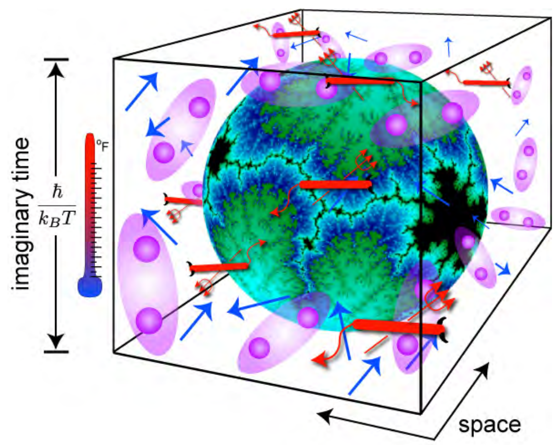
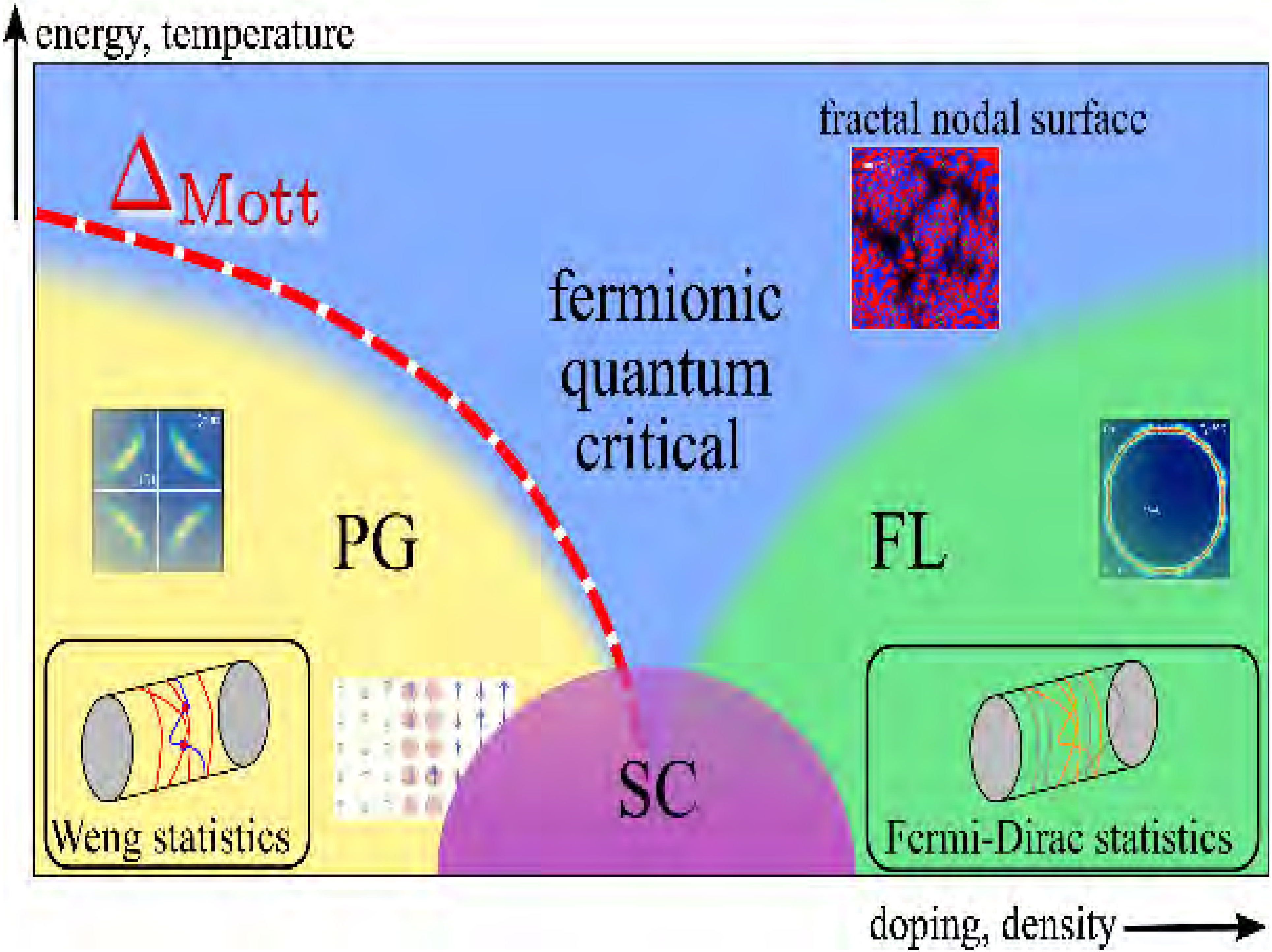
Mott collapse and statistical quantum criticality

Jan Zaanen

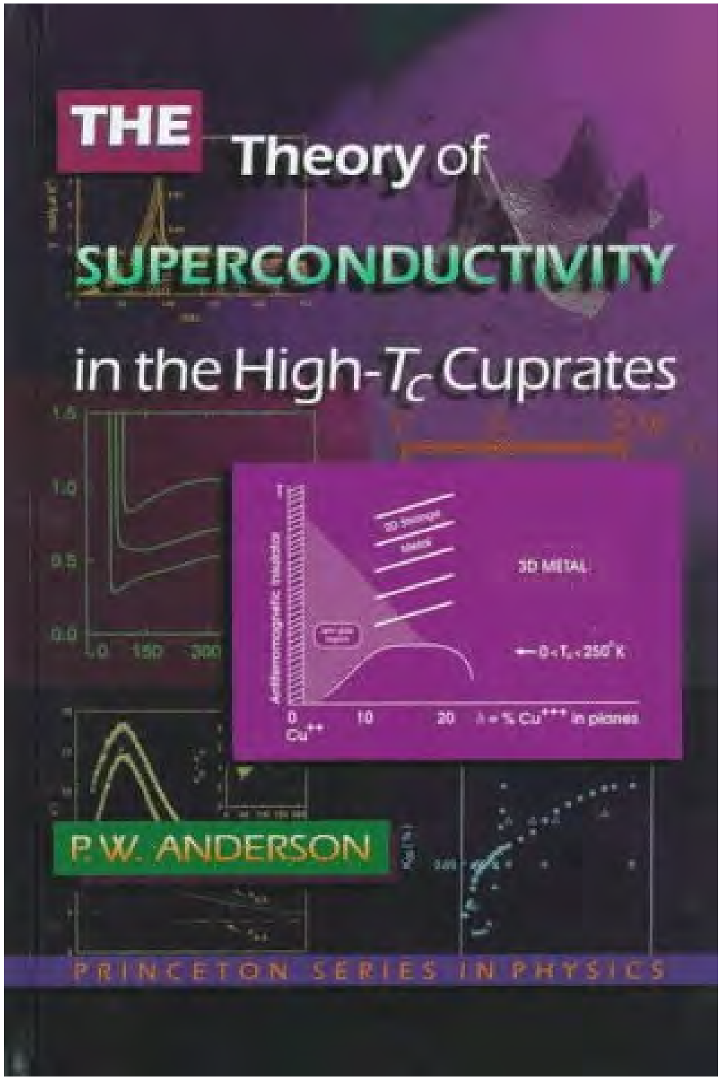


J. Zaanen and B.J. Overbosch, arXiv: 0911.4070 (Phil.Trans.Roy.Soc. A, in press)

Nederlandse Organisatie voor Wetenschappelijk Onderzoek
Universiteit Leiden
Instituut-Lorentz for theoretical physics
FOM

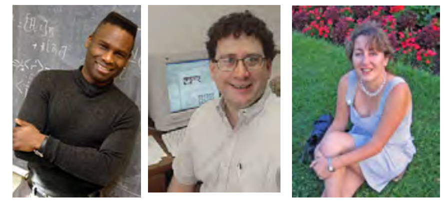

energy, temperature
Δ_{Mott}
fractal nodal surface
fermionic quantum critical
PG
FL
SC
Weng statistics
Fermi-Dirac statistics
doping, density

Who to blame …


THE Theory of
SUPERCONDUCTIVITY
in the High-T_c Cuprates
3D METAL
P. W. ANDERSON
PRINCETON SERIES IN PHYSICS

Plan

1. The idea of Mott collapse



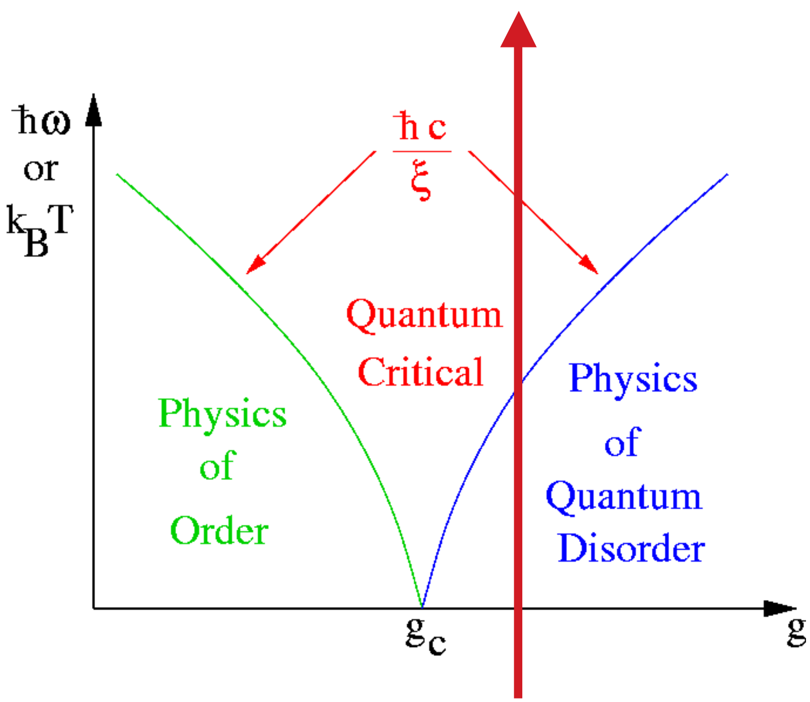
2. Mottness versus “Weng statistics”

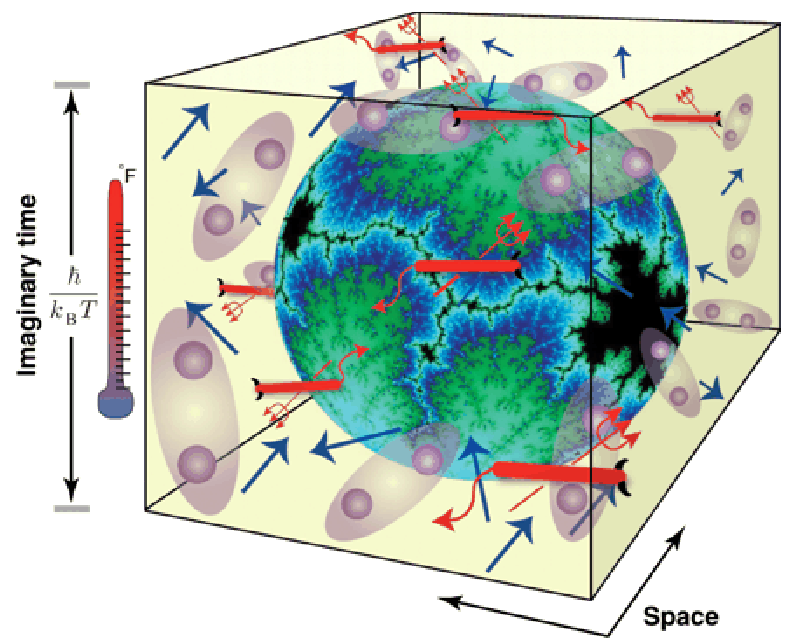
3. Statistics and t-J numerics

4. Fermions, scale invariance and Ceperley’s path integral

Quantum Phase transitions

Quantum scale invariance emerges naturally at a zero temperature continuous phase transition driven by quantum fluctuations:

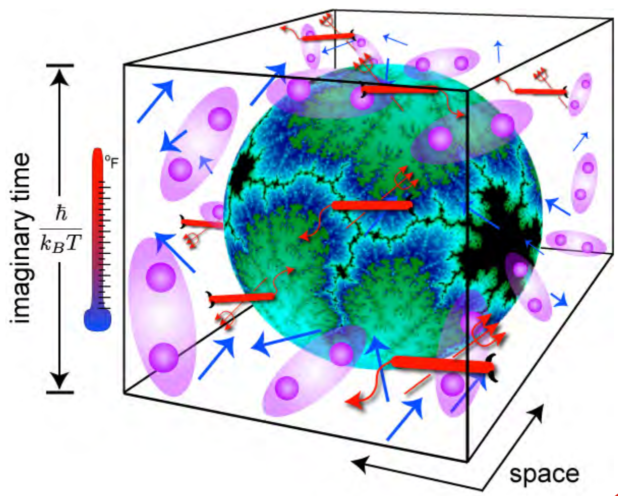


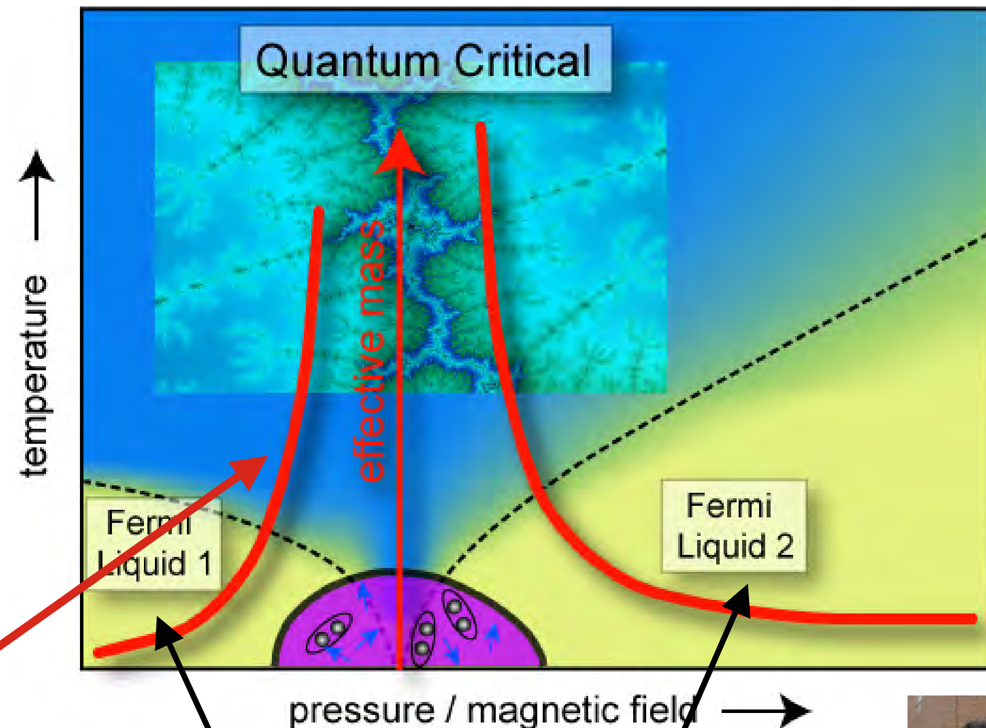


JZ, Science 319, 1205 (2008)

Fermionic quantum phase transitions in the heavy fermion metals

JZ, Science 319, 1205 (2008)





QP effective mass

$$m^* = \frac{1}{E_F}$$

$$E_F \to 0 \Rightarrow m^* \to \infty$$

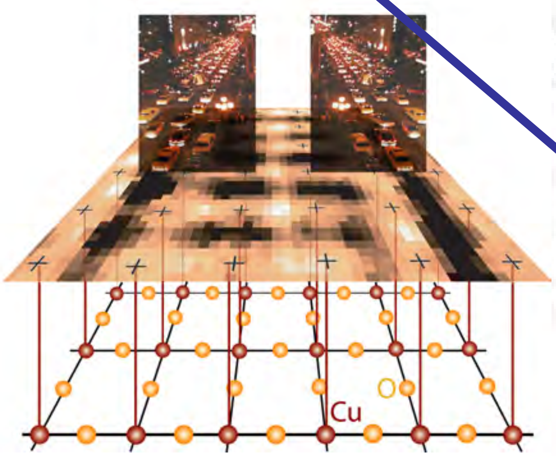
Paschen et al., Nature (2004)

‘bad players’

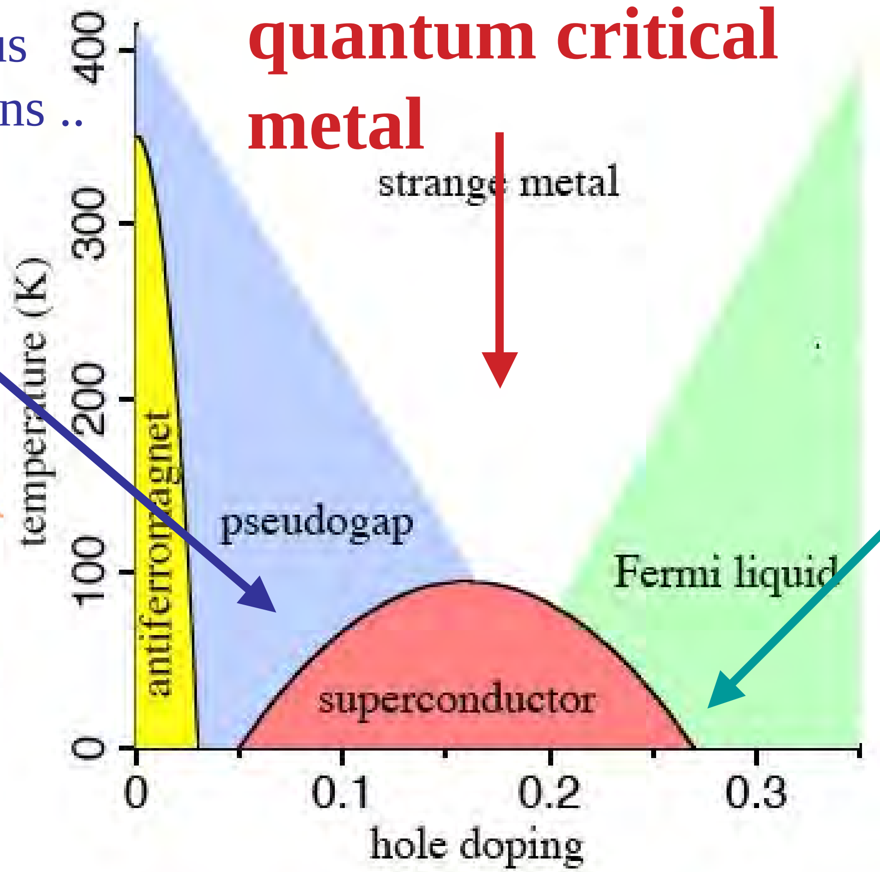
Phase diagram high Tc superconductors

Mystery quantum critical metal

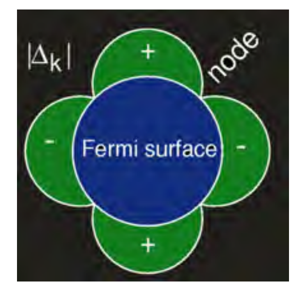
'Stripy stuff', spontaneous currents, phase fluctuations ..



JZ, Science **315**, 1372 (2007)

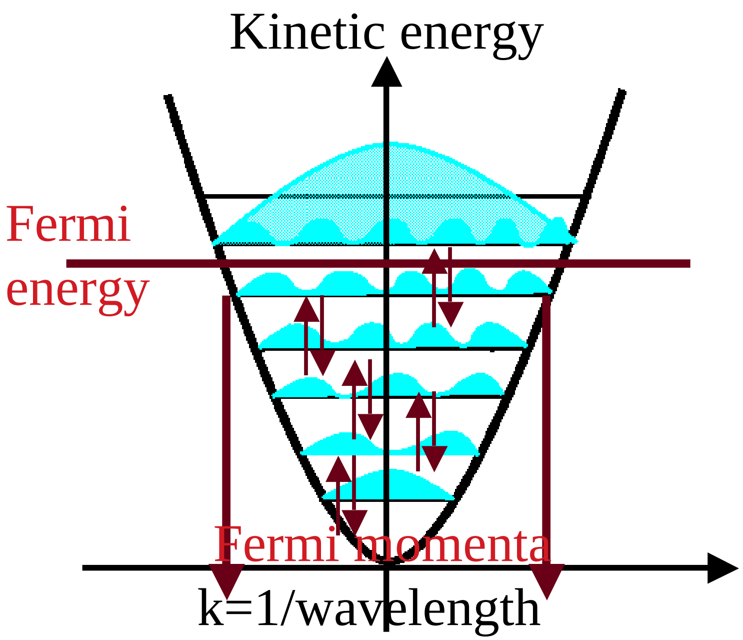


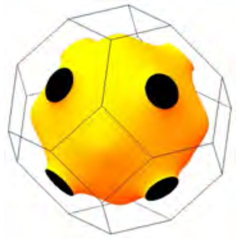
The return of normalcy



$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |vac.\rangle$$

The quantum in the kitchen: Landau's miracle





Fermi surface of copper

Electrons are waves

Pauli exclusion principle: every state occupied by one electron

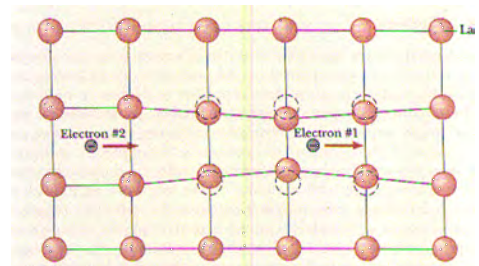
Unreasonable: electrons strongly interact !!

Landau's Fermi-liquid: the highly collective low energy quantum excitations are like electrons that do not interact.

