Lifshitz and Schrödinger Algebras and Dynamical Realizations [1]

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[1] Based on G. Gibbons, J. Gomis, C. Pope, arXiv:0910.3220[hep-th], Phys.Rev. D82 (2010) 065002
R. Casalbuoni, J. Gomis J. Gomis, and K. Kamimura work in progress.

Outline

AdS/CFT correspondence

BMN sector of $AdS_5 \times S^5$[2]
Strings in a maximal Susy plane wave
Susy plane algebra is obtained by contraction of $AdS_5 \times S^5$[3]

Non-relativistic limit of $AdS_5 \times S^5$[4]
Symmetry algebra String super Newton-Hooke (NH) atring algebra

[2] Berenstein, Maldacena, Nastase (02)
[3] Hatsuda, Kamimura, Sakaguchi (02)
[4] Gomis, Gomis, Kamimura 05

Theories with violation of Lorentz symmetry

Examples

Lifshitz theories of gravity[5]
Anisotropic general relativity

Very Special Relativity[6]
The relativity group is a subgroup of Lorentz group preserving a light-like direction

Deformations of Very Special Relativity[7]
General Very special Relativity

[5] Horava (09)
[6] Cohen and Glashow (06)
[7] Gibbons, Gomis, Pope (07)

Lifshitz scalar field theories

In non-relativistic condensed matter theories with k spatial dimensions, one is interested in the behaviour of physical quantities under *Lifshitz scaling*

$$t \to \lambda^z t\,, \qquad \mathbf{x} \to \lambda \mathbf{x}$$

where t is the time variable and $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ is the spatial position vector.
Consider the action[8]

$$S = \frac{1}{2}\int dt\, d^k x \left(\dot{\phi}^2 - \phi(\triangle)^z \phi\right)$$

where $\triangle = \vec{\nabla}^2$. The scaling dimension of the Lifshitz scalar $[\phi] = \frac{k-z}{2}$.

Compared to the relativistic scalar in the same space-time the Lifhsitz scalar has an improved UV behavior.
For z=3 ϕ is dimensionless in 3+1 dimensions, therefore any non-linear polinomial interaction are power counting renormalizable.

[8] Lifshitz (41)

Non-relativistic adS/Condensed Matter Correspondence

Relativistic metrics with non-relativistic isometries like the Lifshitz and Schrödinger symmetries. These metrics could be dual we to some non-relativistic theories in CMP[9]

[9] Son (08), Balasubramanian McGreevy (08), Herzog, Rangamani, Ross (08)

Lifshitz algebra

If D generates scalings or dilatations we may combine this with space translations P_a, spatial rotations, M_{ab} and time translations H, to obtain the *Lifshitz Algebra*, $lif_z(k)$ in k spatial dimensions,

$$[D, M_{ab}] = 0\,, \qquad [D, P_a] = P_a\,, \qquad [D, H] = zH\,,$$

If $a = 1, 2, \ldots, k$ $lif_z(k)$ has dimension $\frac{1}{2}k(k+1)+2$, then the quotient $lif_z(k)/so(k)$ has dimension $k+2$ and it represents the Lifshitz spacetime .

Lifshitz spacetime

Lifshitz spacetime is a $k+2$ dimensional spacetime equipped with a metric invariant under the left action of the $(k+2)$-dimensional group generated by P_i, H and D. A Maurer-Cartan basis f is

$$e^r = \frac{dr}{r}\,, \quad e^a = \frac{dx^a}{r}\,, \quad e^0 = \frac{dt}{r^z}\,.$$

The Lifshitz metric is then

$$ds^2_{k+2} = L^2\left\{-\frac{dt^2}{r^{2z}} + \frac{dx^a dx^a}{r^2} + \frac{dr^2}{r^2}\right\},$$

with Killing vector fields corresponding to

$$M_{ab} = -(x_a\partial_b - x_b\partial_a)\,, \quad P_i = -\partial_a\,, \quad H = -\partial_t\,, \quad D = -(zt\partial_t + x^a\partial_a + r\partial_r)\,.$$

Lifshitz spacetime

The boundary metric at infinity is obtained by taking out a factor of r^2 and letting $r \to 0$:

$$ds^2_{k+2} = \frac{L^2}{r^2}\left\{-\frac{dt^2}{r^{2(z-1)}} + dx^a dx^a + dr^2\right\}$$

