^2 \widetilde{\vartheta}_{\omega,q}(p) + (\omega^2 - q^2 - p^2)\,\widetilde{V}_{\omega,q}(p) = 0\,,$$
$$-\,i\omega < \mathcal{O}_1 >^2 \widetilde{\vartheta}_{\omega,q}(p) + (\omega^2 - q^2 - p^2)\,\widetilde{A}_{\omega,q}(p) = 0\,.$$

The system defines completely the behaviour of the fluctuations and determines p as a function of q and ω

At infinity we expect solutions of the form

$$e^{i\frac{p}{r}} \sim 1 + i\frac{p}{r}$$

Then according to AdS/CFT correspondence:

$p = 0$	corresponds to	source
$p \neq 0$	corresponds to	current

For $p = 0$ we have three energy branches with dispersions

$$\omega = q, \quad \omega = q, \quad \omega = \sqrt{< \mathcal{O}_1 >^2 + q^2}$$

The first two modes correspond to massless photonic-like modes that we anticipated to find since gauge invariance still persists

However for q=0 the last mode has a mass equal to $< \mathcal{O}_1 >$ and basically corresponds to a gapped phason-like mode which originates from phason-gauge coupling

The emergence of the gap denotes the pinning of the CDW this is quite peculiar since we had not `initially' considered any modulated source that could `trap' the CDW, which implies that the resulting pinned CDW has an intrinsic origin.

The presence of a non-vanishing term in the Lagrangian $V(x)\partial_x\vartheta(x)$ demands that

If $\vartheta(x) \sim \sin(kx)$ then $V(x) \sim \cos(kx)$

This means that when the phase transition occurs, both the scalar potential and the phason field become finite and modulated by the same wavevectors $\pm k$ and the relative phase of the two periodic modulations is `locked' to $\frac{\pi}{2}$

Since these periodicities coincide, the CDW is commensurate

Conductivity

The conductivity is defined by Ohm's law

$$\sigma(\omega) = \frac{A^{(1)}_{\omega,0}}{i\omega A^{(0)}_{\omega,0}}$$

determined from the asymptotic expansion

$$\widetilde{A}_{\omega,0}(r) = \sum_p \widetilde{A}_{\omega,0}(p) e^{ip/r} = \sum_{\nu,p} A^{(\nu)}_{\omega,0}/r^\nu = \sum_p \widetilde{A}_{\omega,0}(p)\left(1 + i\frac{p}{r} + \dots\right)$$

We find two pairs of branches with

$$p = \pm\sqrt{\omega(\omega \pm < \mathcal{O}_1 >)}$$

By choosing only the ingoing contributions $p_{-,\pm}$ we obtain the dynamical conductivity

$$\sigma(\omega) = \sum_{\epsilon=\pm} \frac{c_\epsilon p_{-,\epsilon}}{\omega}$$

where

$$c_\epsilon = \widetilde{A}_{\omega,0}(p_{-,\epsilon})/\widetilde{A}^{(0)}_{\omega,0}$$

The factors c_ϵ will be determined by demanding that

$$\lim_{\omega\to\infty}[\sigma(\omega)-1]\to 0$$

Faster than $1/\omega$

in order for the Kramers-Kronig relations to hold or equivalently to ensure causality. This implies

$$c_\epsilon = \frac{1}{2}$$

Kramers-Kronig relations require the fulfillment of the Ferrell-Glover-Tinkham (FGT) sum rule, dictating that

$$\int \mathrm{d}\omega \mathrm{Re}[\sigma(\omega)] = constant$$

Remarks

- In the superconducting case, the FGT sum rule demands the presence of a $\delta(\omega)$ in $\mathrm{Re}[\sigma(\omega)]$ giving rise to a supercurrent.

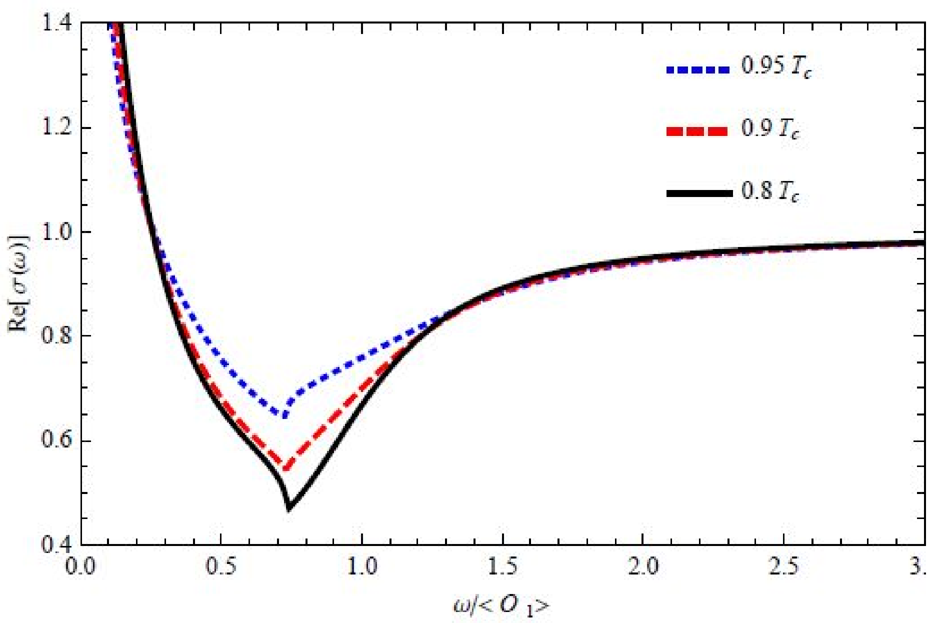
- In our case, this rule is satisfied exactly, without the need of $\delta(\omega)$
  The latter reflects the absence of the Froelich supercurrent, which may be attributed to the commensurate nature of the CDW.