BCS theory: fermions turning into bosons

Bardeen Cooper Schrieffer

Fermi-liquid + attractive interaction

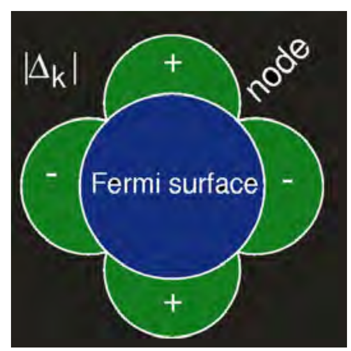


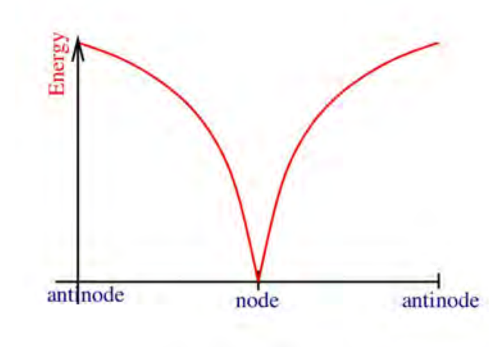
Quasiparticles pair and Bose condense:

Ground state

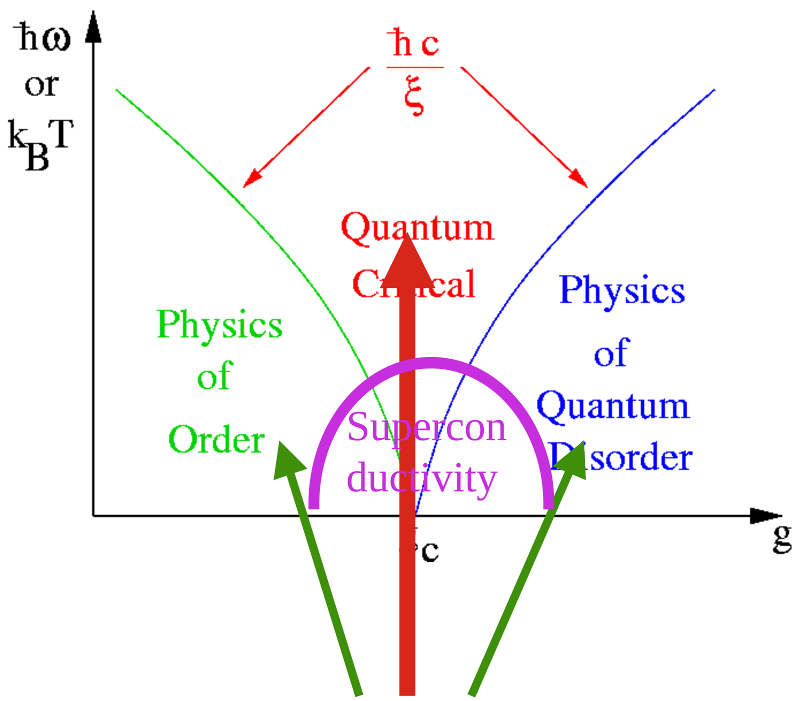
$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |vac.\rangle$$

D-wave SC: Dirac spectrum





Fermions and Hertz-Millis-Moriya-Lonzarich



Fermi gas

Bosonic (magnetic, etc.) order parameter drives the phase transition

Electrons: fermion gas = heat bath damping bosonic critical fluctuations

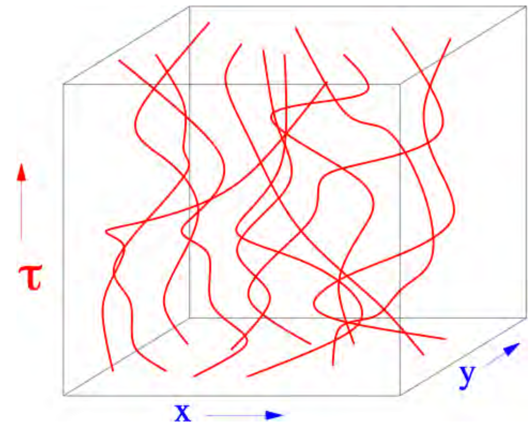
Bosonic critical fluctuations 'back react' as pairing glue on the electrons

Fermion sign problem

Imaginary time path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\,\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

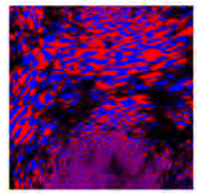
- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

Mottness and quantum statistics

‘Weng’ statistics

‘Resonating Valence Bond’ orderly physics: stripes competing with d-wave superconductivity

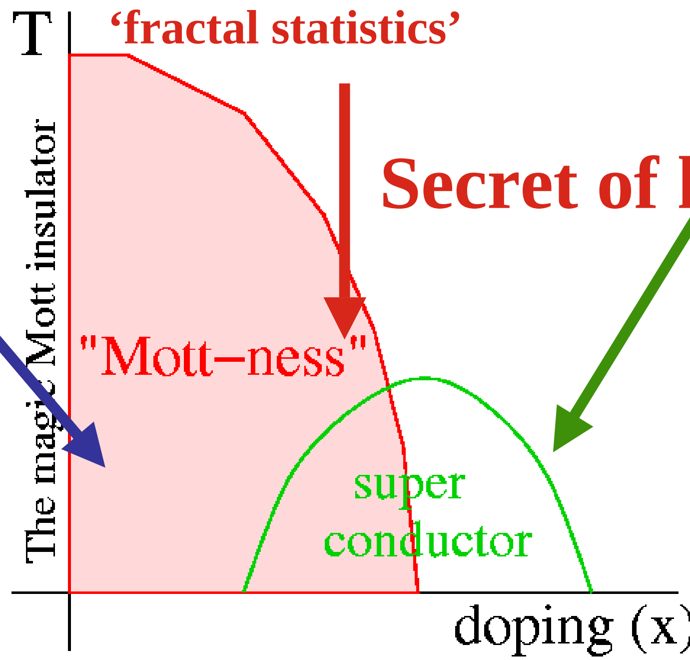
‘Statistical criticality’



Incompatible statistics merge in scale invariant ‘fractal statistics’

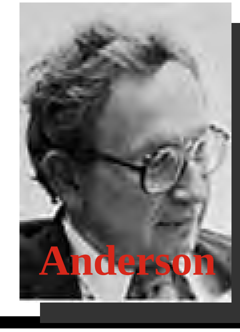
Fermi-Dirac statistics

Implies Fermi-liquid and BCS superconductivity

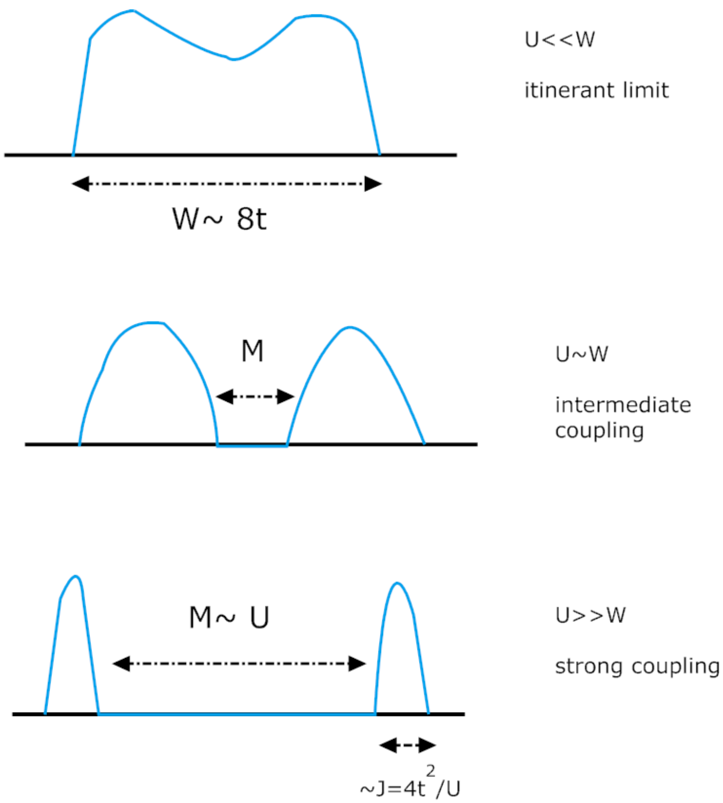


Secret of high Tc??

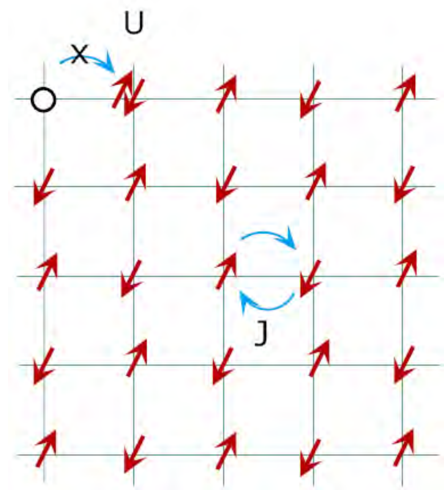
Cuprates start as doped Mott-insulators



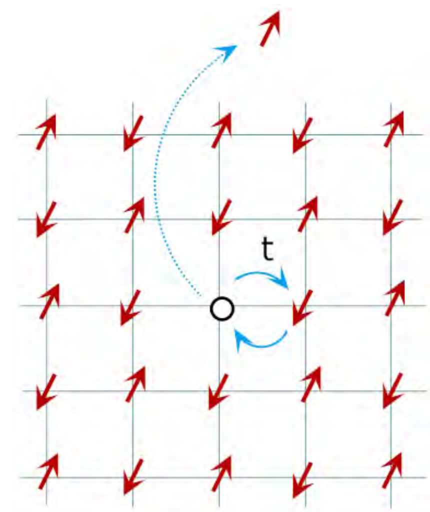
$$H = t\sum_{ij} c^{+}_{i\sigma} c_{j\sigma} + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$



Mott insulator

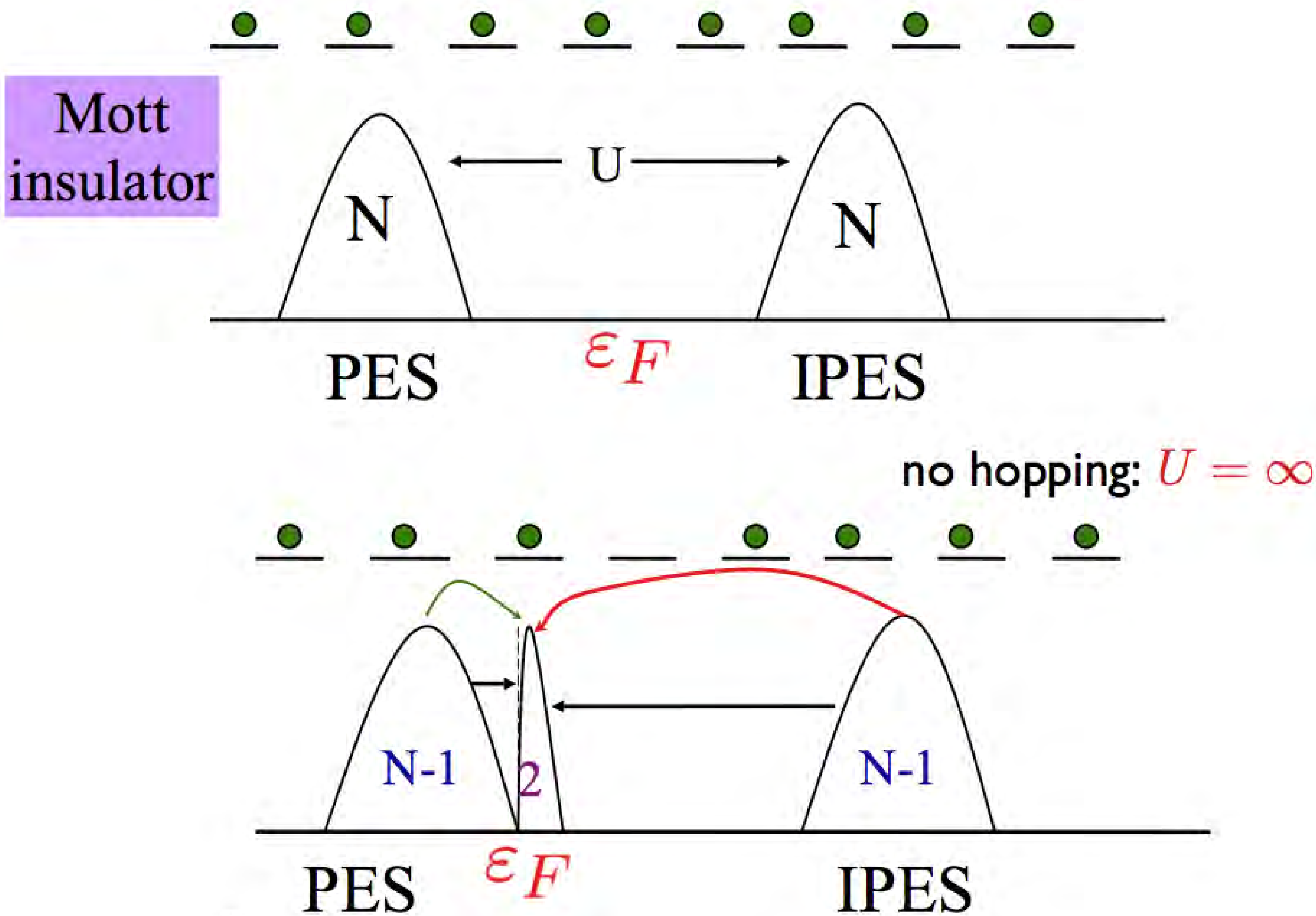


Doped Mott insulator

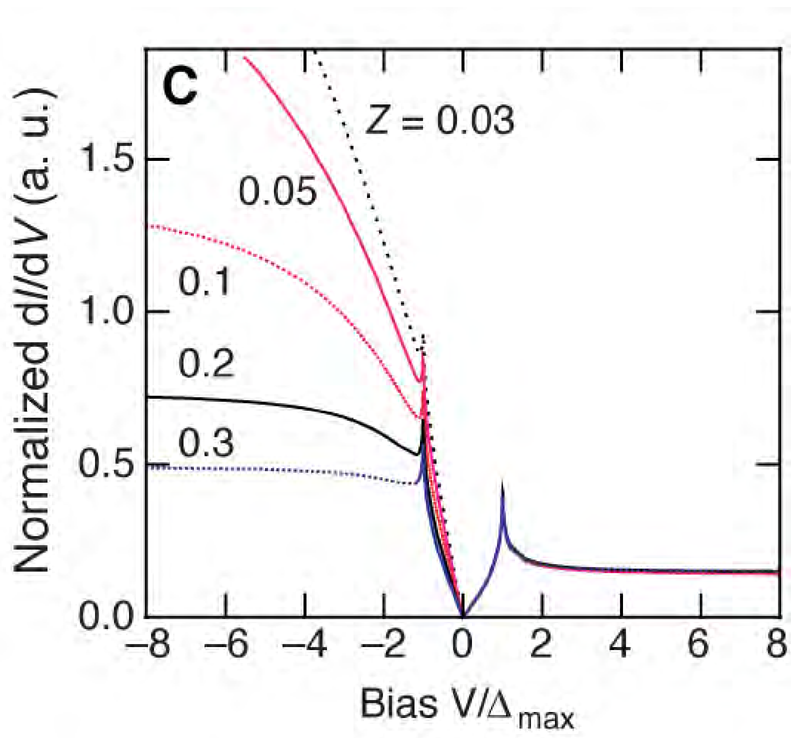


$$H_{t-J} = t\sum_{ij} \hat{c}^{+}_{i\sigma} \hat{c}_{j\sigma} + J\sum_{ij} \vec{S}_i \cdot \vec{S}_j$$

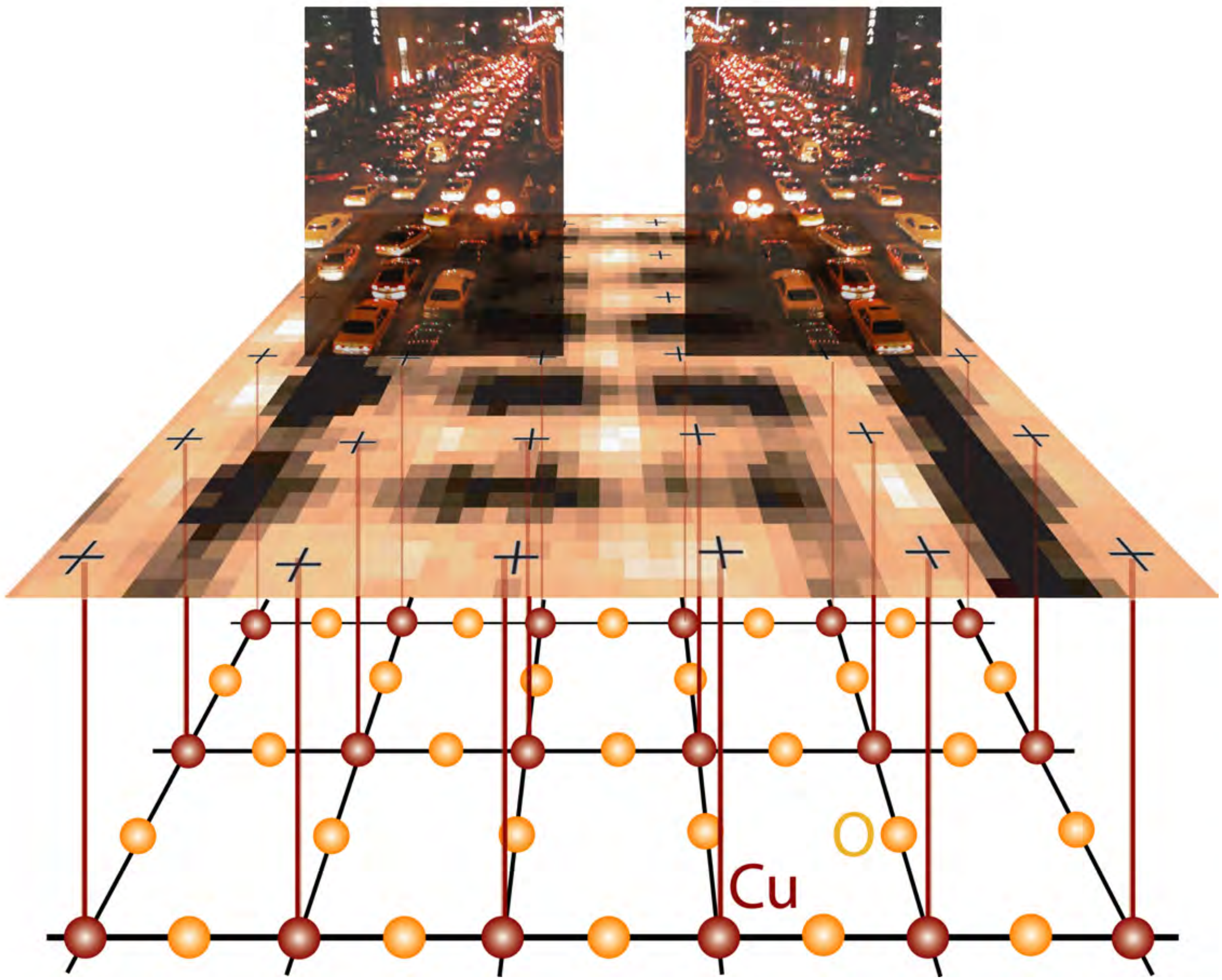
Mottness and Hilbert space dimensionality



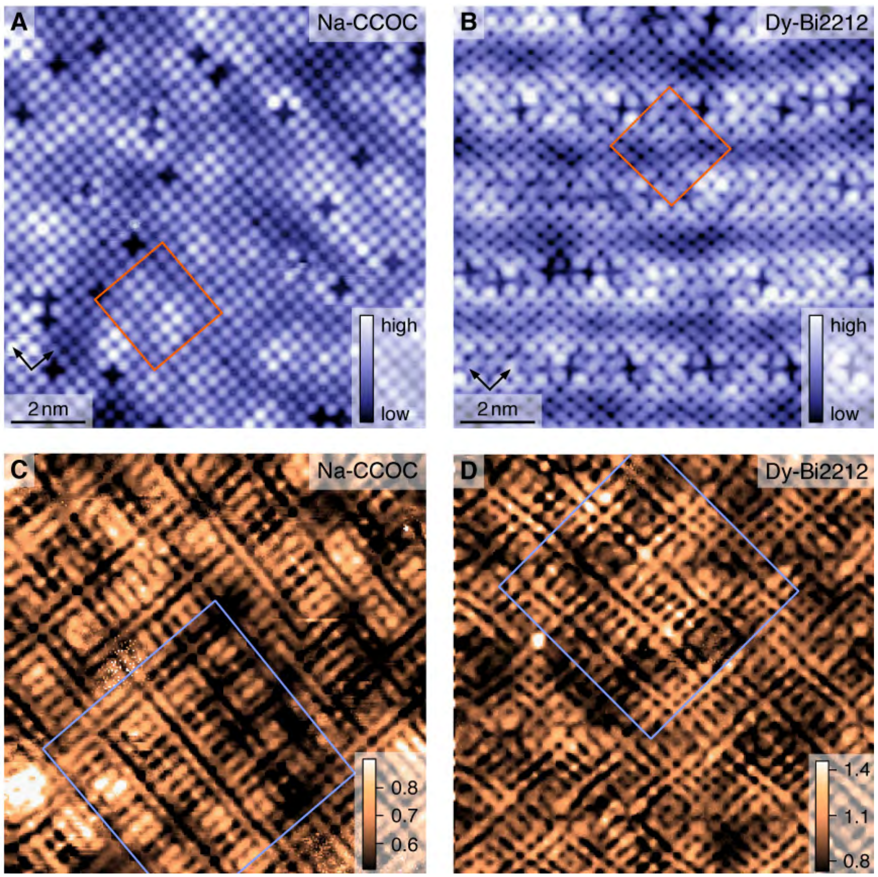
Mott-maps and highway ramps

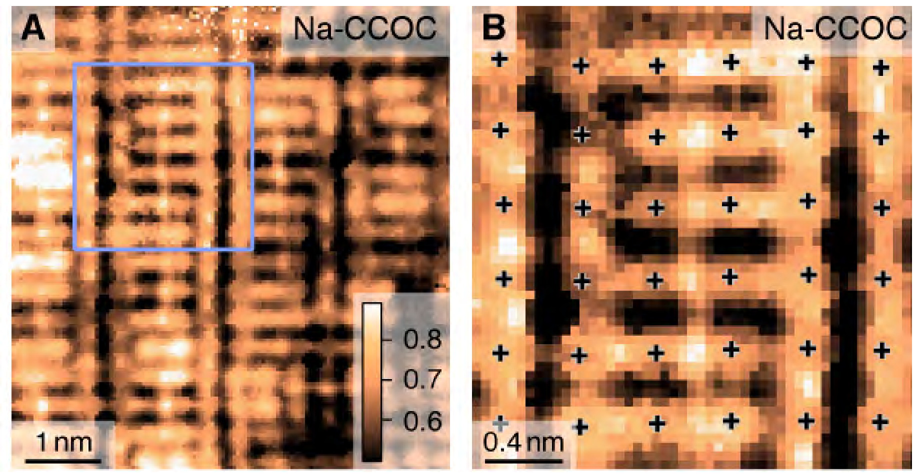


Phil Anderson


O
Cu

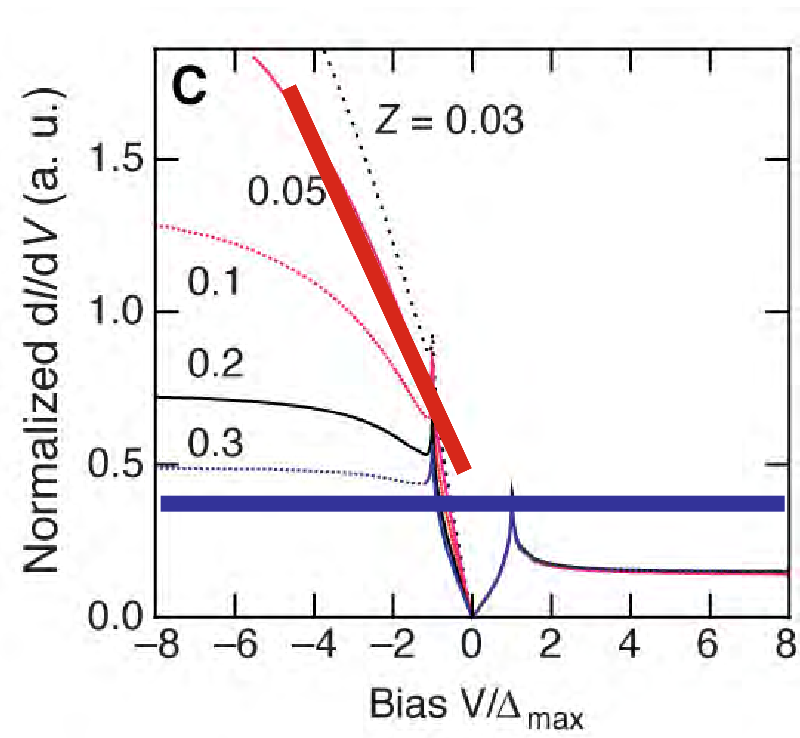
Probing rush hour in the electron world





Seamus Davis et al.:
Science, march 9 2007
(JZ, Perspective)