Thus

$$ds^2_{\text{boundary}} = dx^a dx^a - r^{2(1-z)} dt^2 \,,$$

the speed is $c(r) = r^{(1-z)}$, and

- If $z > 1$, we obtain infinite speed (the boundary lightcone opens out to a plane), Galilean theories
- If $z = 1$, we obtain finite speed (the boundary lightcone remains a cone), Relativistic theories
- If $z < 1$, we obtain zero speed (the boundary lightcone closes up to a half line), Carroll theories

The boost-extended Lifshitz algebra

One may extend the Lifshitz algebra to include boosts. The scaling dependence of K_a is then determined by its commutation relations. Since K_a is a vector we have K_a.

$$[K_c, M_{ab}] = -(\delta_{ca}K_b - \delta_{cb}K_a)$$

For the *Galilei* group,

$$\begin{aligned}[K_a, P_b] &= 0\,,\\ [K_a, H] &= P_a\,,\end{aligned}$$

which implies that we must take

$$[D, K_a] = (1-z)K_a\,.$$

For the *Carroll* group

$$\begin{aligned}[K_a, P_b] &= \delta_{ab}H\,,\\ [K_a, H] &= 0\,,\end{aligned}$$

which implies that we must take

$$[D, K_a] = (z-1)K_a\,.$$

In the case of the Poincaré group there is no choice, and one must take $z = 1$.

The Schrödinger and Extended Schrödinger algebras

In k spatial dimensions, the centrally extended $(\frac{1}{2}k(k+1)+k+3)$ dimensional *Schrödinger algebra* which we denote $\tilde{sch}_z(k)$, is obtained by adjoining Galilean boosts K_i, and a central term N to the of translations, rotations and time translations, such that

$$\begin{aligned}
\left[M_{ab}, K_c\right] &= \left(\delta_{ac}K_b - \delta_{bc}K_a\right), \\
\left[P_a, K_b\right] &= -\delta_{ab}N, \\
\left[H, K_a\right] &= -P_a.
\end{aligned}$$

One then adjoins a dilatation D,

$$\left[D, K_a\right] = (1-z)K_a, \qquad \left[D, N\right] = (2-z)N.$$

Schrödinger group

If $k = 3$ this is 12-dimensional, whereas what has been called the Schrödinger group, i.e. the conformal symmetry group of the free Schrödinger (corresponding to $z = 2$) is 13 dimensional. This is because the special conformal or temporal inversion operator has been left out.

This transformation sometimes called expansion is given

$$\vec{x}' = \frac{\vec{x}}{1 - kt}$$
$$t' = \frac{t}{1 - kt}$$

where the k is the parameter of the expansion. The infinitesimal generator of this special conformal transformation is given by

$$C = t^2 \frac{\partial}{\partial t} + t\, x^i \frac{\partial}{\partial x^i}$$

Schrödinger group

The new commutation relations are

$$\begin{aligned} [C, P_a] &= B_a, \qquad [C, B_a] = 0, \\ [C, H] &= -D, \qquad [C, D] = -C. \end{aligned}$$

(H, C, D) form the conformal algebra in one dimensions $SO(1,2)$.

Notice the difference with the Galilean Conformal algebra obtained by contraction from the relativistic conformal algebra which has 15 generators

ISIM and DISIM$_b$

Cohen and Glashow have made the proposal that the local laws of physics need not be invariant under the full Lorentz group, generated by $M_{\mu\nu}$, but rather, under a SIM(2) subgroup,

$$M_{+a}, M_{ab}, M_{+-} = M_{03},$$

(with *i* and *b* ranging over the values 1 and 2) . This they referred to as *Very Special Relativity*. Taking the semi-direct product with the translations (P_+, P_-, P_a) gives an 8-dimensional subgroup of the Poincaré group called ISIM(2)

$$\begin{aligned} &[M_{+-}, P_\pm] = \mp P_\pm\,, \quad [M_{+-}, M_{+a}] = -M_{+a}\,, \\ &[J, P_a] = \epsilon_{ab} P_b\,, \quad [J, M_{+a}] = \epsilon_{ab} M_{+b}\,, \\ &[M_{+a}, P_-] = P_a\,, \quad [M_{+a}, P_b] = -\delta_{ab} P_+. \end{aligned}$$

where $J \equiv M_{ab}$.