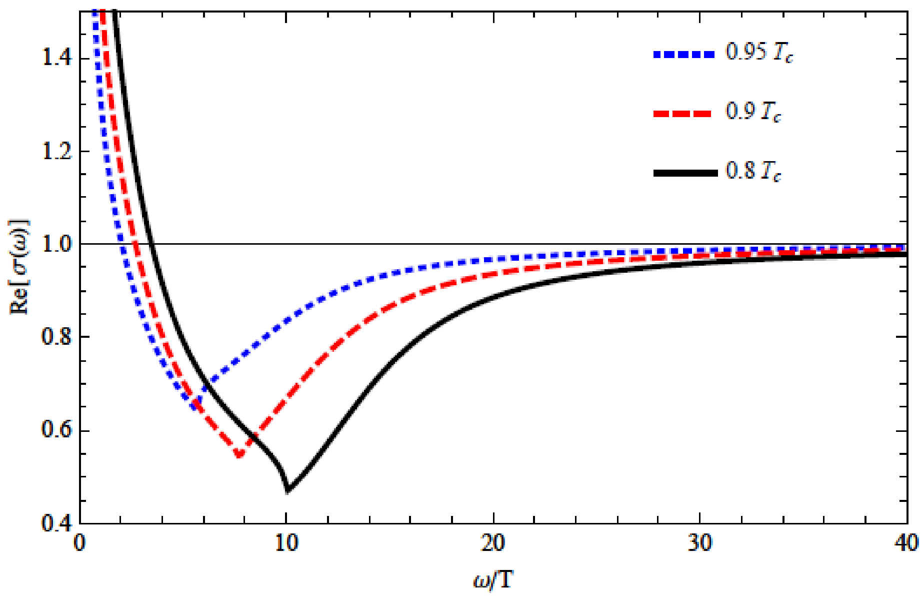
- A Drude peak is a manifestation of the presence of a $\delta(\omega)$. The absence of a Drude peak also demonstrates the translational symmetry downgrading by the commensurate CDW. If translation symmetry was intact, a Drude peak should also appear. The system is not translationally invariant anymore, since something inhomogeneous has been generated and no Drude peak appears.

- In the same time, there is a remnant translational symmetry that prevents the
  Froelich supercurrent to appear and gaps the phason.

S. Hartnoll, C. Herzog, G. Horowitz
JHEP 0812, 15 (2008)

The real part of conductivity





The numerically calculated real part of the conductivity $Re[\sigma(\omega)]$ versus normalized frequency with the condensate $\omega/\langle\mathcal{O}_1\rangle$ and temperature ω/T
In both plots, we clearly observe a `dip' that arises from the CDW formation and softens with

$$T \to T_c \text{ since } \langle\mathcal{O}_1\rangle \to 0$$

At T=Tc we retrieve the normal state conductivity $Re[\sigma(\omega)] = 1$

Conclusions

The AdS/CFT correspondence allows us to calculate quantities of strongly coupled theories (like transport coefficients, conductivity) using weakly coupled gravity theories

We presented a holographic charge density wave model where

- The charged density is dynamically modulated
- The charge density wave is commensurate
- The conductivity shows no Fröhlich supercurrent

Further study

- All fields to be spatially dependent
- Include overtones
- Extend the analysis for the 1/r2 condensate
- Backreaction (beyond the probe limit)

Discussion

- Extension to Spin Density Waves (SDW) and to Pair Density Waves (PDW)

- Can CDW be generated from a modulated Fermi Liquid?

- The holographic view of coexistence of phases

- Coexistence of CDW and Superconductivity. Modulated superconductivity? Striped superconductors?

R. Flauget, F. Pajer and S. Papanikolaou
Phys. Rev. D83, 064009, (2001)