Mott-maps and highway ramps



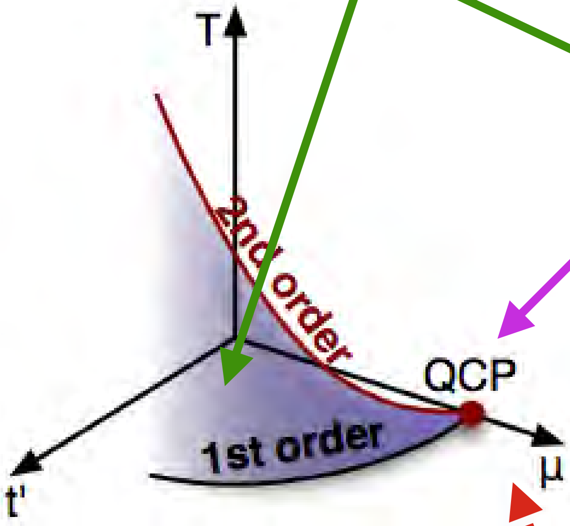
Phil Anderson

Mott collapse: Hubbard model

DCA results for Hubbard model at intermediate couplings (U = 0.75W):

Non-fermi liquid 'Mott fluid'

Quantum critical state, very unstable to d-wave superconductivity



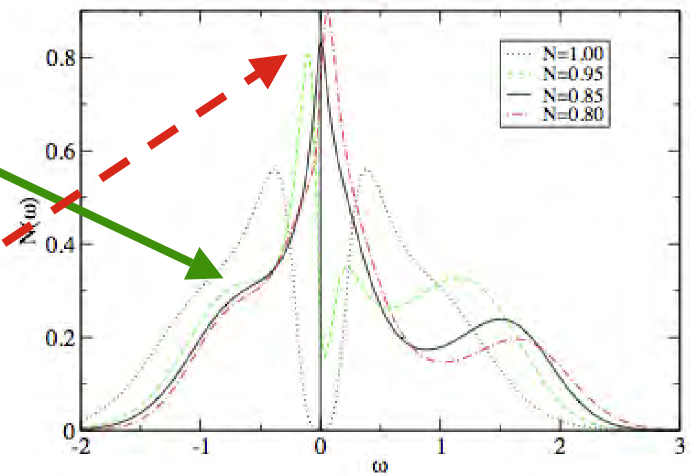
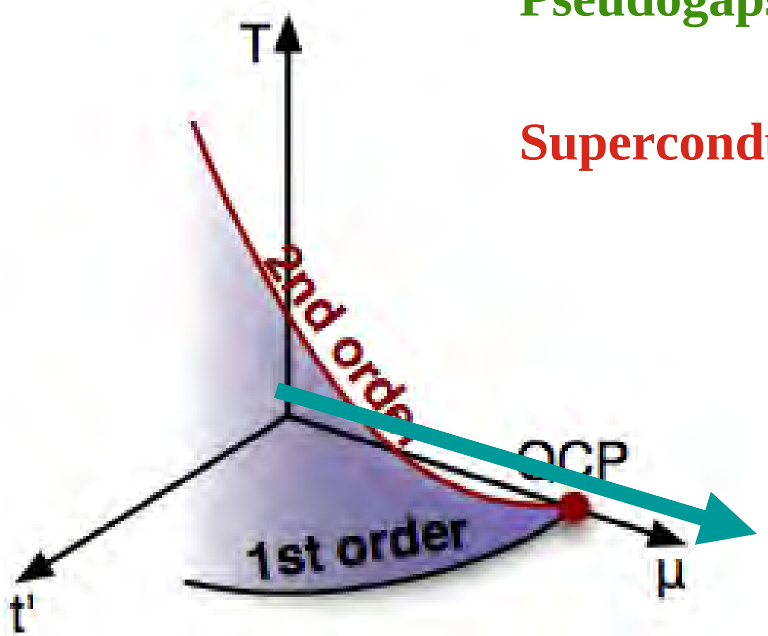


FIG. 2: The single-particle density of states (DOS) when $U = 1.5$, $W = 2$, and $T = 0.01429$. Note that the low-energy DOS is roughly symmetric around $\omega = 0$ for the critical filling $N = 0.85$ where $Z_0(\mathbf{k})$ fits a MFL form. This is consistent with the observation of rough particle-hole symmetry in the cuprates in the proximity of optimal doping[6].

Fermi-liquid at 'high' dopings

Mott collapse: Hubbard model

DCA results for Hubbard model at intermediate couplings (U = 2W):



Pseudogaps

Superconducting Tc

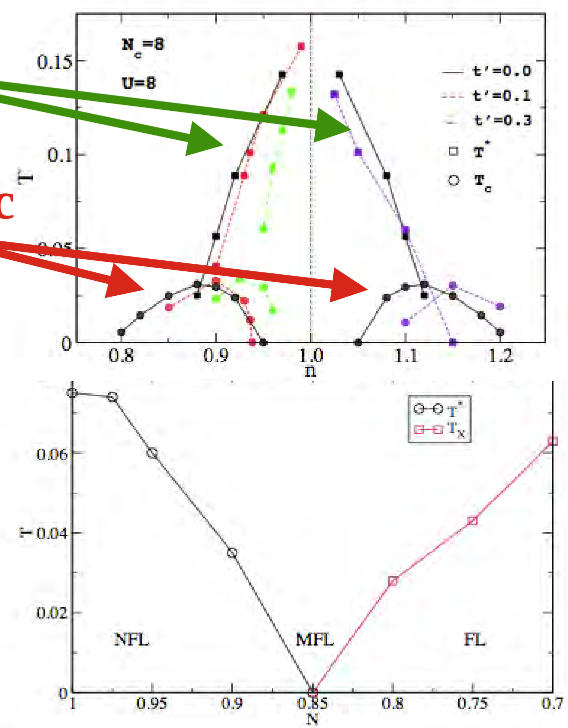
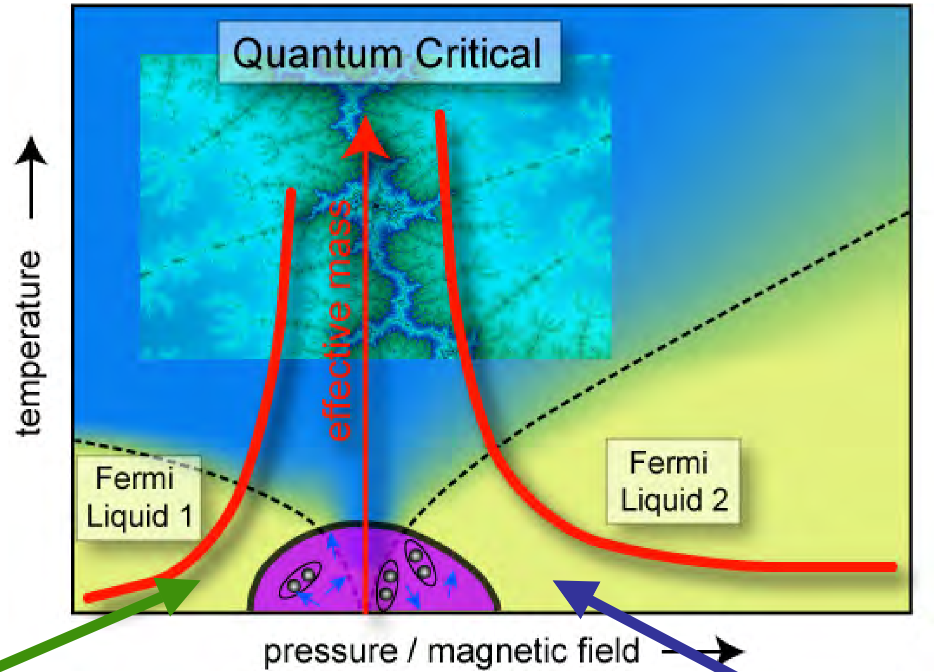


FIG. 4: The pseudogap temperature T^*, identified from the peak in the susceptibility and the emergence of the PG in the DOS shown in Fig. 3 and the FL to MFL crossover temperature identified by fitting Eq. 9 to the Matsubara quasiparticle data shown in Fig. 1.

Catherine's 'selective Mott transition'

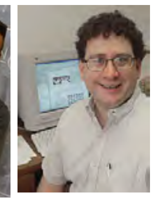
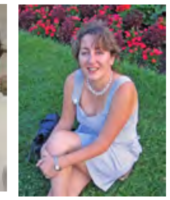


"RKKY" = f-electrons are Mott-localized

"Kondo"= f-electrons are effectively unprojected.

Plan

1. The idea of Mott collapse

2. Mottness versus “Weng statistics”

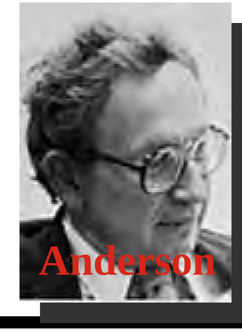
3. Statistics and t-J numerics

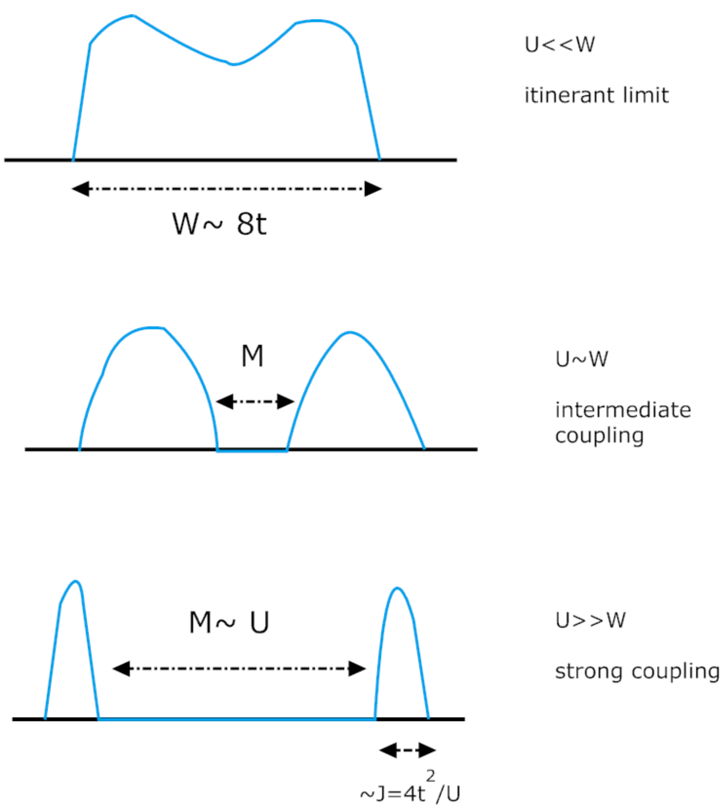
  

4. Fermions, scale invariance and Ceperley’s path integral

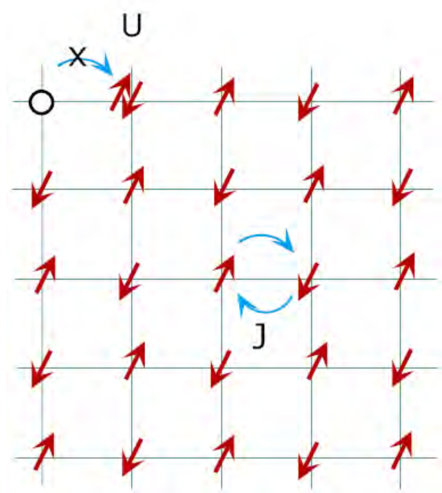
Cuprates start as doped Mott-insulators



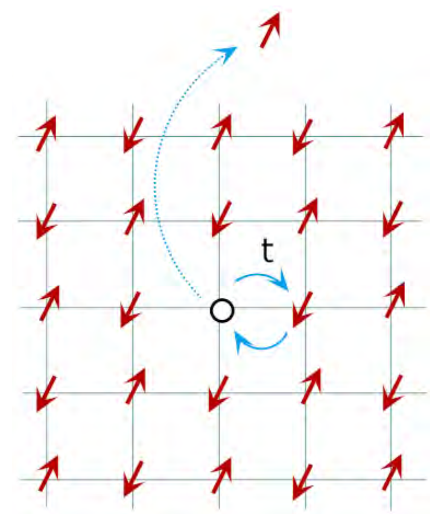


$$H = t\sum_{ij} c^{+}_{i\sigma} c_{j\sigma} + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Mott insulator

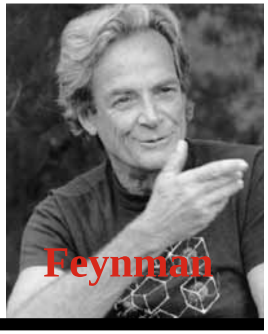


Doped Mott insulator

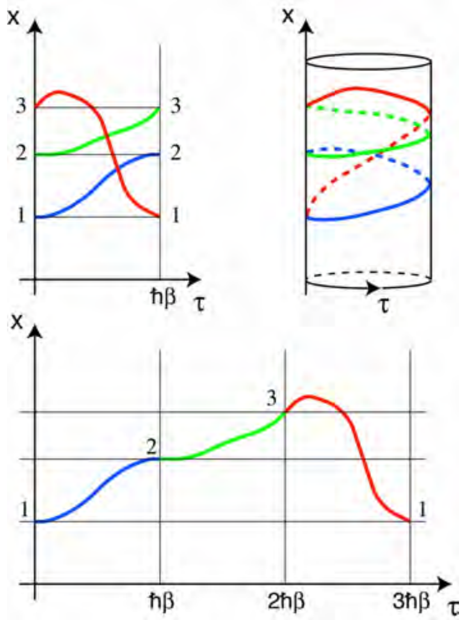


$$H_{t-J} = t\sum_{ij} \hat{c}^{+}_{i\sigma} \hat{c}_{j\sigma} + J\sum_{ij} \vec{S}_i \bullet \vec{S}_j$$

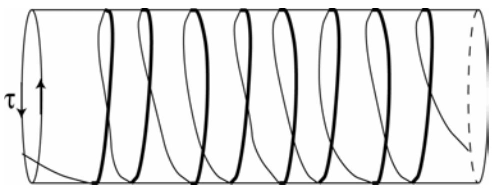
Quantum statistics and path integrals



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$
$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$



Bose condensation: Partition sum dominated by infinitely long cycles

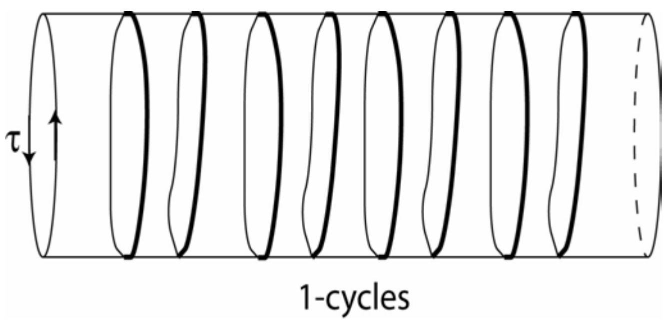


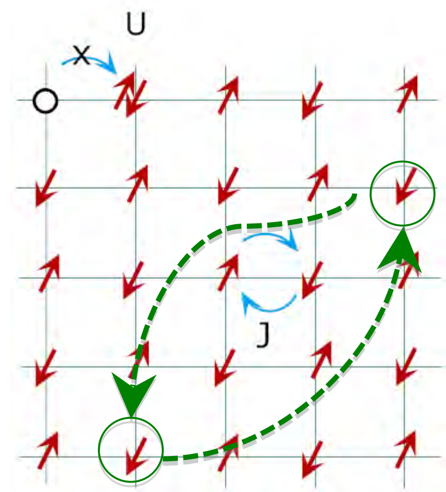
Cycle decomposition

Fermions: infinite cycles set in at T_F, but cycles with length w and w+1 cancel each other approximately. Free energy pushed to E_F!

Mott insulator: the vanishing of Fermi-Dirac statistics

Mott-insulator: the electrons become distinguishable, stay at home principle!





Spins live in tensor-product space. “Spin signs” are like hard core bosons in a magnetic field, can be gauged away on a bipartite lattice (“Marshall signs”)

$$Z_{t-J} = \sum_c \tau_c \mathcal{Z}[c]$$

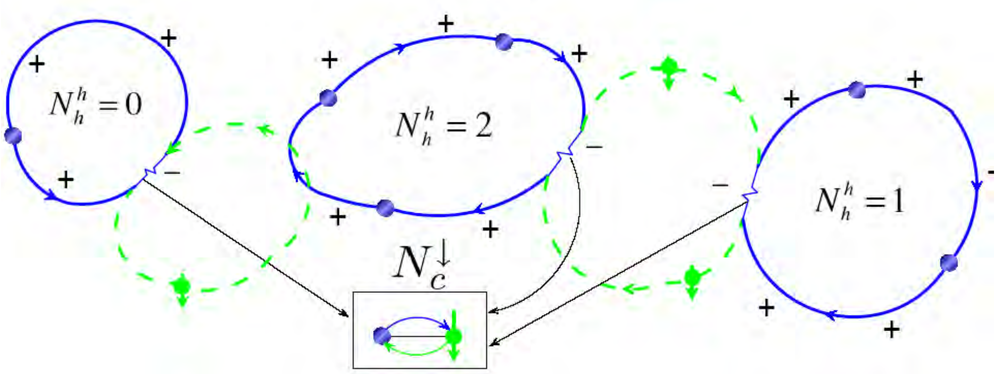
$$\mathcal{Z}[c] \geq 0 \qquad \tau_c = +1$$

Doped Mott-insulator: Weng statistics

t-J model: spin up is background, spin down's ('spinons') and holes ('holons') are **hard core bosons**.

Exact Partition sum: $Z_{t-J} = \sum_c \tau_c \mathcal{Z}[c]$ $\quad \mathcal{Z}[c] = \left(\frac{2t}{J}\right)^{M_h[c]} \sum_n \frac{(\beta J/2)^n}{n!} \delta_{n, M_h + M_{\uparrow\downarrow} + M_Q}$

The **sign** of any term is set by: $\boxed{\tau_c \equiv (-1)^{N_h^{\downarrow}[c] + N_h^h[c]}}$

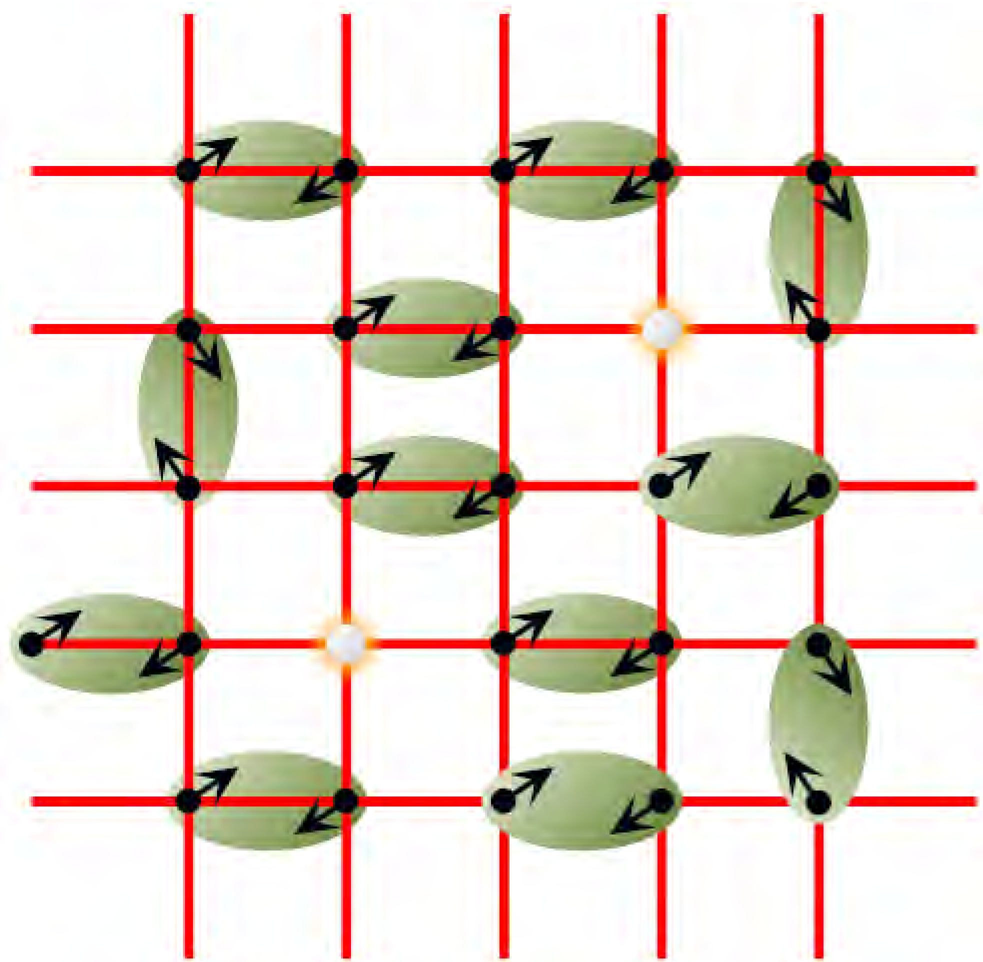


$N_h^h[c]$ **The (fermionic) number of holon exchanges**

$N_h^{\downarrow}[c]$ **The number of spinon-holon 'collisions'**

arXiv: 0802.0273

RVB: the statistical rational

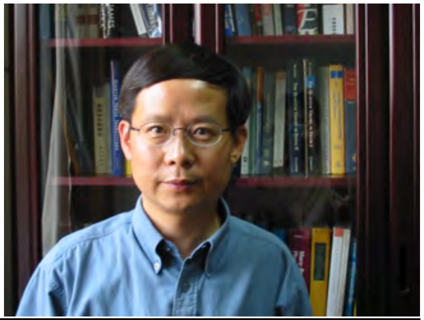


Resonating valence bond states:

Quantum liquid 'organizing away' the Weng 'collision signs', lowers the energy!