DISIM$_b$(2)

In order to see if very special relativity with curved space, one can find the deformations of ISM(2) algebra. One obtain DISIM$_b$(2)[10]

$$[M_{+-}, P_\pm] = -(b \pm 1)P_\pm \,, \quad [M_{+-}, P_a] = -bP_a \,,$$

which does not describe a curved since the translations commute. Howvever one can show that[11]

$$\tilde{sch}_z(k) \equiv disim_b(k) \,, \qquad b = \frac{1}{1-z} \,.$$

To see this, one must identify the generators as follows;

$$H \leftrightarrow P_- \,, \qquad N \leftrightarrow -P_+ \qquad P_a \leftrightarrow P_a \,, \qquad K_a \leftrightarrow M_{+a} \,.$$

and

$$D \leftrightarrow (z-1)M_{+-} \,.$$

[10] Gibbons, Gomis, Pope (07)
[11] Gibbons, Gomis, Pope (09)

Dynamical realizations. Lifshitz particle

The starting point is the Lifshitz algebra with generators $H, \vec{P}, M_{ab}, D$ associated to spacetime translations, rotations and dilatations. The algebra is given by

$$\begin{aligned}
[M_{ab}, M_{cd}] &= -i\left(\eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc}\right), \\
[P_a, M_{bc}] &= -i\left(\eta_{ab}P_c - \eta_{ac}P_b\right),
\end{aligned}$$

$$[D, H] = -iz\,H, \quad [D, P_a] = -i\,P_a$$

We consider the coset $\frac{\text{Lifshitz}}{\text{Rotations}}$ that we locally parametrize as

$$g = e^{-iHt}\, e^{iP_a x^a}\, e^{iD\sigma}$$

Lifshitz particle

The Maurer Cartan is given by

$$\Omega = -ig^{-1}dg = HL_H + P_a L_P^a + DL_D$$

where

$$\begin{aligned} L_H &= -e^{-z\sigma}\, dt, \\ L_P^a &= e^{-\sigma}\, dx^a, \\ L_D &= d\sigma. \end{aligned}$$

The global symmetries are

$$\begin{aligned} H &\;;\; \delta t = \epsilon^0, \\ P_a &\;;\; \delta x^a = \epsilon^a, \\ D &\;;\; \delta\sigma = \epsilon, \delta t = z\epsilon t, \delta x_a = \epsilon x_a \end{aligned}$$

Lifshitz particle

An action invariant under Lifshitz and diffeomorphism with the lowest order in derivative is given by

$$L = a\,L_H + b\sqrt{L_P^2} \;\; = \;\; -a\,e^{-z\sigma}\,(\dot{t}) + b\,e^{-\sigma}\sqrt{(\dot{x}^a)^2}.$$

where the coefficients a and b are dimension full parameters, $[a] = M^z, [b] = M$
The momenta are

$$\begin{aligned} p_t &+ a\,e^{-z\sigma} = 0, \\ p_a &= b\,e^{-\sigma}\frac{\dot{x}_a}{\sqrt{(\dot{x}^a)^2}}, \quad \rightarrow \quad p_a^2 - b^2\,e^{-2\sigma} = 0, \\ p_\sigma &= 0. \end{aligned}$$

We have two second class constraints

$$p_\sigma = 0, \qquad e^{-z\sigma} = -\frac{p_t}{a},$$

Lifshitz particle

If $a = b^z$ the first class constraint can be written as

$$\phi = \frac{1}{2}\left(p_t^2 - (\vec{p}^2)^z\right) = 0.$$

There is a certain degree of arbitrariness in presenting the power of p_t.

Canonical form of the action

$$L = -p_t(\sigma, \vec{x})\, \dot{t} + \vec{p}(\sigma, \vec{x})\, \dot{\vec{x}}$$

that we can write as[12]

$$L_C = -p_t\, \dot{t} + \vec{p}\, \dot{\vec{x}} - \frac{e}{2}\left(p_t^2 - (\vec{p}^2)^z\right)$$

[12] S.Kalyana Rama, (2010), D. Capasso, A.P. Polychronakos (2009) T. Suyama (2009) L. Sindoni (2009), A. Mosaffa (2010), M. Eune, W. Kim (2010)]

Quantization Lifshitz particle

Following the Dirac's procedure of quantization we should impose the first class constraint on the physical states. We have

$$(\partial_t^2 - \triangle^z)\phi(\vec{x}, t) = 0$$

which gives the equation of motion of the free Lifshitz scalar.