Weng statistics compatible with a d-wave superconducting ground state

Field theory: “Mutual Chern-Simons”



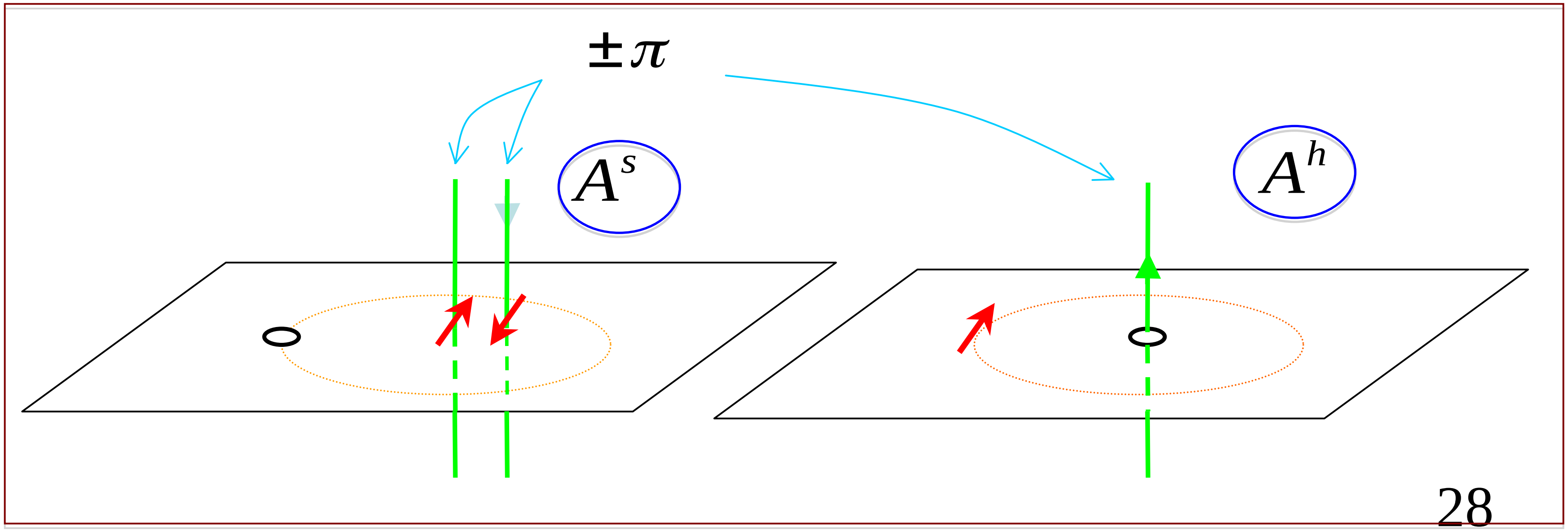
Coherent state description:

$$H_t = -t\sum_{\langle ij\rangle\sigma} h_i^+ h_j \left(e^{iA_{ij}^s - i\phi_{ij}^0}\right) b_{j\sigma}^+ b_{i\sigma}\left(e^{i\sigma A_{ji}^h}\right) + h.c.$$

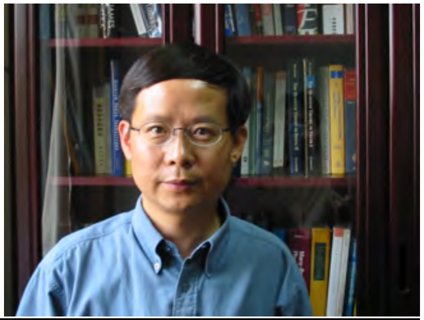
- holes: hard core bosons
- spins: Schwinger bosons

$$H_J = -\frac{J}{2}\sum_{\langle ij\rangle} \hat{\Delta}_{ij}^{s\,+} \hat{\Delta}_{ij}^s \qquad \hat{\Delta}_{ij}^s = \sum_\sigma e^{-i\sigma A_{ij}^h} b_{i\sigma} b_{j-\sigma}$$

Statistics: mutual flux π attachments!



The subtly different d-wave superconductor



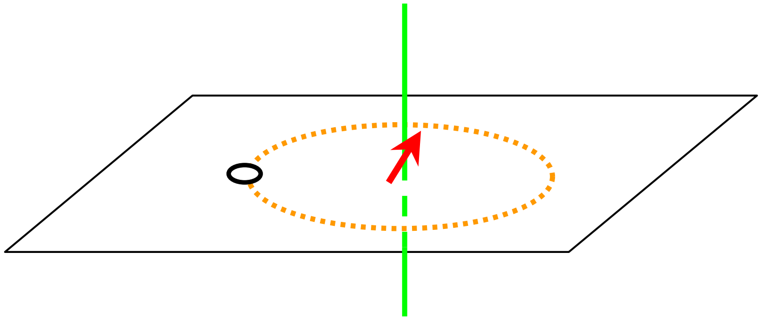
$$H_t = -t \sum_{\langle ij\rangle \sigma} h_i^+ h_j \left(e^{iA_{ij}^s - i\phi_{ij}^0}\right) b_{j\sigma}^+ b_{i\sigma} \left(e^{i\sigma A_{ji}^h}\right) + h.c. \qquad H_J = -\frac{J}{2} \sum_{\langle ij\rangle} \hat{\Delta}_{ij}^{s\,+} \hat{\Delta}_{ij}^s \qquad \hat{\Delta}_{ij}^s = \sum_\sigma e^{-i\sigma A_{ij}^h} b_{i\sigma} b_{j-\sigma}$$

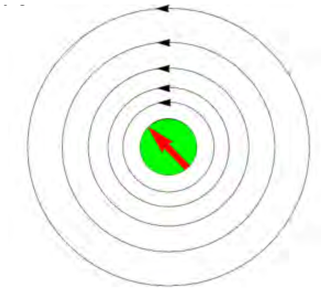
Charge e condensate: $\left\langle h_i^+ \right\rangle \neq 0$

Spins Arovas-Auerbach massive RVB: $\left\langle \hat{\Delta}_{ij}^s \right\rangle \neq 0$

=> d-wave superconductor supporting massless Bogoliubov excitations!

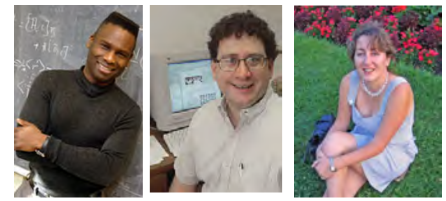
Topologically subtly different from BCS superconductor: $\frac{h}{2e}$ **vortex carries S=1/2 quantum number.**





Plan

1. The idea of Mott collapse



2. Mottness versus “Weng statistics”

3. Statistics and t-J numerics

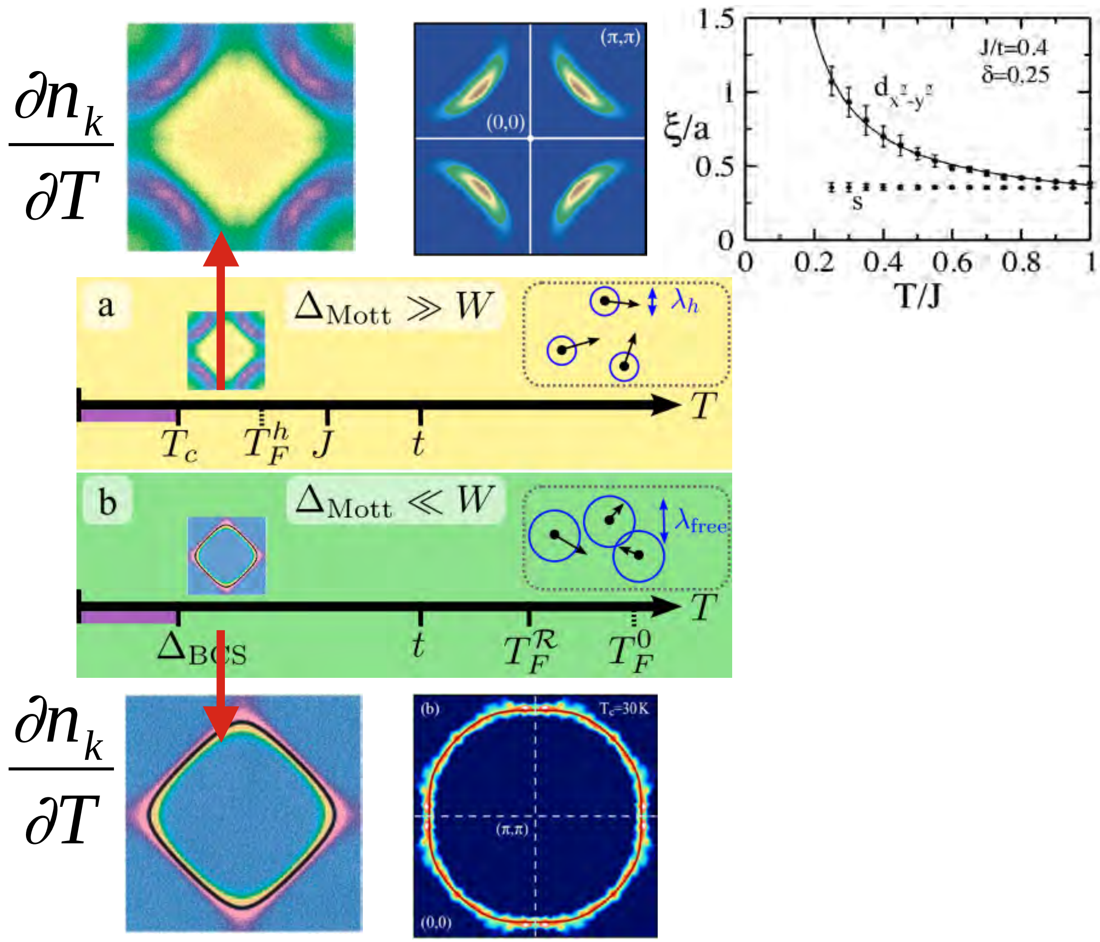
4. Fermions, scale invariance and Ceperley’s path integral

t-J numerics: high T expansions

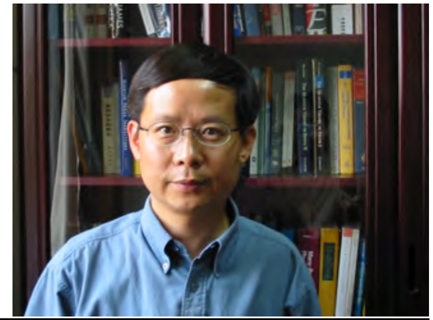
12-th order in 1/T, down to T = 0.2 J … (PRL 81, 2966 (1998)

- Contrast in momentum distribution ($\partial_k n_k$, $\partial_T n_k$) tiny compared to equivalent free fermion problem

- Fermi-arcs develop with pairing correlations: no big Fermi surface, on its way to the d-wave ground state!



t-J numerics: high T predictions



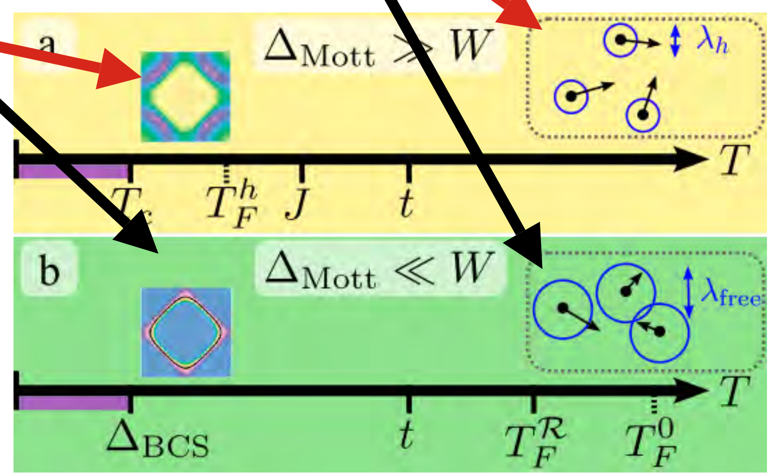
Take J << t, low hole density:

Free case: $$\lambda_{free} = a\sqrt{2zt/k_B T},\ \lambda_{free}(T_F) \approx r_s$$

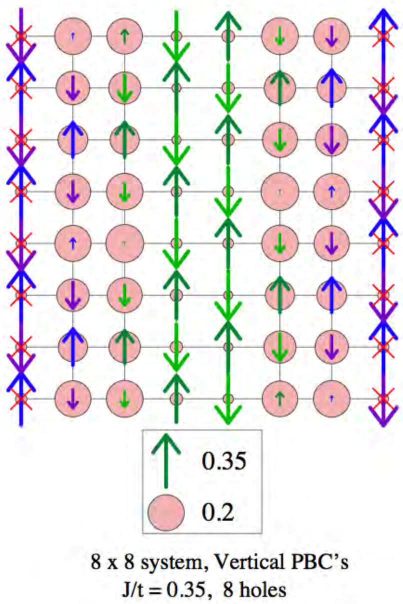
t-J model: hole thermal de Broglie wavelength limited by spin-spin correlation length through 'collision signs'!

Free case: below the Fermi temperature the high T expansion is strongly oscillating because of 'hard' Fermion signs.

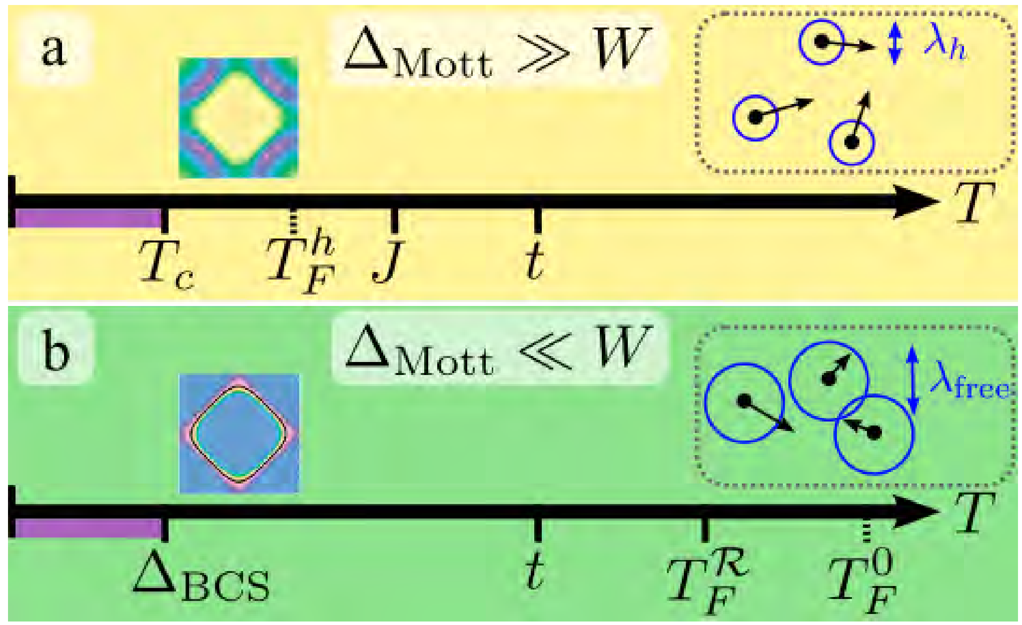
t-J model: positive contributions increasingly outnumber negative ones since 'signs are organized away'!



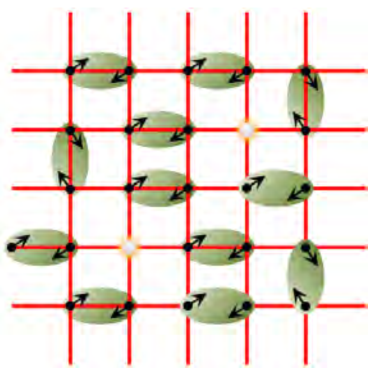
t-J numerics: the DMRG stripes



T=0



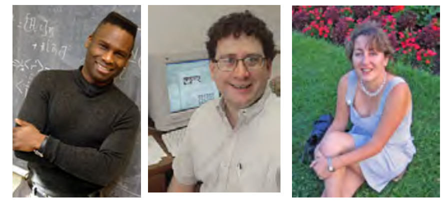
"Crystallized RVB"



Weng statistics implies much less 'delocalization pressure' compared to Fermi-Dirac: competing 'localizing' instabilities spoil the superconducting fun!

Plan

1. The idea of Mott collapse



2. Mottness versus “Weng statistics”

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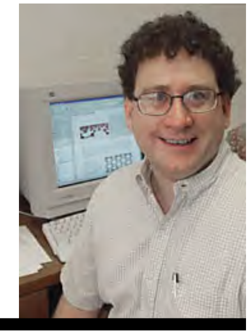
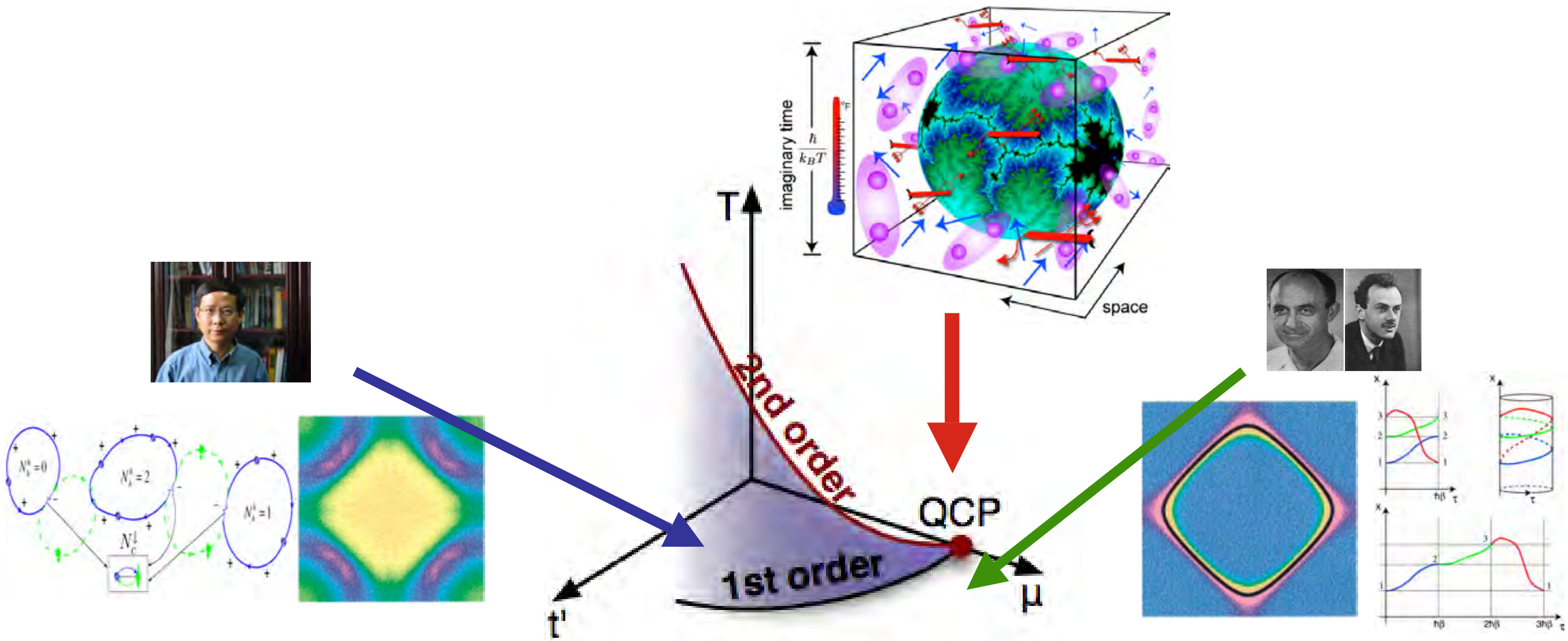
4. Fermions, scale invariance and Ceperley’s path integral

Statistical quantum criticality



Weng- and Fermi-Dirac statistics microscopically incompatible: Mott collapse should turn into a first order phase separation transition …

But Mark claims a quantum critical end point !?

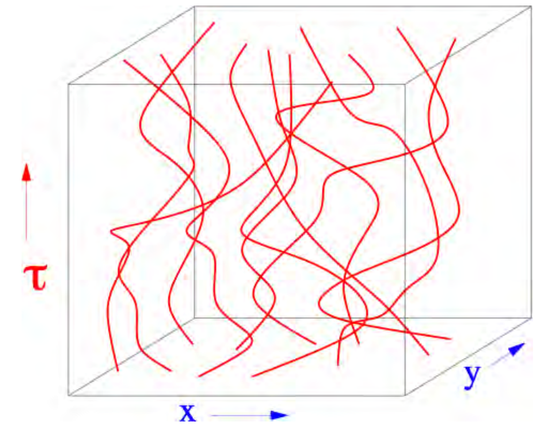


Fermionic sign problem

Imaginary time path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1, \ldots, \mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$
$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

- negative Boltzmann weights
- non probablistic!!!