Lifshitz particle

Since the σ variable is non dynamical we can use their equations of motion to eliminate it. The lagrangian depending only on x's and t is[13]

$$L \sim \left(\dot{\vec{x}}\dot{\vec{x}}\right)^{\frac{z}{2(z-1)}} \dot{t}^{\frac{1}{1-z}}$$

which is a Finslearian lagragian. The Finslerian line element is given by

$$ds = \left(\delta_{ij} dx^i dx^j\right)^{\frac{z}{2(z-1)}} \, dt^{\frac{1}{1-z}}$$

[13] Romero, Cuesta, Garcia, Vergara 2009

Lifshitz particle

Note there are not WZ terms.

If we consider higher order derivatives we could have add the extrinsic curvature term of the spatial dimensional curve $\vec{x}(\tau)$

$$\frac{1}{(\dot{x}^2)}\sqrt{(\ddot{x}\dot{x})^2 - (\ddot{x}^2)(\dot{x}^2)}.$$

The lagrangian in this case will be

$$L = -a\,e^{-z\sigma}\,(\dot{t}) + b\,e^{-\sigma}\,\sqrt{(\dot{x}^a)^2} + \frac{1}{(\dot{x}^2)}\sqrt{(\ddot{x}\dot{x})^2 - (\ddot{x}^2)(\dot{x}^2)}.$$

Lifshitz particle

Note that if we add to the Lifshitz algebra, the boost generators $\vec{K}$ and the "central" element N, we obtain the schrodinger$_z$ algebra which is isomorphic to DISim(b). We know that in this case the line element is also Finslearian.[14]

$$L = -m(-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu)^{(1-b)/2} (-n_\rho \dot{x}^\rho)^b \,.$$

The Schrodinger algebra is generated by $(H, P_a, B_a, J_a, Z, D, C)$. Since (B_a, J_a, Z, C) have a closed subalgebra we can consider a coset $\frac{Schrodinger}{(B_a, J_a, Z, C)}$. For z=2 the particle invariant under the Schrodinger algebra is

$$L = \frac{\dot{x}^2}{\dot{t}}$$

[14] Bogoslovsky(06), Gomis, Gibbons, Pope (07)

z=1 Lifshitz particle

For z=1 we should consider the initial lagrangian and $(\dot{t}) + \sqrt{(\dot{x}^a)^2} \neq 0$, $a = b$, as

$$L = \; = \; e^{-\sigma}(-(\dot{t}) + \sqrt{(\dot{x}^a)^2}) = \frac{e^{-\sigma}}{((\dot{t}) + \sqrt{(\dot{x}^a)^2})}(-(\dot{t})^2 + (\dot{x}^a)^2)$$
$$= \; \frac{e^{-\sigma}}{((\dot{t}) + \sqrt{(\dot{x}^a)^2})}(\dot{x}_\mu \dot{x}^\mu) \equiv \frac{\dot{x}_\mu \dot{x}^\mu}{2\tilde{e}}.$$

It is the massless relativistic particle lagrangian if we regard the coefficient as ein-bein.

Note that in this case dilatation invariance implies conformal invariance

Uniqueness of Lifshitz particle action

Another possible invariant action, for $z > 1$, is

$$L = \sqrt{\beta L_P^2 - \alpha L_H^2} = \sqrt{\beta e^{-2\sigma}\, \dot{\vec{x}}^2 - \alpha\, e^{-2z\sigma}\, \dot{t}^2}.$$

where α, β are positive dimensionfull parameters $[\alpha] = M^{2z}, [\beta] = M^2$.

$$\begin{aligned} p_t &= \frac{-\alpha\, e^{-4z\sigma}\, \dot{t}}{L}, \\ p_a &= \frac{\beta\, e^{-2\sigma}\, \dot{x}_a}{L}, \quad \rightarrow \quad \phi = \frac{1}{\beta}\left(\frac{p_a}{e^{-\sigma}}\right)^2 - \frac{1}{\alpha}\left(\frac{p_t}{e^{-z\sigma}}\right)^2 - 1 = 0 \\ p_\sigma &= 0. \end{aligned}$$