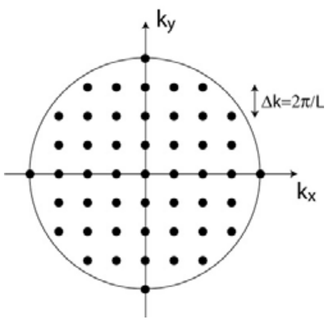
The nodal hypersurface

Antisymmetry of the wave function

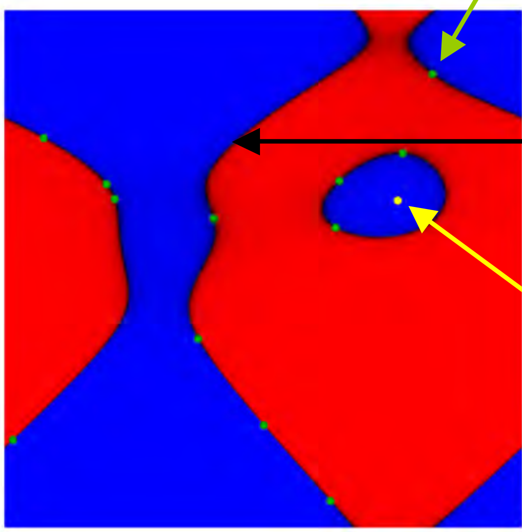
$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_j, \ldots, \mathbf{r}_N) = -\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_j, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_N)$$

Free Fermions

$$\Psi_0(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\mathbf{r}_j}\right)_{ij}$$



d=2



Pauli hypersurface

$$P = \bigcup_{i\neq j} P_{ij}$$

$$P_{ij} = \{\mathbf{R} \in \mathbb{R}^{Nd} | \mathbf{r}_i = \mathbf{r}_j\}$$

$$\dim P = Nd - d$$

Nodal hypersurface

$$\Omega = \{\mathbf{R} \in \mathbb{R}^{Nd} | \Psi(\mathbf{R}) = 0\}$$

$$\dim \Omega = Nd - 1$$

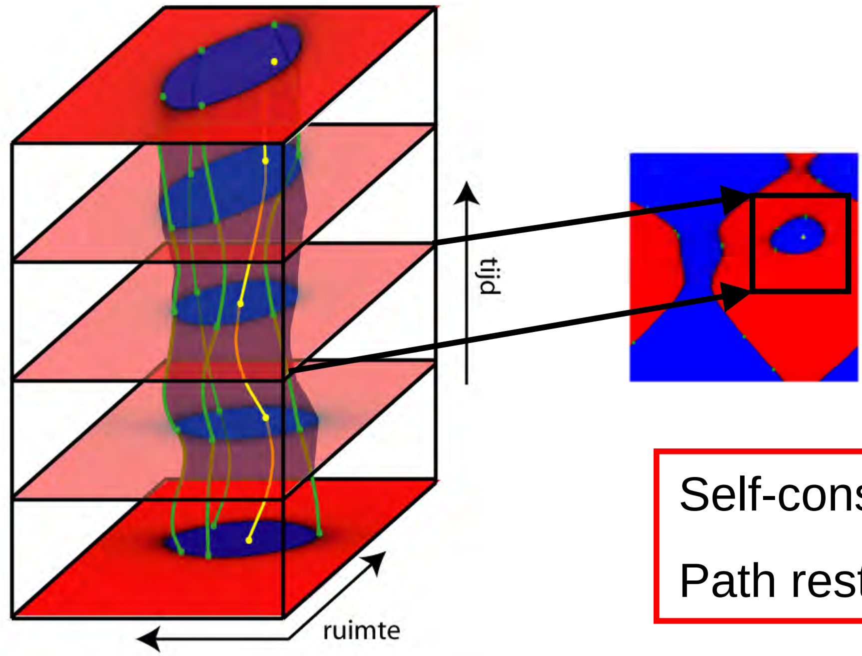
Test particle

Constrained path integrals

Formally we can solve the sign problem!!

$$\rho_F(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P},\mathrm{even}} \int_{\gamma:\mathbf{R}\to\mathcal{P}\mathbf{R}}^{\gamma\in\Gamma(\mathbf{R},\mathcal{P}\mathbf{R})} \mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

$$\Gamma(\mathbf{R},\mathbf{R}') = \{\gamma:\mathbf{R}\to\mathbf{R}' \,|\, \rho_F(\mathbf{R},\mathbf{R}(\tau);\tau)\neq 0\}$$



Ceperley, J. Stat. Phys. (1991)

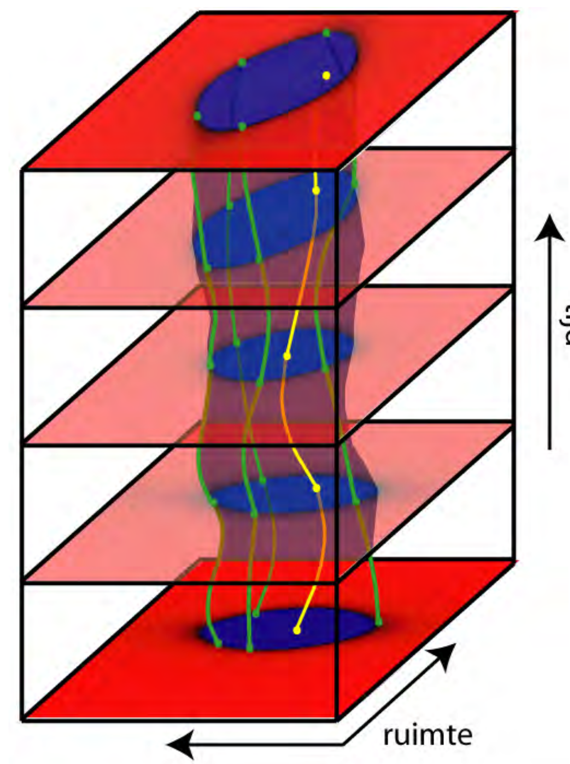
Self-consistency problem:

Path restrictions depend on ρ_F !

Reading the worldline picture

Average node to node spacing

$$\sim r_s = \left(\frac{V}{N}\right)^{1/d} = n^{-1/d}$$



Persistence length

$$l^2(\tau) = \langle(\mathbf{r}_i(\tau) - \mathbf{r}_i(0))^2\rangle = 2d\mathcal{D}\tau = 2d\frac{\hbar}{2m}\tau$$

Collision time

$$l^2(\tau_c) \simeq r_s^2 \to \tau_c \simeq \frac{1}{2d}\frac{2m}{\hbar}n^{-2/d}$$

Associated energy scale

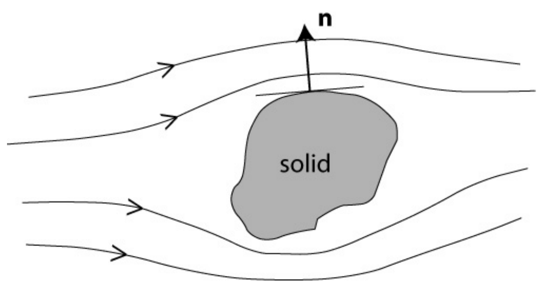
$$\hbar\omega_c = \frac{\hbar}{\tau_c} \simeq d\frac{\hbar^2}{2m}n^{2/d} \simeq E_F$$

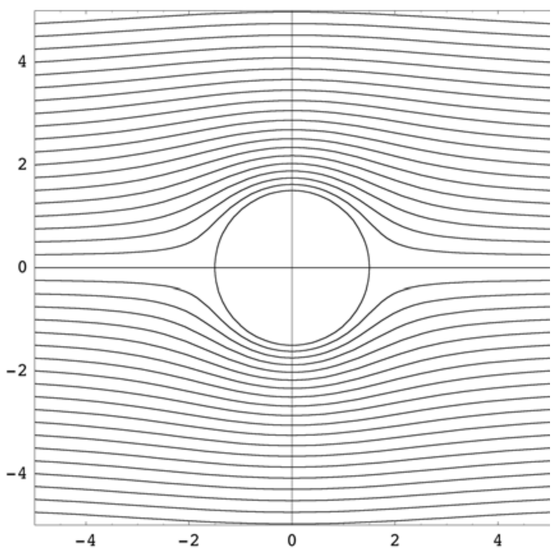
Key to fermionic quantum criticality

At the QCP scale invariance, no E_F → Nodal surface has to become fractal !!!

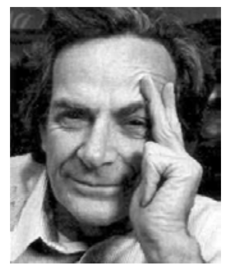
Hydrodynamic backflow

Classical fluid:
incompressible flow





Feynman-Cohen: mass enhancement in ^{4}He



Wave function ansatz for „foreign“ atom moving through He superfluid with velocity small compared to sound velocity:

$$g(\mathbf{r}) \sim \frac{\mathbf{kr}}{r^3} \quad \rightarrow \quad \Psi = \phi \exp[i\mathbf{k}\left(\mathbf{r}_A + \sum_{i \neq A} \frac{\mathbf{r}_i - \mathbf{r}_A}{r_{iA}^3}\right)]$$

Backflow wavefunctions in Fermi systems

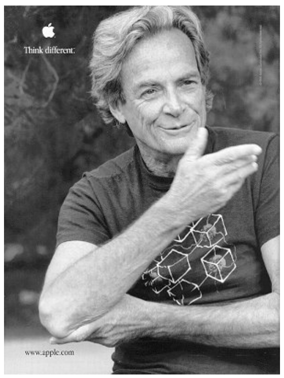
$$\begin{aligned} \psi_{bf}(\mathbf{R}) &\sim \mathrm{Det}\left(e^{i\mathbf{k}_i \tilde{\mathbf{r}}_j}\right)_{ij} \\ \tilde{\mathbf{r}}_j &= \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l) \end{aligned}$$

Widely used for node fixing in QMC

→ Significant improvement of variational GS energies

Frank’s fractal nodes …

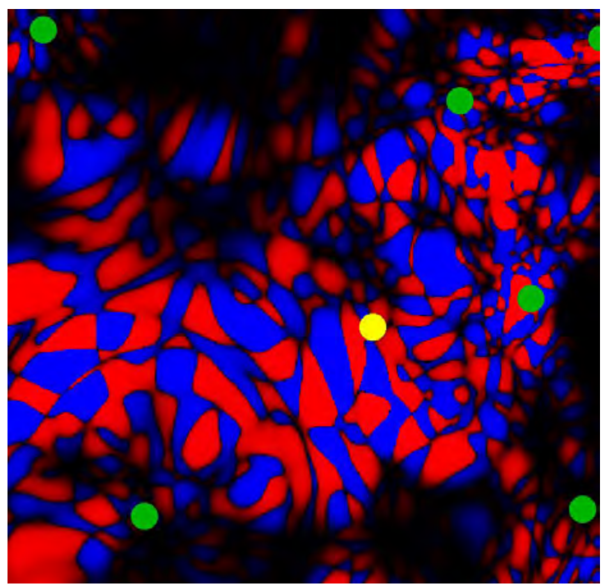
Feynman‘s fermionic backflow wavefunction:

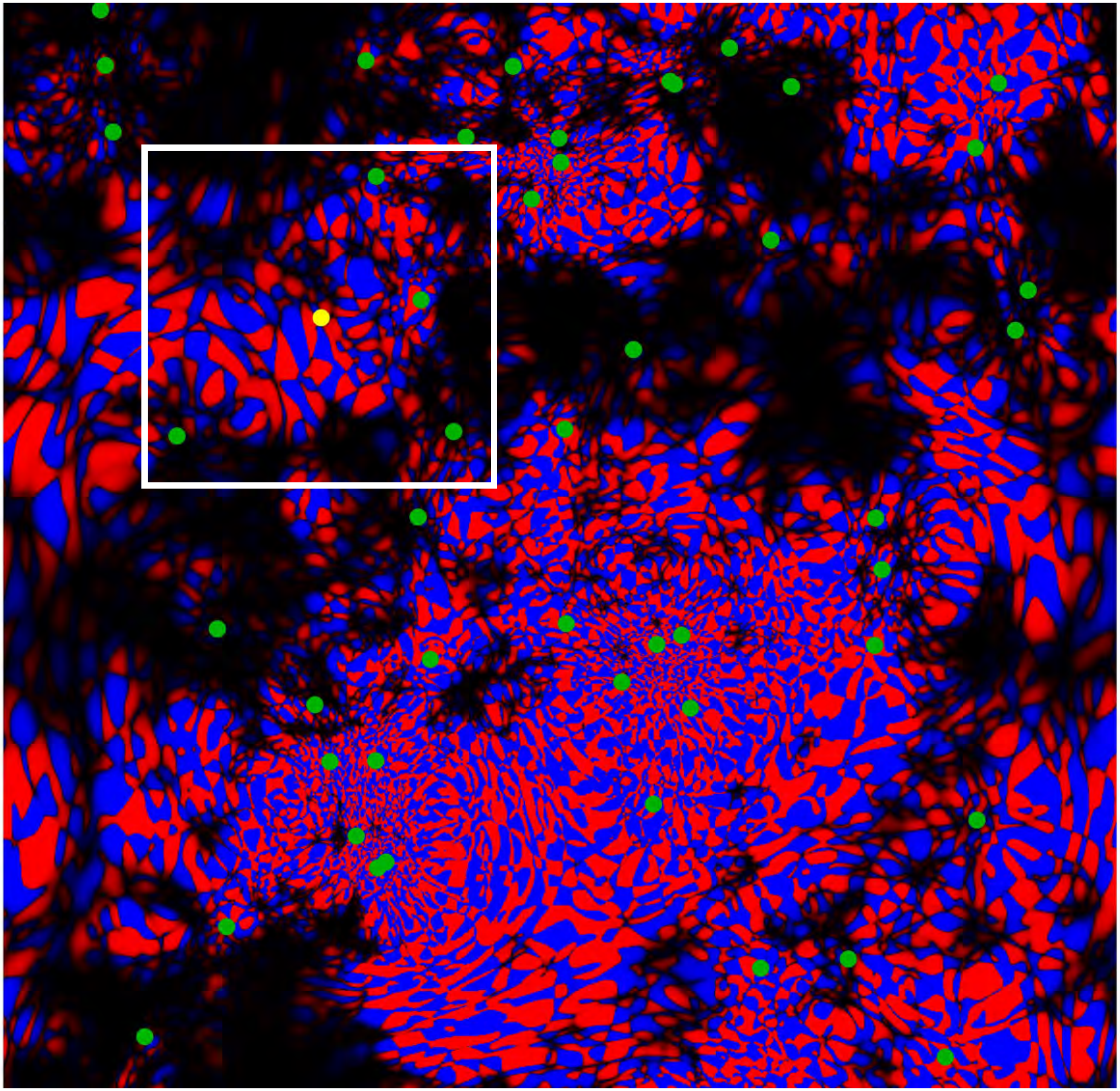


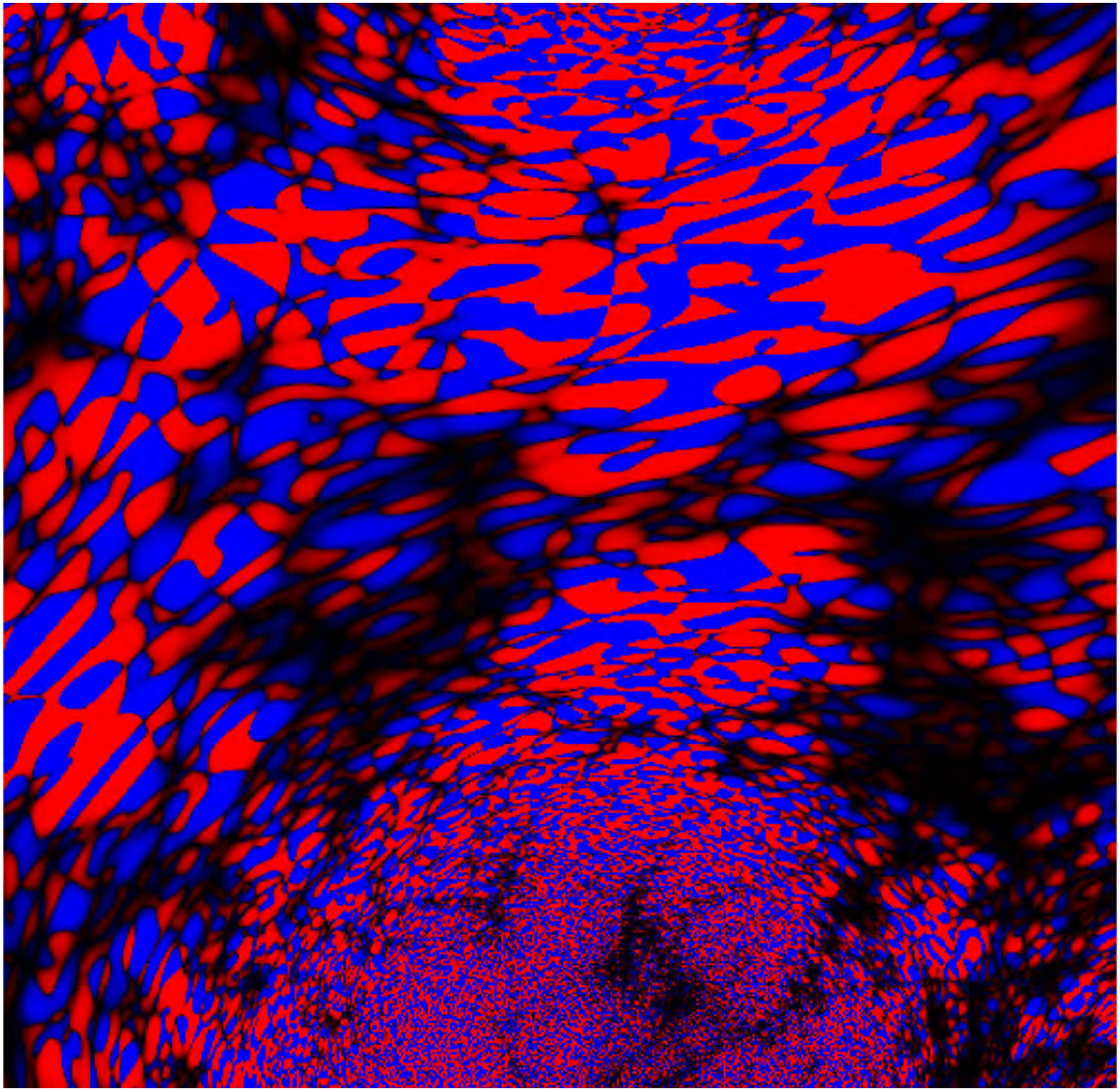
$$
\begin{aligned}
\psi_{bf}(\mathbf{R}) &\sim \operatorname{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij} \\
\tilde{\mathbf{r}}_j &= \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)
\end{aligned}
$$

$$
\eta(r) = \frac{a^3}{r^3 + r_0^3}
$$

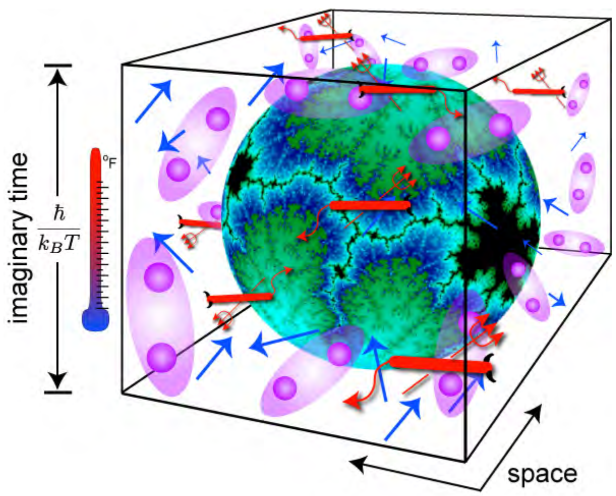
Frank Krüger



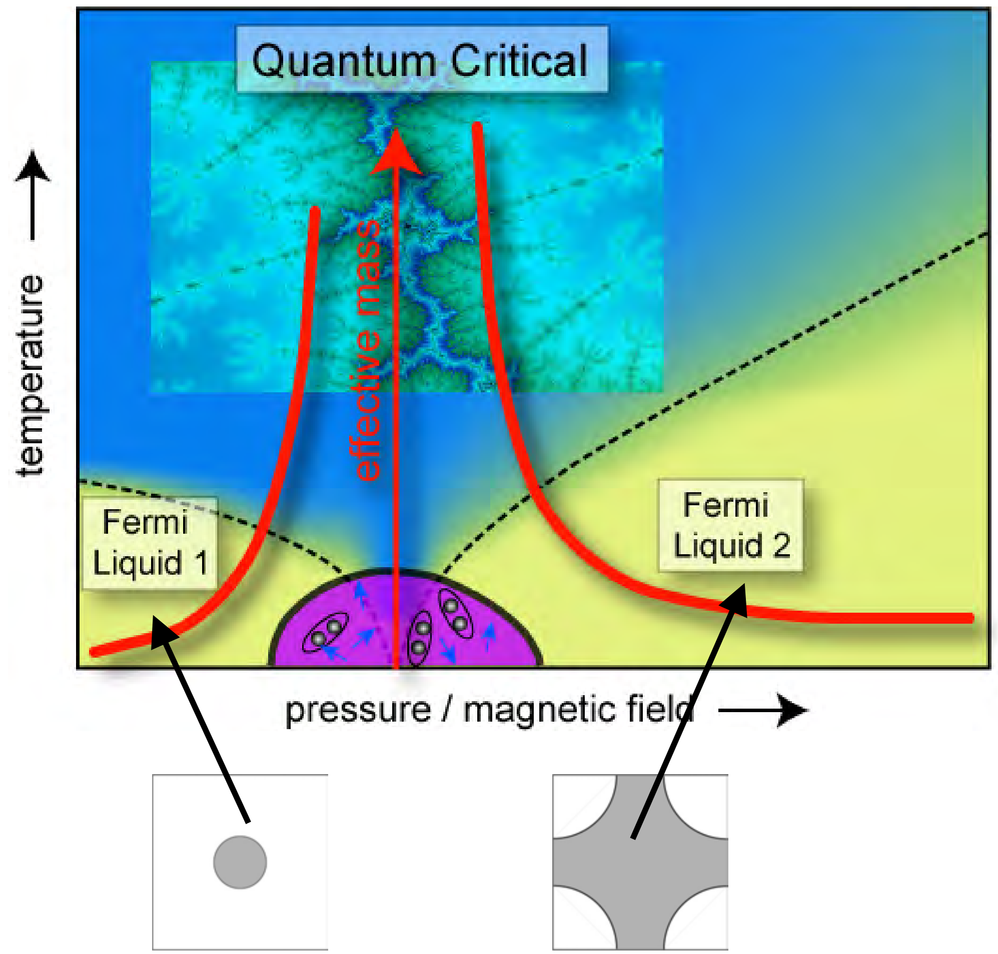




Fermionic quantum phase transitions in the heavy fermion metals



JZ, Science 319, 1205 (2008)



Paschen et al., Nature (2004)

Turning on the backflow

Nodal surface has to become fractal !!!