Uniqueness of Lifshitz particle

There appear two primary constraints and one secondary from $\dot{p}_\sigma = 0$

$$\frac{1}{\beta}(\frac{p_a}{e^{-\sigma}})^2 - z\frac{1}{\alpha}(\frac{p_t}{e^{-z\sigma}})^2 = 0.$$

If we choose

$$\alpha = \beta^z z^z (\frac{1}{z-1})^{\frac{1}{z-1}}$$

we have the first class constraint

$$p_t^2 - (\vec{p}^2)^z = 0$$

It gives the same dispersion relation as the previous lagrangian

Lifshitz Lagrangian with further breaking of anisotropy

Let further break the anisotropy among the spatial directions

$$t \to e^{z\epsilon} t, \qquad x^1 \to e^{z_1 \epsilon} x^1, \quad x^i \to e^{\epsilon} x^i \quad (j = 2, ...d-1).$$

the coset is given by

$$g = e^{-itH} e^{ix^1 P_1} e^{ix^i P_i} e^{i\sigma D}$$

The MC forms are

$$\begin{aligned} L_H &= -e^{-z\sigma}\, dt, \\ L_P^1 &= e^{-z_1\sigma}\, dx^1, \\ L_P^i &= e^{-\sigma}\, dx^i, \\ L_D &= d\sigma. \end{aligned}$$

An invariant lagrangian is

$$L = a\, L_H + bL_P^1 + c\sqrt{L_P^2} = -a\, e^{-z\sigma}\, (\dot{t}) + b\, e^{-z_1\sigma}\, (\dot{x}_1) + c\, e^{-\sigma}\, \sqrt{(\dot{x}^i)^2}.$$

Lifshitz particle

there are two first class constraints, $a = c^z, a = b$

$$\phi = \frac{1}{2}\left(p_t^2 - (p_a^2)^z\right), \quad p_t = (-p_1)^{\frac{z}{z_1}}.$$

For the quantization one should impose the two first class constraints on the physical states.

Bosonic Lifshitz string

To construct a Lifshitz string we first consider the analogous of the string Galilei algebra. We distinguish among the longitudinal directions, x^0, x^1, and transverse directions of the string.

A Lifshitz string algebra is

$$[D, P_a] = P_a, \quad [D, H] = izH, \quad [D, P_1] = izP_1$$

The string theory is invariant under

$$x^\mu \to e^{z\epsilon} x^\mu, \qquad x^a \to e^{\epsilon} x^a, \quad (\mu = 0, 1;\ a = 2, ...d-1).$$

We consider the coset $\frac{\text{Lifshitzstring}}{\text{TransverseRotations}}$ that we local parametrize as

$$g = e^{ix^\mu P_\mu} e^{ix^a P_a} e^{i\sigma D}$$

Bosonic Lifshitz string

The Maurer Cartan is given by

$$\Omega = e^{\mu} P_{\mu} + e^{i} P_{i} = -e^{-z\sigma} dx^{\mu} P_{\mu} + e^{\sigma} dx^{a} P_{a} + d\sigma D$$

An action invariant under Lifshitz and diffeomorphism with the lowest order in derivative is given by

$$\begin{aligned} S &= S_{\|} + S_{\perp} = \int d\tau d\sigma \left(a\, e^{-2z\rho} \sqrt{-\det g_{\|}} + b\, e^{-2\rho} \sqrt{\det g_{\perp}} \right), \\ g_{\| ij} &= \eta_{\mu\nu} \partial_i x^{\mu} \partial_j x^{\nu} = -(\epsilon_{\mu\nu} \partial_i x^{\mu} \partial_j x^{\nu} \epsilon^{ij})^2, \\ g_{\perp ij} &= \eta_{ab} \partial_i x^{a} \partial_j x^{b}. \end{aligned}$$

Bosonic Lifshitz string

The Hamiltonian becomes

$$\mathcal{H} = \int d\sigma \left(\lambda^0_{\parallel} H^* + \lambda_{\parallel} T_{\parallel} + \lambda^1_{\perp} T_{\perp} \right)$$

where

$$\begin{aligned} H^* &= \frac{1}{2}(p_{\parallel}^2 + a^2 \left(\frac{p_{\perp}^2}{b^2 \, {x'_{\perp}}^2} \right)^z {x'_{\parallel}}^2) = 0, \\ T_{\parallel} &= p_{\parallel} x'_{\parallel} = 0, \\ T_{\perp} &= p_{\perp} x'_{\perp} = 0, \end{aligned}$$

One can prove that $\left(\frac{p_{\perp}^2}{b^2 \, {x'_{\perp}}^2} \right)$ is a constan of motiont. For z=1 does not reproduce the tensionless string.

Finslearian string

Now we would like to construct the Lifshitz string in terms in the x's and t space,

$$e^{-\sigma} = \left(\frac{-b\sqrt{detg_{ij}}}{a\epsilon_{\mu\nu}\partial_i x^\mu \partial_j x^\nu \epsilon^{ij}} \right)^{\frac{1}{2(z-1)}}$$

using this expression we get

$$L* \sim (\sqrt{detg_{ij}})^{\frac{z}{z-1}} (\epsilon_{\mu\nu}\partial_i x^\mu \partial_j x^\nu \epsilon^{ij})^{\frac{1}{z-1}}$$

which for z=2 gives

$$L* \sim \frac{detg_{ij}}{\epsilon_{\mu\nu}\partial_i x^\mu \partial_j x^\nu \epsilon^{ij}} = \frac{detg_{ij}}{\sqrt{-detg_{\mu\nu}}}$$

Non-relativistic String

It does not coincide with the non-relativistic string[15]

$$S = -\frac{1}{2} T \int d^2\sigma \sqrt{-\det g}\, g^{ij} \partial_i X^a \partial_j X^b \delta_{ab}$$

where g^{ij} is the inverse of the two dimensional metric

$$g_{ij} = \partial_i x^\mu \partial_j x^\nu \eta_{\mu\nu},$$

the worldsheet coordinates are τ and σ, $\sigma^i = (\tau, \sigma), i = 1, 2$.
The NR string can be obtained as semiclassical expansion along an straight Wilson line[16]

[15] Gomis Ooguri (2000), Guijosa et al (2000) Gomis, Gomis, Kamimura (2005)

[16] Sakaguchi and Yoshida (2007)

Other bosonic string actions

We could also consider the NG like string action

$$S = \int d\tau d\sigma \sqrt{-\det g_{mn}} = \int d\tau d\sigma \sqrt{(\dot{y}y')^2 - \dot{y}^2 y'^2},$$

where

$$\begin{aligned} \dot{y}^2 &= -e^{-2z\sigma}\dot{t}^2 + e^{-2\sigma}\dot{\vec{x}}^2 \\ y'^2 &= -e^{-2z\sigma}t'^2 + e^{-2\sigma}\vec{x}'\vec{x}' \\ \dot{y}y' &= -e^{-2z\sigma}\dot{t}t' + e^{-2\sigma}\dot{\vec{x}}\vec{x}' \end{aligned}$$

For z=1

$$S = \int d\tau d\sigma\, e^{-\sigma} \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2},$$

whic reproduces the Poincare tensionless string

Super Lifhsitz algebras

The superLifshitz algebra in 10 dimensions is given

$$
\begin{aligned}
[M_{ab}, M_{cd}] &= -i\left(\eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc}\right), \\
[P_a, M_{bc}] &= -i\left(\eta_{ab}P_c - \eta_{ac}P_b\right), \\
[Q_\pm, M_{ab}] &= \frac{i}{2}Q_\pm\Gamma_{ab}, \\
\{Q_-, Q_-\} &= -2\left(C\Gamma^0\mathbf{P}_-\right)H, \\
\{Q_+, Q_-\} &= 2\left(C\Gamma^a\mathbf{P}_-\right)P_a,
\end{aligned}
$$

and

$$[D, H] = -in_H\, H, \quad [D, P_a] = -in_P\, P_a, \quad [D, Q_\pm] = -in_\pm\, Q_\pm.$$