↓

Try backflow wave functions

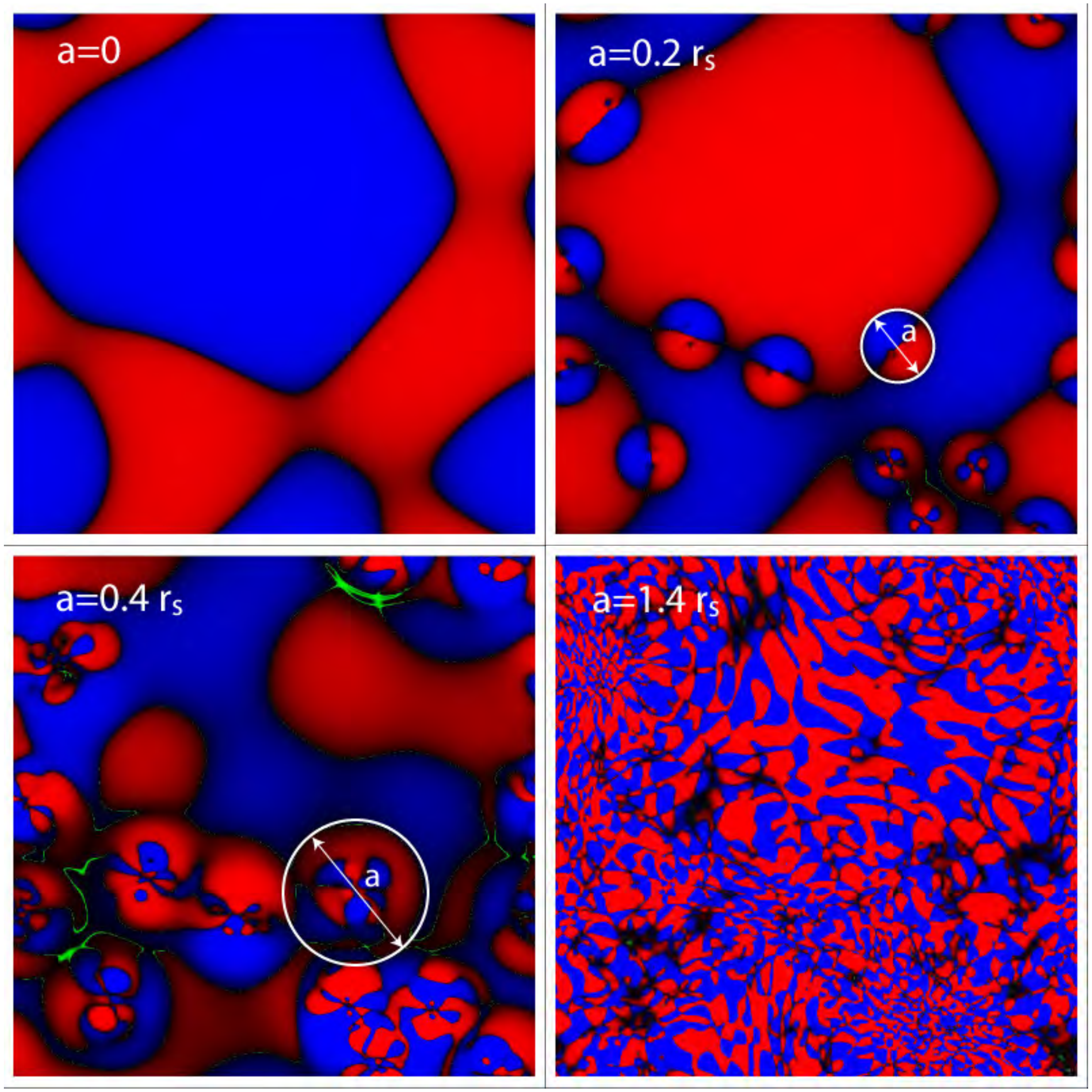
$$\psi_{bf}(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

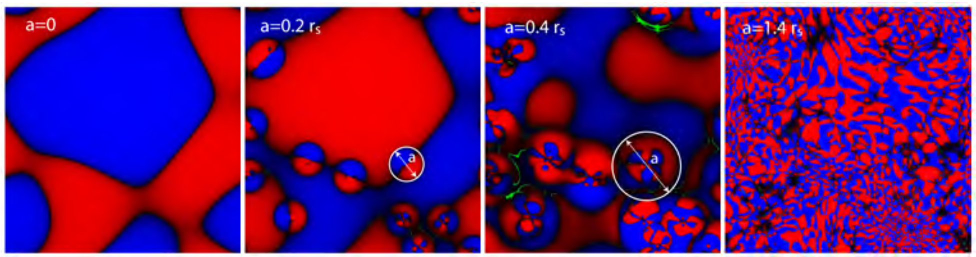
$$\eta(r) = \frac{a^3}{r^3 + r_0^3}$$

Collective (hydrodynamic) regime:

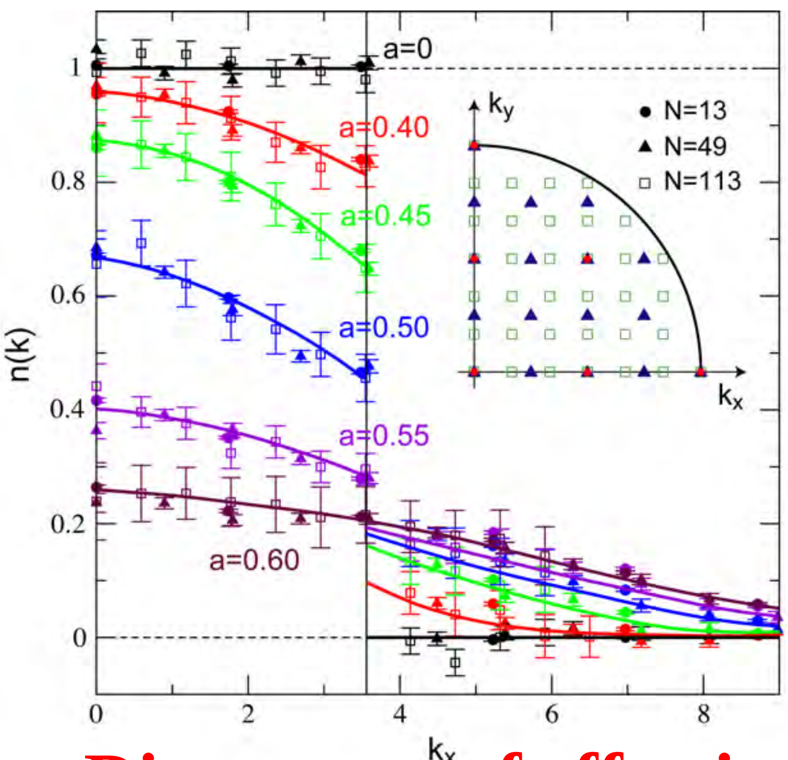
$$a \gg r_s$$

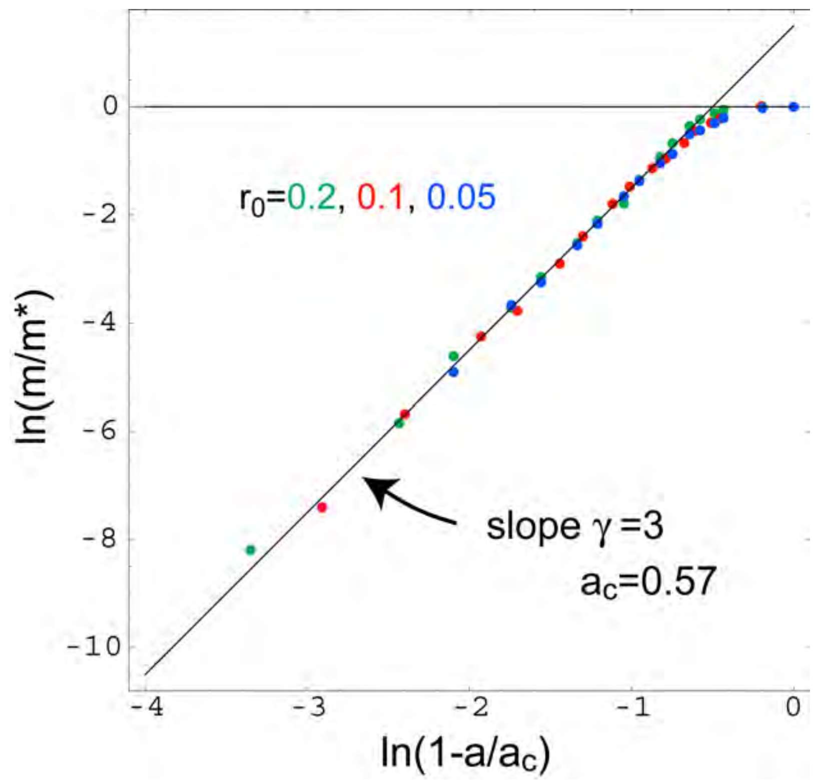


MC calculation of n(k)

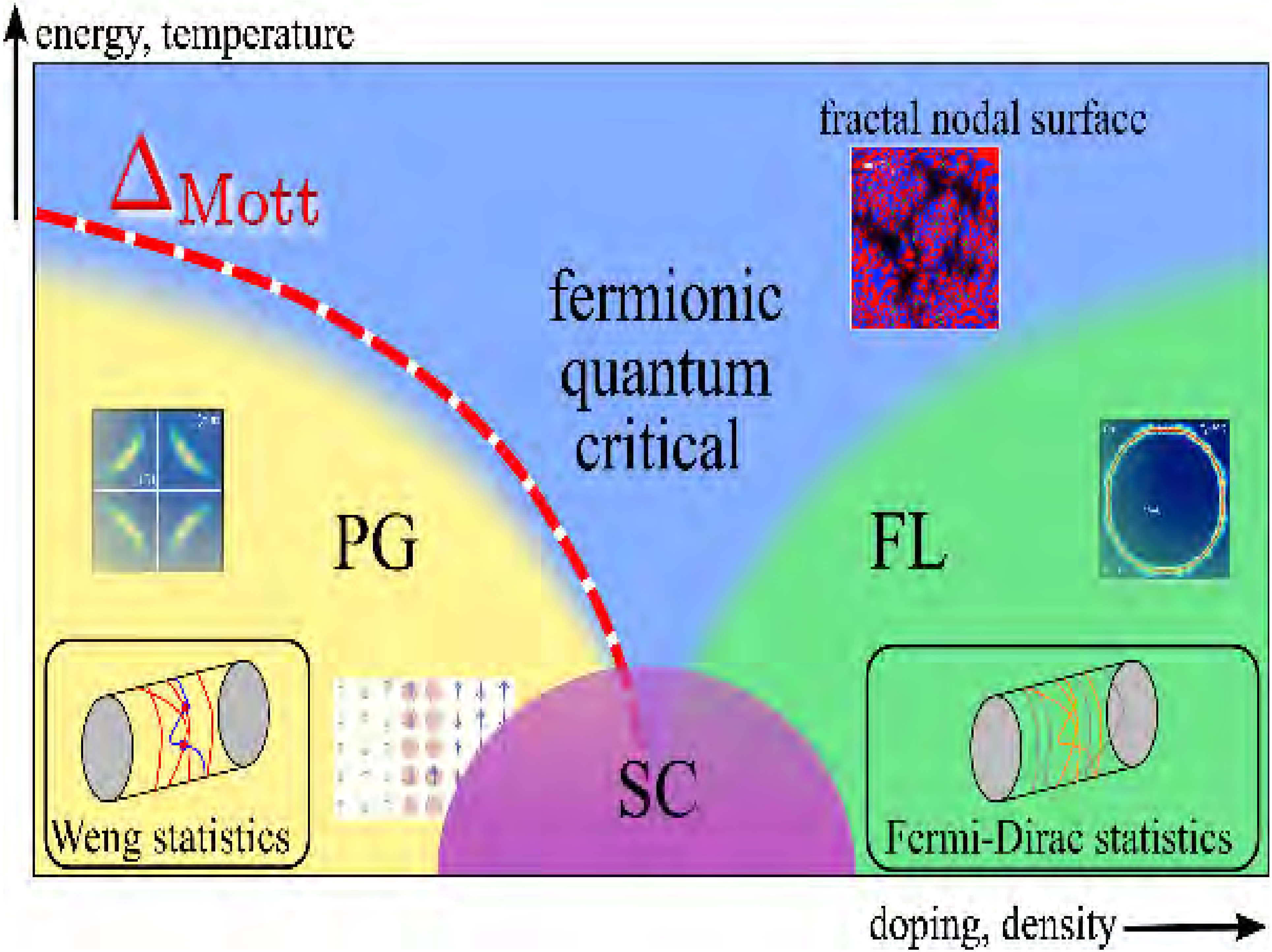


$$\frac{m}{m^*} \propto \left(1 - \frac{a}{a_c}\right)^3$$





Divergence of effective mass as $a \rightarrow a_c$



energy, temperature
Δ_{Mott}
fractal nodal surface
fermionic quantum critical
PG
FL
SC
Weng statistics
Fermi-Dirac statistics
doping, density

Further reading.

Overview: J. Zaanen and B.J. Overbosch, arXiv: 0911.4070 (Phil.Trans.Roy.Soc. A, in press).

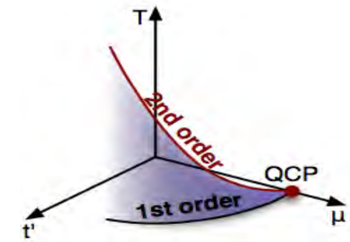
Weng statistics: K. Wu, Z.Y. Weng and J. Zaanen, PRB 77, 155102 (2008); arXiv:1102.2941.

Mott collapse and "conformal" superconductivity: S.-X. Yang et al, PRL 106, 047004 (2011).

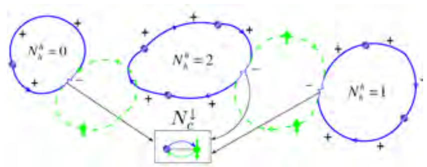
Fractal nodes: F. Kruger and J. Zaanen, PRB 78, 035104 (2008)

In summary …

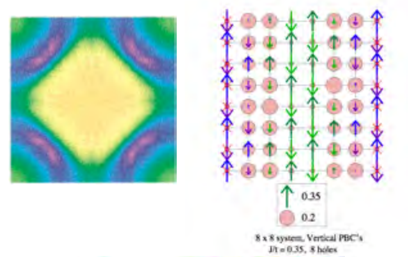
1. Mott collapse: cuprates, heavy fermions? DCA is convincing!



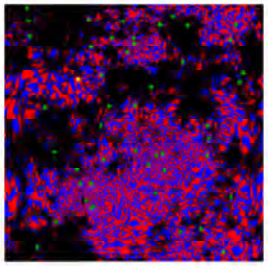
2. Mottness and Weng statistics



3. Statistics and t-J numerics



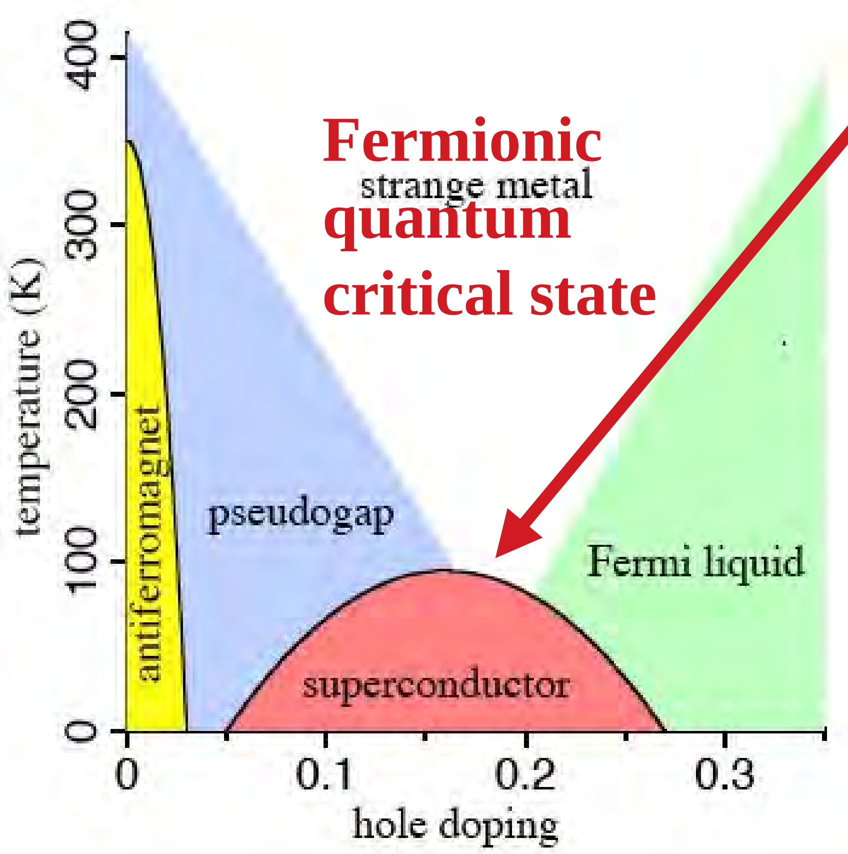
4. Ceperley's path integral: quantum statistics can go scale invariant



Why Tc is high …

JZ, Nature 430, 512(2004)

BCS type transition: pairs form at Tc



But BCS wisdoms like:

$$2\Delta = \hbar\omega_{boson}e^{-1/\lambda}$$

$$2\Delta \approx 3.5k_BT_c$$

Need the Fermi energy!

Why is Tc high?

"Because there is superglue binding the electrons in pairs"

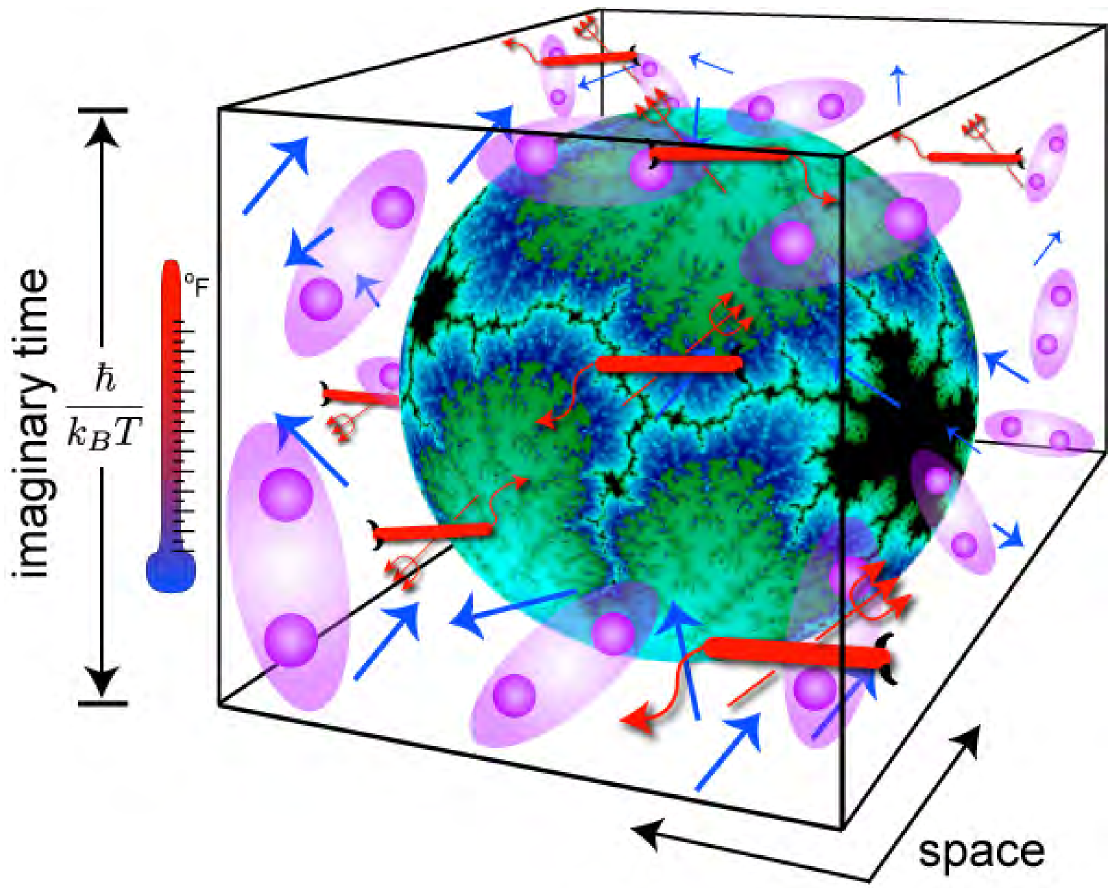
Wrong!

The superfluid density is tiny, it is very easy to 'bend and twist' a high Tc superconductor. **Its cohesive energy sucks.**

Tc's are set by the competition between the two sides …

The theory of the mechanism should explain why the free energy of the metal is seriously BAD.

Superconductivity born from a fermionic critical state …



The glue-ish essence of BCS

Let's believe  'retarded glue" =>

Gap equation: $$\underbrace{1 - g}_{\text{Glue strength}}\, \chi_{pp}(k_B T, \underbrace{\Delta}_{\text{SC gap}}, \underbrace{\hbar\omega_B}_{\text{Glue frequency}}) = 0$$

The pair susceptibility of the Fermi liquid is a logarithm because of EF:

$$\chi_{pp}(k_B T, \cdots) = \ln\left(\frac{E_F \to \hbar\omega_B}{k_B T}, \cdots\right)$$

$$\Rightarrow k_B T_c = \hbar\omega_B e^{-1/\lambda}, 2\Delta \approx 3.5 k_B T_c$$

Hitting the critical stuff with glue

J.-H. She

Gap equation: $1 - g\chi_{pp}(k_B T, \Delta, \hbar\omega_B) = 0$

Glue strength **SC gap** **Glue frequency**

Cooper instability of the fermionic quantum critical state?

The pair susceptibility has just to be the most divergent one!

Form largely fixed by scaling (non-conserved currents):

$$\chi_{pp}(\omega, T) \propto \frac{Z}{T^{(2-\eta_{pp})/z}} \Phi_{pp}\left(\frac{\hbar\omega}{k_B T}\right)$$

$$\hbar\omega \geq k_B T: \quad \chi_{pp} \propto \frac{1}{(i\omega)^{(2-\eta_{pp})/z}}$$

$$\hbar\omega \leq k_B T: \quad \chi_{pp} \propto \frac{1}{T^{(2-\eta_{pp})/z}} \frac{1}{1 - i\omega\tau_\hbar}$$

Scaling versus the BCS gap

J.-H. She

Gap equation: $1 - g\chi'(\omega = 0) = 0$

$$\chi'(\omega = 0) = 2\int_0^{2\omega_D} d\omega' \frac{\chi''(\omega')}{\omega'} \quad \Rightarrow 1 - \frac{g}{\omega_c}\int_{2\bar{\Delta}_0}^{2\hbar\bar{\omega}_B} \frac{d\bar{\omega}}{\bar{\omega}^{\alpha}} = 0$$

Fermi-liquid: $\omega_c = E_F, \lambda = \frac{g}{E_F}, \alpha = 1 \quad \Rightarrow \Delta_0 = 2\hbar\omega_B e^{-1/\lambda}$

Critical case: $\lambda = \frac{g}{\omega_c}, \ \alpha = \frac{2 - \eta_{pp}}{z} + 1 \quad \Rightarrow$

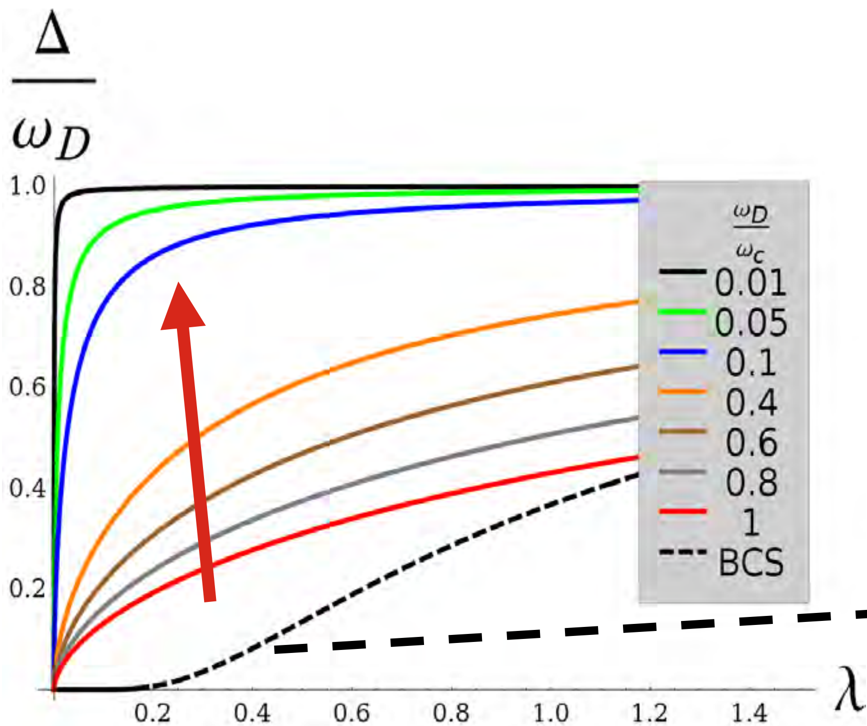
'Huang's equation': $$\Delta_0 = 2\hbar\omega_B \left(\frac{\lambda}{\lambda + \left(2\omega_B/\omega_c\right)^{(2-\eta_{pp})/z}} \right)^{\frac{z}{2-\eta_{pp}}}$$

Huang's equation at work

J.-H. She

$$\Delta_0 = 2\hbar\omega_B \left(\frac{\lambda}{\lambda + \left(2\omega_B/\omega_c\right)^{(2-\eta_{pp})/z}} \right)^{\frac{z}{2-\eta_{pp}}}$$

Strongly interacting critical state, e.g. 1+1D Ising: $\eta_{pp} = 1/4, \quad z = 1$



Increasing retardation: more bang for the bucks!

Standard BCS

Huang's equation versus high Tc

J.-H. She

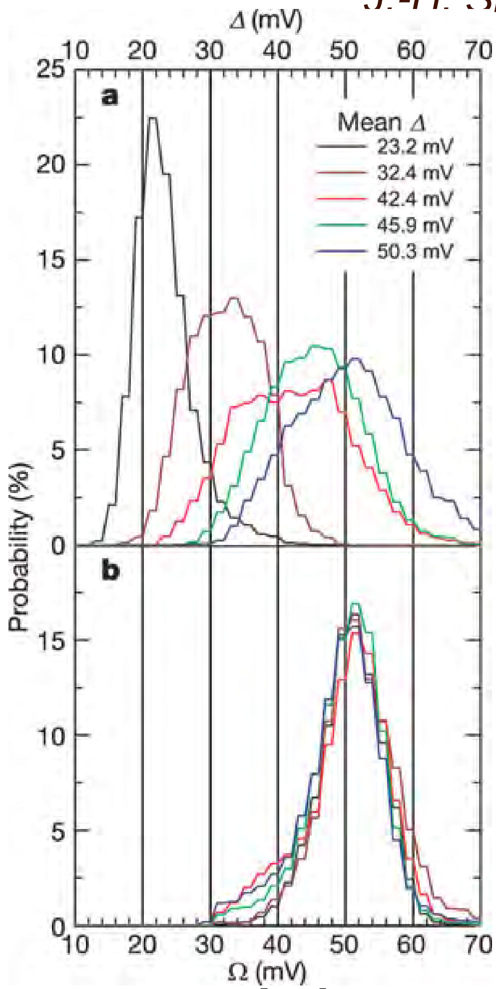
E.g. 1+1D Ising: $\eta_{pp} = 1/4, \quad z = 1$

Typical phonon-, cut-off energy: $\frac{\omega_B}{\omega_c} = \frac{50\ meV}{500\ meV}$

Typical gap: $\Delta_0 = 40\, meV$

Fermi-liquid: $\lambda \approx 1.1$

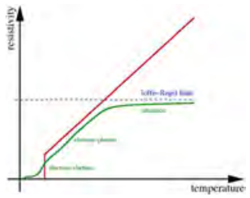
Critical case: $\lambda \approx 0.13$!!!

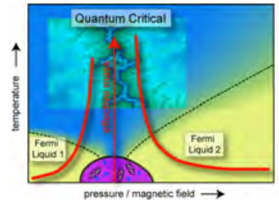


Davis, Balatsky

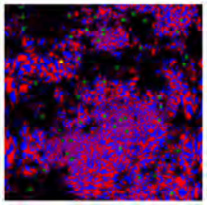
In summary

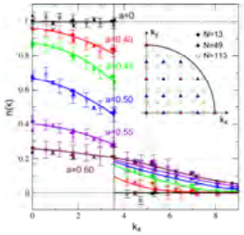
1. High Tc's normal state, heavy fermions: experiment demonstrates the existence of a mysterious scale invariant state formed from fermions.





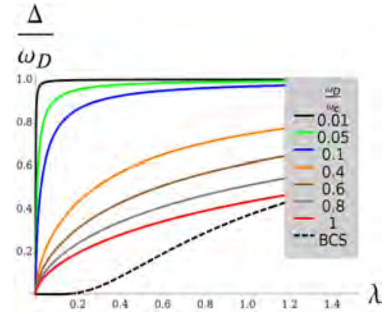
2. Fermionic quantum criticality is not governed by Wilsonian RNG: the fractal nodal surface, …



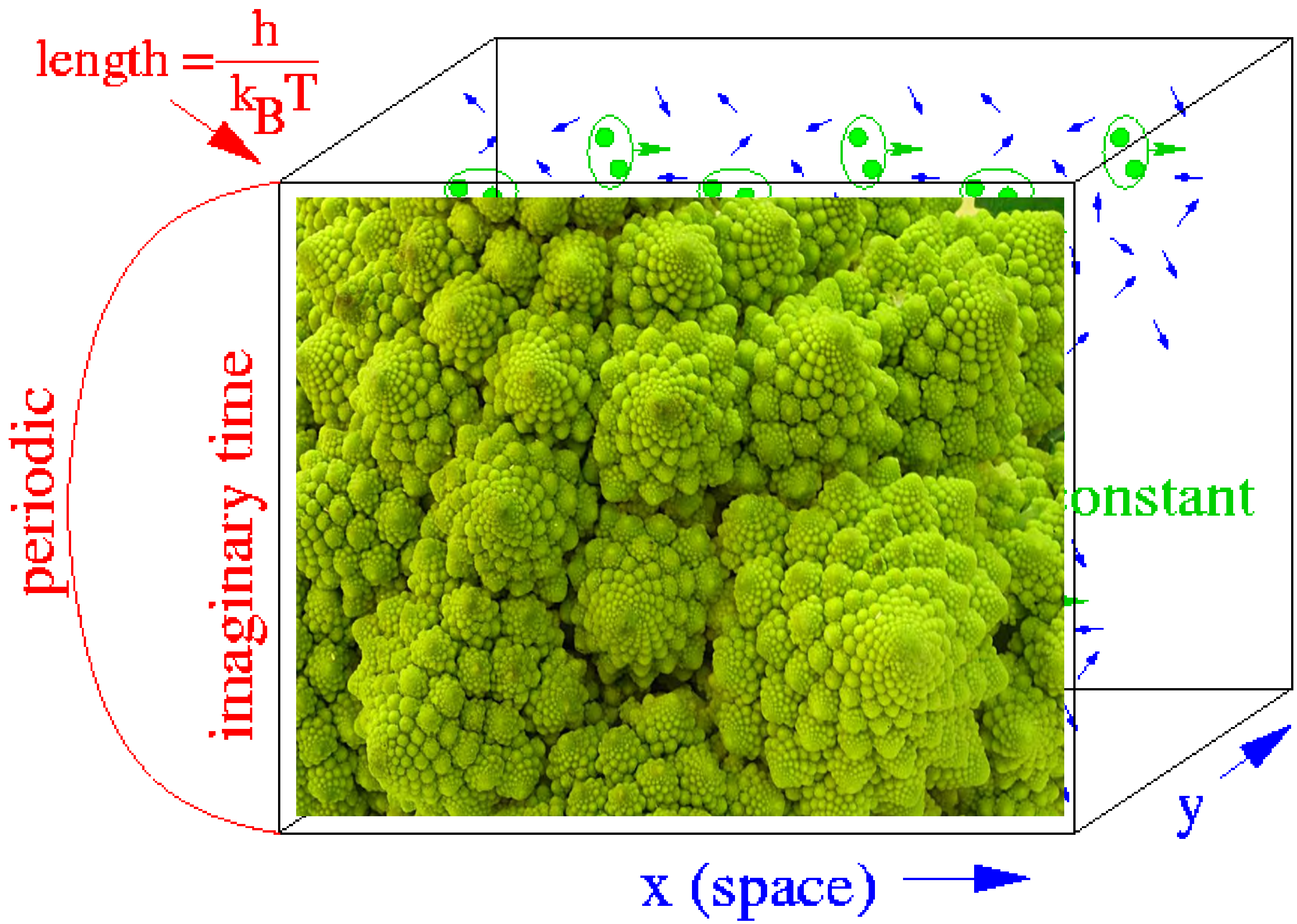


3. Quantum critical BCS: moderate glue yields a high Tc !

$$\Delta_0 = 2\hbar\omega_B\left(\lambda/\left(\lambda + \left(2\omega_B/\omega_c\right)^{(2-\eta_{pp})/z}\right)\right)^{\frac{z}{2-\eta_{pp}}}$$



Quantum criticality or 'conformal fields'



Fractal Cauliflower (romanesco)

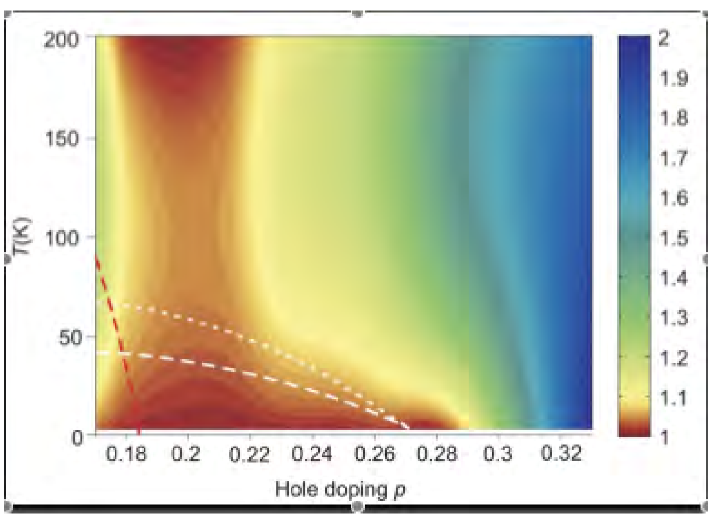
Where is the cuprate quantum critical point? …

High precision resistivities La2-xSrxCuO4.
Science **323**, 603 (2009)

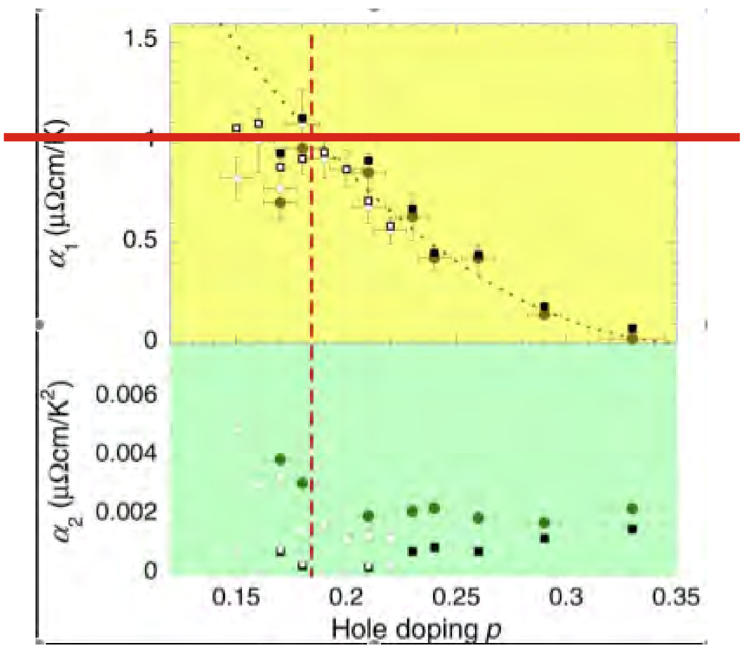
Hussey
et al.

$$\rho \propto T^{\alpha}$$

$$\rho = \rho_0 + \alpha_1 T + \alpha_2 T^2$$



$$\tau_{\hbar} = \frac{\hbar}{k_B T}$$



Fermion hunting club …

She

Sadri

Overbosch

Krueger,
Urbana

Mukhin,
Moscow

Weng,
Beijing

Mitas,
Raleigh

Fisher,
Microsoft

Ceperley,
Urbana

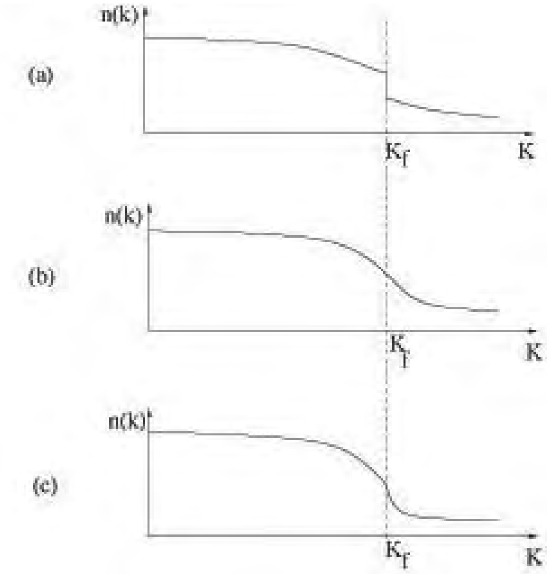
Further reading/playing:

http://demonstrations.wolfram.com/DressedMultiParticleElectronWaveFunctions/

http://physics.aps.org/

arXiv:0802.0273, 0802.2455, **0804.2161**, 0807.1279

Senthil's critical Fermi-surface



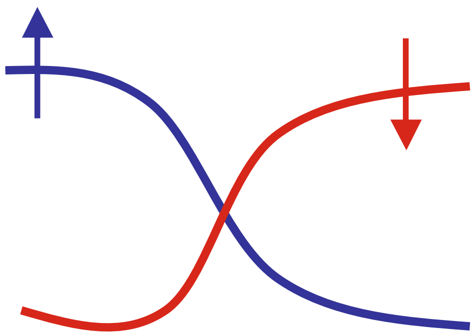
$$A\left(\vec{K},\omega,T,g\right) = \frac{c_0}{|\omega|^{\alpha(k_{//})/z(k_{//})}} F\left(\frac{c_1\omega}{k_\perp^{z(k_{//})}}, \frac{\omega}{T}, k_\perp |g - g_c|^{-\nu(k_{//})} \right)$$

arXiv:0803.4092

BCS nodal structure

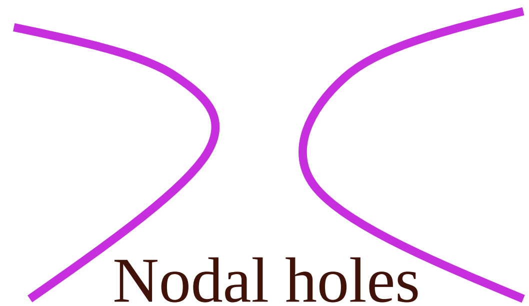
Fermi liquid:

$$\Psi_{FL} = \Pi_k c^+_{k\uparrow} c^+_{k\downarrow} \left|vac.\right\rangle$$



BCS state:

$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}\right) \left|vac.\right\rangle$$



Nodal holes
=> BCS log

Conjecture: the 'thermodynamic measure' of the BCS nodal holes is larger for fractal nodal structure, because the latter is more configuration space filling.

Temperature dependence of nodes

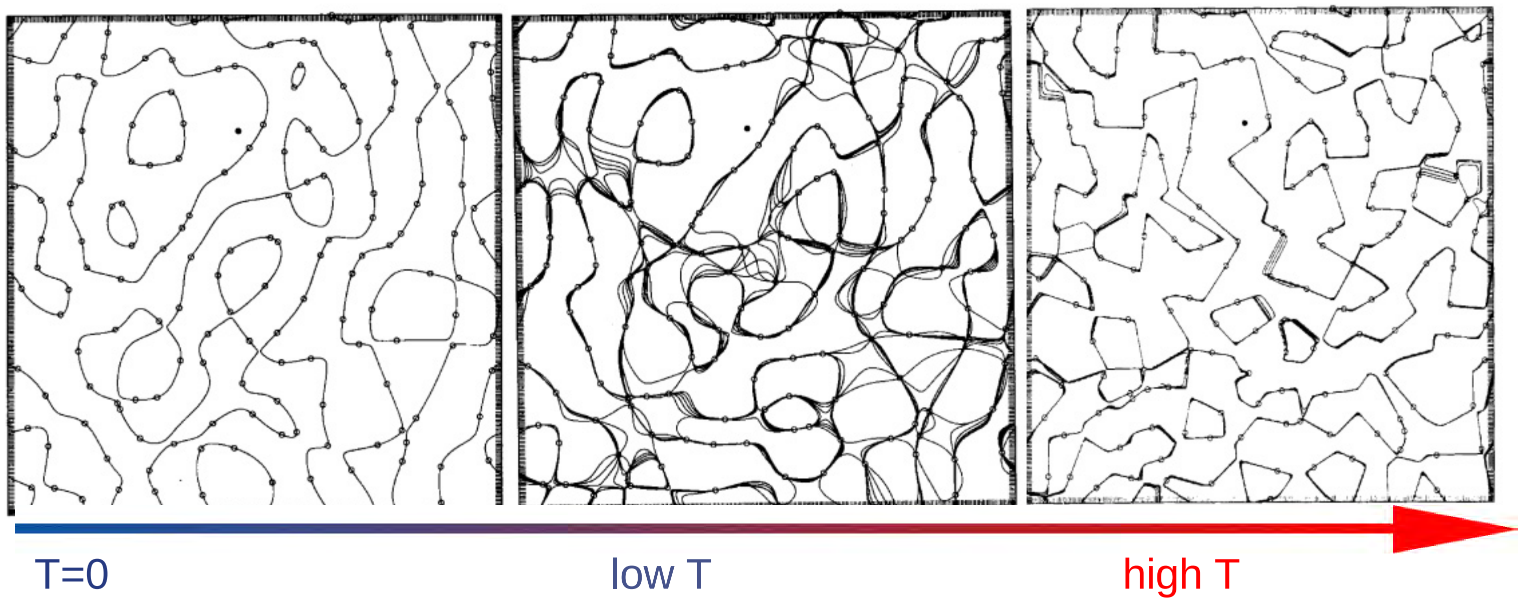
The nodal hypersurface at finite temperature

$$\Omega(\mathbf{R}_0;\beta) := \left\{\mathbf{R} \in \mathbb{R}^{Nd} | \rho_F(\mathbf{R}_0,\mathbf{R};\beta) = 0\right\} \qquad \dim\Omega = Nd - 1$$

Free Fermions

$$\rho_F(\mathbf{R}_0,\mathbf{R};\beta) = (4\pi\lambda\beta)^{-dN/2}\mathrm{Det}\left[\exp\left(-\frac{(\mathbf{r}_i - \mathbf{r}_{j0})^2}{4\lambda\beta}\right)\right]_{ij} \qquad \lambda = \frac{\hbar^2}{2m}$$

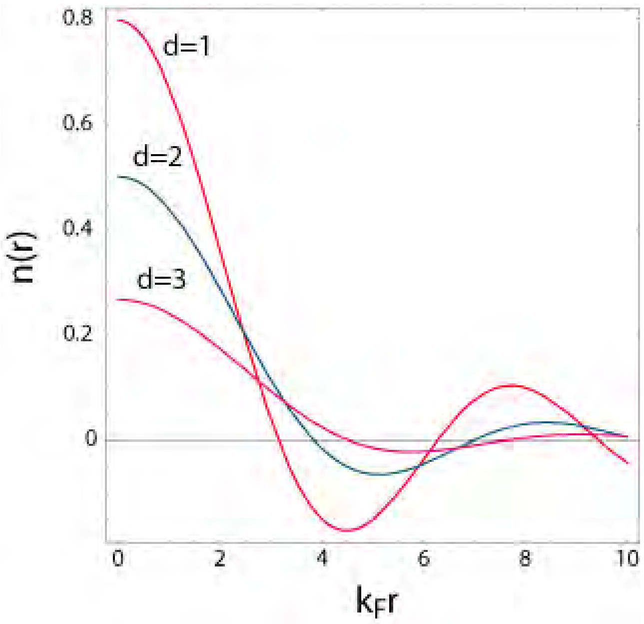
$$\rho_F(\mathbf{R}_0,\mathbf{R},\beta) \stackrel{T\to 0}{\to} \Psi_0^*(\mathbf{R}_0)\Psi_0(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\mathbf{r}_j}\right)_{ij}$$



Fermi liquid's nodal pocket

Average distance to the nodes

$$n(\mathbf{r}) = \int \mathrm{d}\mathbf{R}\Psi^*(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)\Psi(\mathbf{r}_1 + \mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$
$$= \int_{\mathbf{k}} e^{i\mathbf{kr}} n_{\mathbf{k}}$$



Free fermions

$$n(\mathbf{r}) = (k_F r)^{-d/2} J_{d/2}(k_F r)$$

First zero

$$r_0 = (3.14, 3.83, 4.49) k_F^{-1} \sim r_s$$

Just Ansatz or physics?