The Jacobi identities require the weights of D as

$$n_H = 2\, n_-, \qquad n_P = n_- + n_+.$$

For example

$$n_H = z, \quad n_P = 1, \quad n_- = \frac{z}{2} \quad n_+ = 1 - \frac{z}{2}.$$

Super Lifhsitz algebras

We use projection operators

$$\mathbf{P}_{\pm} = \frac{1}{2}(1 \pm \Gamma_*), \qquad (\Gamma_* = \Gamma^0\Gamma_{11},\ \eta = (- + + + ...),\ \eta_{11} = +1),$$

to define

$$Q_{\pm} = Q\mathbf{P}_{\pm}.$$

Super Lifshitz particle in 10 dimensions

$$g = e^{-iHt}\, e^{iP_a x^a}\, e^{iQ_{-\alpha}\theta_-^\alpha}\, e^{iQ_{+\alpha}\theta_+^\alpha}\, e^{iD\sigma}$$

we get

$$\begin{aligned}
L_H &= e^{-n_H\sigma}\,(dt + i\,\bar\theta_-\Gamma^0 d\theta_-),\\
L_P^a &= e^{-n_P\sigma}\,(dx^a + 2i\,\bar\theta_+\Gamma^a d\theta_-),\\
L^{ab} &= 0,\\
L_+^\alpha &= e^{-n_+\sigma}\,d\theta_+,\\
L_-^\alpha &= e^{-n_-\sigma}\,d\theta_-,\\
L_D &= d\sigma.
\end{aligned}$$

and satisfy the MC equation

Super Lifshitz particle in 10 dimensions

Global symmetries are

$$
\begin{array}{rcl}
H & ; & \delta t = \epsilon^0, \\
P_a & ; & \delta x^a = \epsilon^a, \\
Q_+ & ; & \delta\theta_+^\alpha = \epsilon_+^\alpha,\ \delta x^a = 2i\bar{\theta}_-\Gamma^a\epsilon_+ \\
Q_- & ; & \delta\theta_-^\alpha = \epsilon_-^\alpha,\ \delta t = i\bar{\theta}_-\Gamma^0\epsilon_-,\ \delta x^a = 2i\bar{\theta}_+\Gamma^a\epsilon_- \\
D & ; & \delta\sigma = \epsilon,\ \delta\theta_\pm = n_{\pm\epsilon}\,\theta_\pm,\ \delta t = n_{H\epsilon}\,t,\ \delta x^a = n_{P\epsilon}\,x^a.
\end{array}
$$

A possible invariant action for a particle is (using second parametrization)

$$
\begin{aligned}
L = a\,L_H + b\sqrt{L_P^2} \quad &= \quad a\,e^{-n_H\sigma}\,(\dot{t} + i\,\bar{\theta}_-\Gamma^0\dot{\theta}_-) + b\,e^{-n_P\sigma}\sqrt{(\dot{x}^a + 2i\,\bar{\theta}_+\Gamma^a\dot{\theta}_-)^2}. \\
&= \quad a\,e^{-n_H\sigma}\,u^t + b\,e^{-n_P\sigma}\sqrt{(u^a)^2}.
\end{aligned}
$$

No WZ term and no kappa symmetry for general z.

Super Lifshitz particle in 10 dimensions

$$
\begin{aligned}
p_t &- a\, e^{-n_H \sigma} = 0, \\
p_a &= b\, e^{-n_P \sigma} \frac{u_a}{\sqrt{(u^a)^2}}, \quad \rightarrow \quad p_a^2 - b^2\, e^{-2n_P \sigma} = 0, \\
\zeta_- &- i\bar{\theta}_- \Gamma^0 p_t - 2i\bar{\theta}_+ \Gamma^a p_a = 0, \\
\zeta_+ &= 0, \\
p_\sigma &= 0.
\end{aligned}
$$

Four of them are second class

$$
p_\sigma = 0, \qquad e^{-n_H \sigma} = \frac{p_t}{a},
$$

$$
\zeta_- = i\bar{\theta}_- \Gamma^0 p_t + 2i\bar{\theta}_+ \Gamma^a p_a, \qquad \zeta_+ = 0,
$$

Super Lifshitz particle in 10 dimensions

and one is the first class

$$\phi = \frac{1}{2}\left(\left(p_t^2\right)^{n_P} - \left(p_a^2\right)^{n_H}\right) = 0.$$

Therefore there is no kappa symmetriy in this model.

Conclusions

We have constructed two actionsion for a Lifshitz bosonic particle that leads to the same Lifshitz dispersion relation

We have also constructed actions for a Lifshitz bosonic strings

We have constructed a Super Lifshitz algebra and a super particle realization