Mott transition, continuous

metal

Finite compressibility

Mott insulator

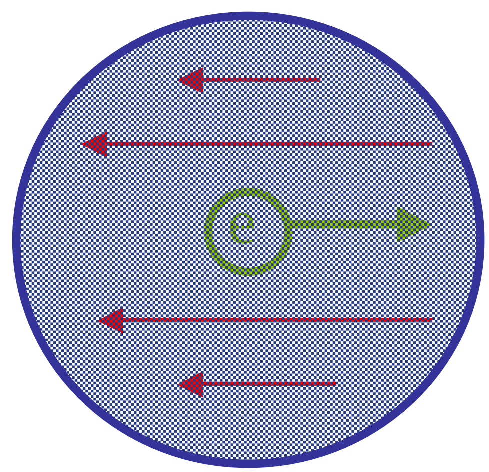
Compressibility = 0

U/W

Quasiparticles turn charge neutral

Neutral QP

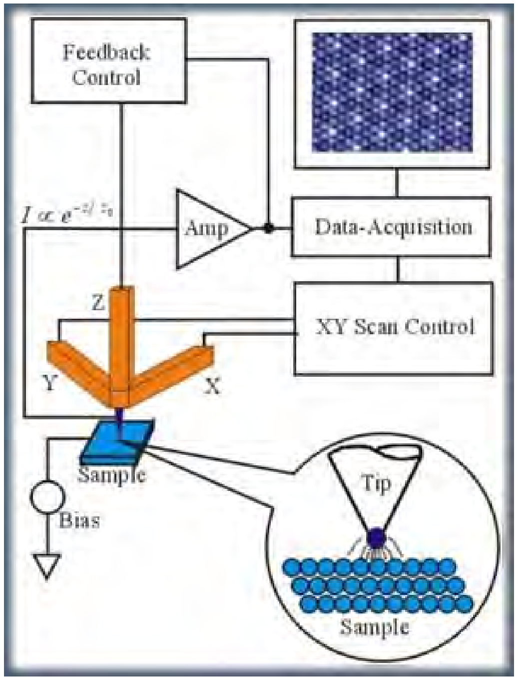
Gabi Kotliar



Backflow turns hydrodynamical at the quantum critical point!

Scanning tunneling spectroscopy

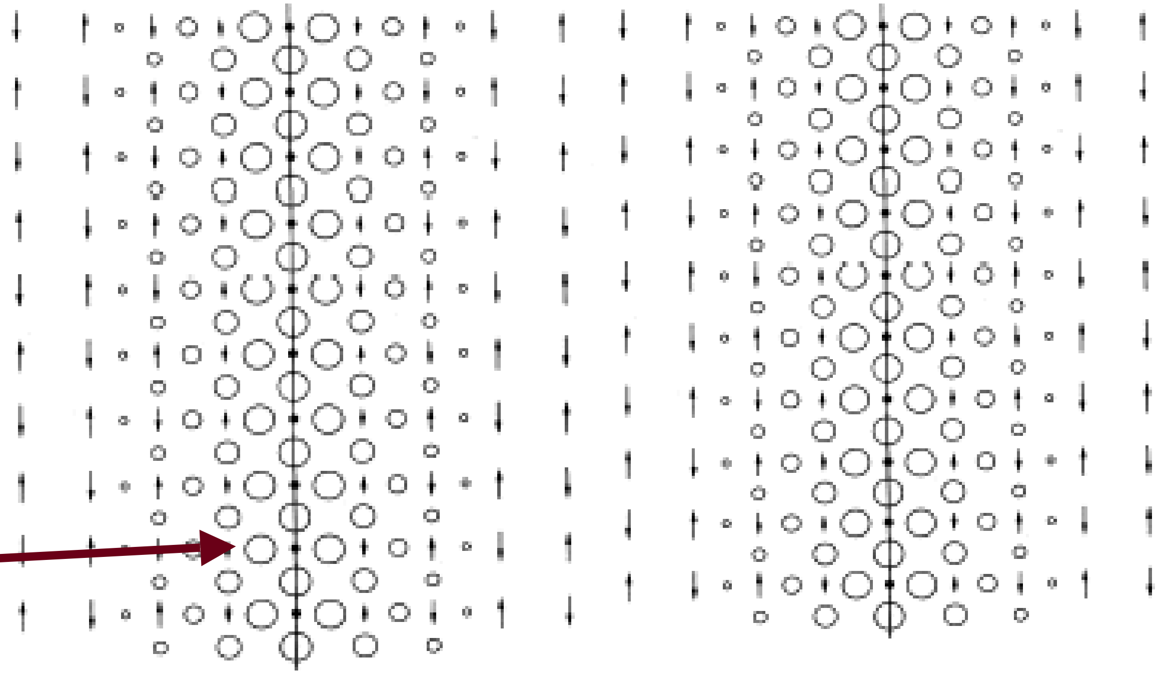
Seamus Davis, Cornell



Zaanen-Gunnarsson (1987)

Large S on 3-band Hubbard model:

Hole density on oxygen



Ceperley path integral: Fermi gas in momentum space

Single particle propagator:

$$g(\boldsymbol{k}, \boldsymbol{k}', \tau) = 2\pi\delta(\boldsymbol{k} - \boldsymbol{k}')e^{-\frac{|\boldsymbol{k}|^2\tau}{2\hbar m}}$$

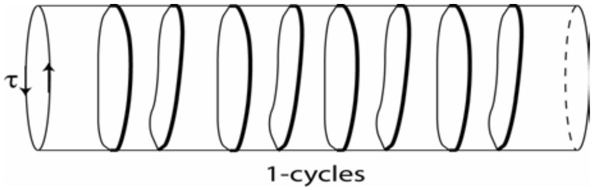
single particle momentum conserved

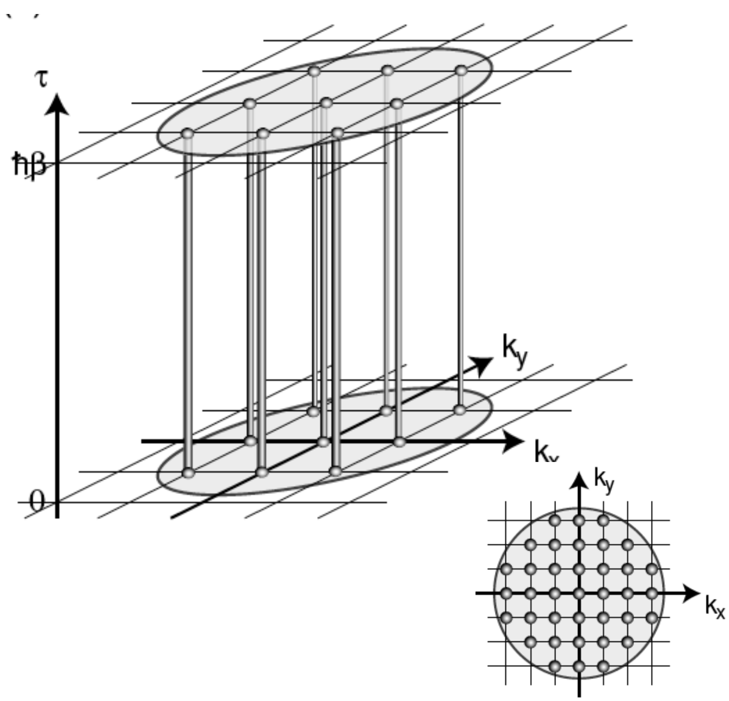
Sergei Mukhin

N particle density matrix:

$$\begin{aligned}\rho_F(\boldsymbol{K}, \boldsymbol{K}', \tau) &= \frac{1}{N!}\sum_{\mathcal{P}}(-1)^{\mathcal{P}}\prod_{i=1}^{N} g(\boldsymbol{k}_i, \boldsymbol{k}'_{p(i)}, \tau) \\ &= \frac{1}{N!}e^{-\sum_{i=1}^{N}\frac{|\boldsymbol{k}_i|^2\tau}{2\hbar m}}\sum_{\mathcal{P}}(-1)^{\mathcal{P}}\prod_{i=1}^{N} 2\pi\delta(\boldsymbol{k}_i - \boldsymbol{k}'_{p(i)}) \\ &= \prod_{\boldsymbol{k}_1\neq\boldsymbol{k}_2\neq\ldots\neq\boldsymbol{k}_N} 2\pi\delta(\boldsymbol{k}_i - \boldsymbol{k}'_i)e^{-\frac{|\boldsymbol{k}_i|^2\tau}{2\hbar m}}\end{aligned}$$

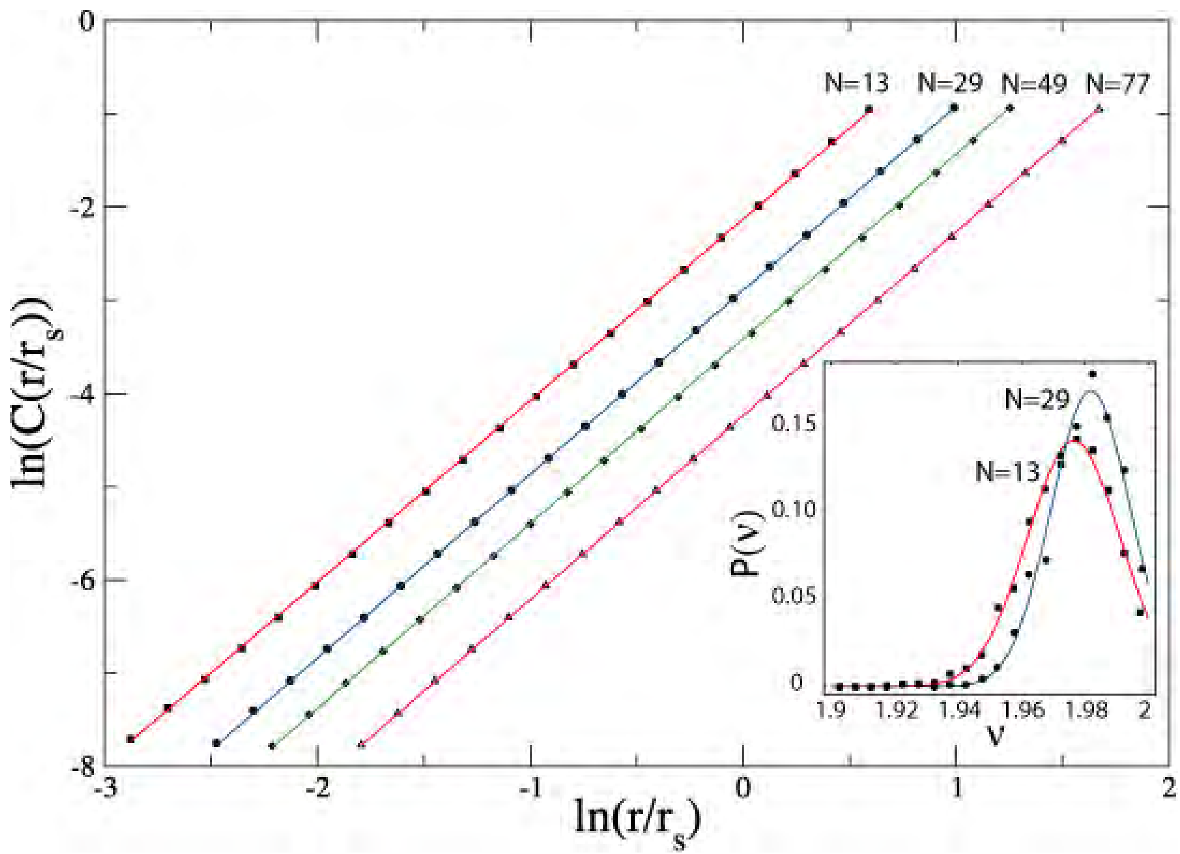
'harmonic potential'





Fractal dimension of the nodal surface

Calculate the correlation integral $C(r) \sim r^{\nu}$ on random d=2 dimensional cuts



$$Nd - 1 < \quad D_H = N\nu_d \quad < Nd$$
$$d - \frac{1}{N} < \qquad \nu_d \qquad < d$$

$N = 13:$ $\nu = 1.976 \pm 0.012$
$\rightarrow D_H = 25 + (0.69 \pm 0.16)$

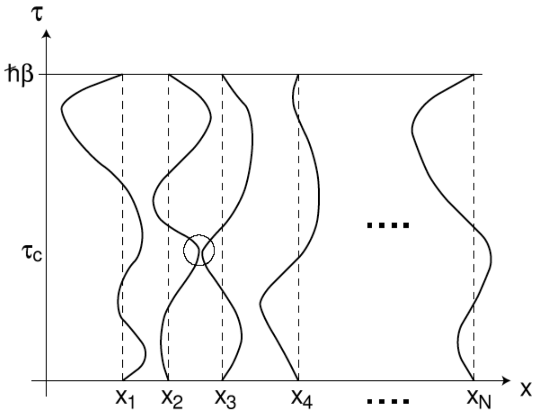
$N = 29:$ $\nu = 1.982 \pm 0.008$
$\rightarrow D_H = 57 + (0.48 \pm 0.23)$

Backflow turns nodal surface into a fractal !!!

Ceperley path integral in 1+1D

Nodal surface (Nd-1)=
Pauli surface (Nd-d)

Statistical physics of hard core polymers (Pokrovsky, Talapov)

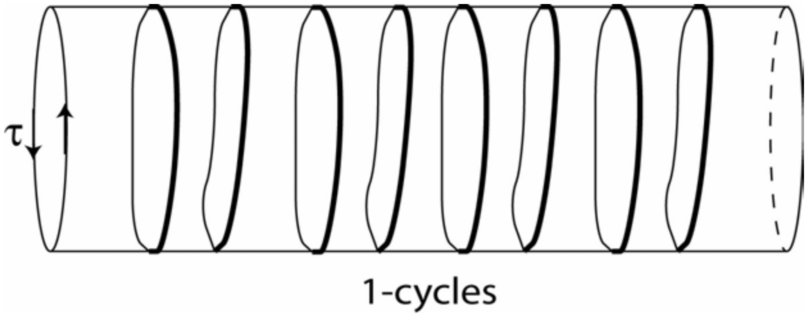


Ordering is preserved, statistics changes in N! relabellings

v

$$x_1(\tau) < x_2(\tau) < \cdots < x_N(\tau), \forall \tau$$

Particles become distinguishable.



Fermi energy: $E_F = \hbar / \tau_{collisions}$

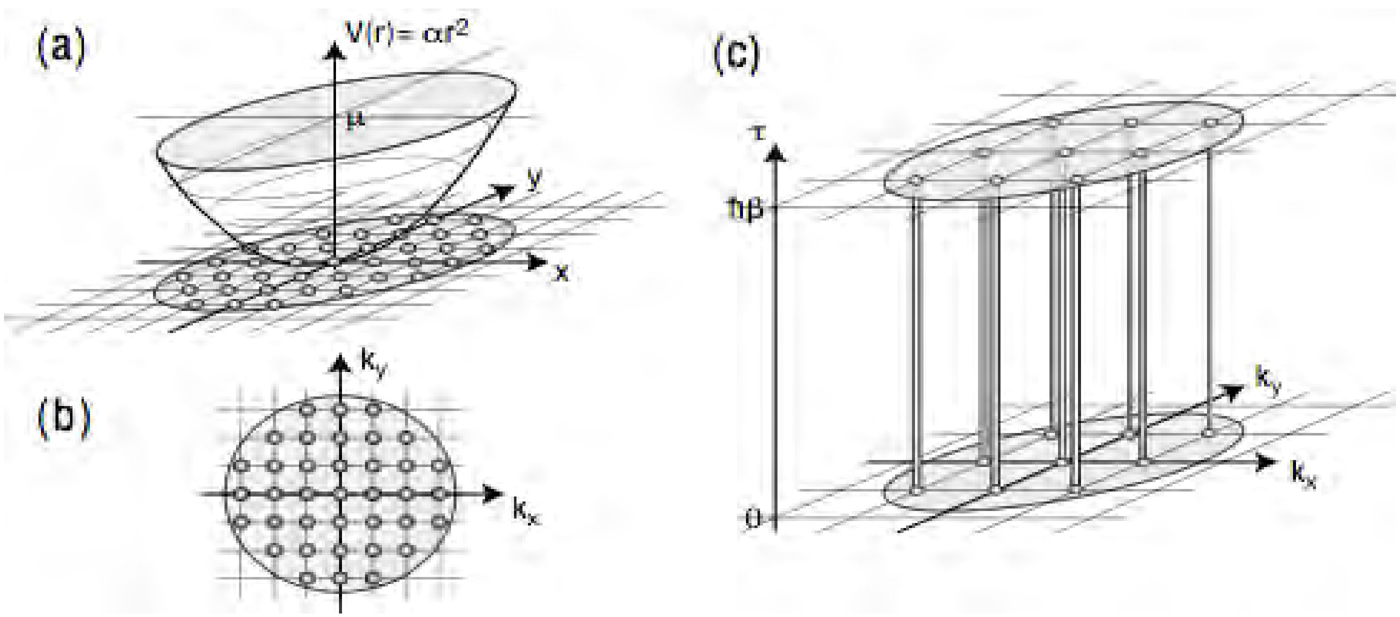
Entropic repulsions: algebraic order, sound velocity = Fermi velocity.

JZ, PRL 2000

Fermi gas = cold atom Mott insulator in harmonic trap!

$$\rho_F(K,K'';\tau) = \prod_{k_1 \neq k_2 \neq \cdots \neq k_N} 2\pi\delta\left(k_i - k_i''\right) e^{-\frac{|k_i|^2 \tau}{2\hbar m}}$$

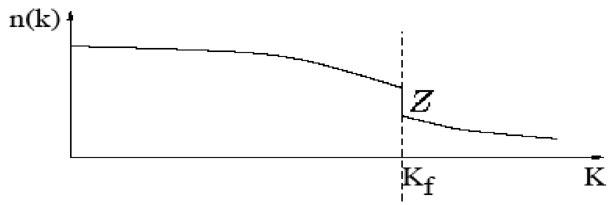
Mukhin, JZ, ..., Iranian J. Phys. (2008)



Switching on interactions

Single particle spectral function: $A(\boldsymbol{k},\omega) = \frac{1}{\pi}\frac{Z\gamma}{(\omega - v_F k_{||})^2 + \gamma^2}$

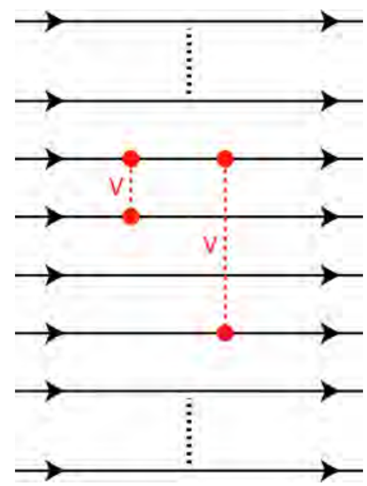
$\gamma \sim \omega^2 \sim k_{||}^2 = (k - k_F)^2$



J.-H. She

Configuration space: all Mott configurations of particles in trap

Fermi-gas: all configurations isolated = nodal surface

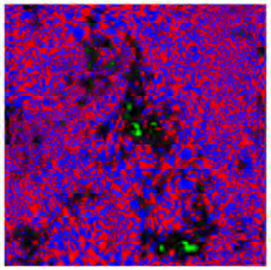


Fermi-liquid: $n_F^{d-1} \times (n - n_F)$ **pieces of (d-1) dimensional** **antinodal surfaces** **each of which has volume** $n_F^{d-2} \times (n - n_F)$

$(\vec{k} = \frac{2\pi}{a}\vec{n})$

Fermi-surface protection: antinodal surface shrinks to a point!

Extracting the fractal dimension



The Hausdorff dimension. The Hausdorff dimension of a metric space X, $\dim_H(X)$, is the infimimum of the numbers α with the following property: For any $\epsilon > 0$ there is a $\delta > 0$ and a cover $\mathfrak{U}$ of X such that the sets $B \in \mathfrak{U}$ all have diameter $|B|$ smaller than δ and

$$\sum_{B \in \mathfrak{U}} (|B|)^\alpha < \epsilon.$$

The correlation integral:

$$\begin{aligned} C(r) &= \lim_{n\to\infty} \frac{1}{n^2} \sum_{i,j=1}^{n} \Theta(r - |\mathbf{r}_i - \mathbf{r}_j|) \\ &= \int_0^r \mathrm{d}^D r' c(\mathbf{r}') \end{aligned}$$

For fractals:

$$C(r) \sim r^\nu, \quad \nu \le \dim_H$$

Inequality very tight, relative error below 1%

Grassberger & Procaccia, PRL (1983)

The Gross list: the 14 Big Questions

1. The origin of the universe?
2. What is dark matter?

•

•

11. What is space-time?

•

14. New states of matter: are there generic non-Fermi liquid states of interacting condensed matter?

Solution of the Fermion sign problem??

2004

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