Unquenched massive flavors and flows in Chern-Simons matter theories

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Collaborators/References

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Outline

- Brief motivation
- Review of the ABJM model
- Addition of flavor
- Backreacted flavored backgrounds
- Observables along the RG flow
  - Entanglement entropy on the disk
  - Entanglement entropy on the strip
  - Wilson loops and quark-antiquark potential
  - Two-point function of high dimension operators
  - Meson spectrum

Brief motivation

- Use AdS/CFT to learn about phenomena with low-energy physics in strongly coupled systems. (FQHE, superconductivity, confinement, chiral symmetry breaking,...)
- Concrete string theory construction: known field theory dual.
- Will give new effective models.
- Universal features?
- Maybe we will gain new understanding.

ABJM Chern-Simons matter theories

- Associated with M2-branes in $\mathbb{C}^4/\mathbb{Z}_k$ in M-theory.
  [Aharony-Bergman-Jafferis-Maldacena]
- Field theory: Chern-Simons matter theories in 2+1 dimensions with gauge group $U(N)_k \times U(N)_{-k}$.
- Bosonic field content:
  - Two gauge fields $A_\mu, \hat{A}_\mu$
  - Four complex scalar fields: $C^{I=1,\dots,4}$, bifundamentals $(N, \bar{N})$
- Action

$$S = k\mathrm{CS}[A] - k\mathrm{CS}[\hat{A}] - kD_\mu C^{I\dagger} D^\mu C^I - V_{\mathrm{pot}}(C)$$

$$V_{\mathrm{pot}}(C) = \text{sextic scalar potential}$$

ABJM Chern-Simons matter theories

- The ABJM has $\mathcal{N} = 6$ SUSY in 3d.
- It has two parameters, which form the 't Hooft coupling $\lambda \sim \frac{N}{k}$:
  - N: rank of the gauge group
  - k: CS level ($1/\sqrt{k} \sim$ gauge coupling)
- It is a CFT with very nice properties
  - partition function and Wilson loops can be obtained from localization

    [Drukker-Mariño-Putrov]
  - has many integrability properties (Bethe ansatz, Wilson loop/amplitude relation, ...)
  - may help understand some cond-mat phenomena which are essentially 3d?
- It is the 3d analogue of $\mathcal{N} = 4$ SYM in 4d

ABJM: SUGRA side

- At low energy, the M-theory description for large $N \to$ 11d SUGRA in $AdS_4 \times \mathbb{S}^7/\mathbb{Z}_k$.
- SUGRA description in type IIA:
  - Represent $\mathbb{S}^7$ as a $U(1)$ bundle over $\mathbb{CP}^3$. Reduce from 11d to 10d along the $U(1)$ fiber φ
  - Get $AdS_4 \times \mathbb{CP}^3$+fluxes, $\mathbb{CP}^3 = \mathbb{C}^4/(z_i \sim \lambda z_i)$.

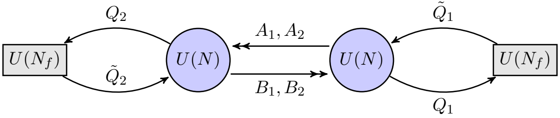
$$ds^2_{10d} = L^2 ds^2_{AdS_4} + 4L^2 ds^2_{\mathbb{CP}^3} \quad , \quad L^4 = 2\pi^2 \frac{N}{k}$$

$$ds^2_{AdS_4} = r^2 dx^2_{1,2} + \frac{dr^2}{r^2} \quad , \quad ds^2_{\mathbb{CP}^3} = \mathrm{Fubini-Study\ metric}$$

$$F_2 = 2kJ \quad , \quad F_4 = \frac{3\pi}{\sqrt{2}} \sqrt{kN} \Omega_{AdS_4} \quad , \quad e^{\phi} = \frac{2L}{k} = 2\sqrt{\pi} \left(\frac{2N}{k^5}\right)^{1/4}$$

- Effective description for $N^{1/5} \ll k \ll N$.

Adding flavor



- Add flavor D6-branes (massless quarks) in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$.

  [Hohenegger-Kirsch]
  [Gaiotto-Jafferis]

- Introduce quarks in the $(N, 1)$ and $(1, N)$ representation:

  $$Q_1 \to (N, 1)\ , \quad Q_2 \to (1, N)\ , \quad \tilde{Q}_1 \to (\bar{N}, 1)\ , \quad \tilde{Q}_2 \to (1, \bar{N})$$

- coupling to vectors ($V, \hat{V}$ vector supermultiplets for $A, \hat{A}$):

  $$Q_1^\dagger e^{-V} Q_1 + Q_2^\dagger e^{-\hat{V}} Q_2 + \text{antiquarks}$$

- coupling to the bifundamentals ($C^I = (A_1, A_2, B_1^\dagger, B_2^\dagger)$):

  $$\tilde{Q}_1 A_i B_i Q_1\ , \quad \tilde{Q}_2 B_i A_i Q_2\ , \quad + \text{quartic terms in } Q, \tilde{Q}'\text{s}$$

Including the backreaction

- The flavors backreact on the geometry:

$$S_{WZ} = T_{D_6} \sum_{i=1}^{N_f} \int_{\mathcal{M}_7^{(i)}} \hat{C}_7 \to T_{D_6} \int_{\mathcal{M}_{10}} C_7 \wedge \Omega$$

- $\Omega = \sum_{N_f} \delta^{(3)}(\mathcal{M}_7)\omega_3$ is a charge distribution 3-form, where ω_3 is the transverse volume element
- Induces a violation of the Bianchi identity of F_2:

$$dF_2 = 2\pi\Omega \qquad \to \quad \delta - \text{function source term}$$

- Einstein equations have also δ-function source terms: very difficult to solve! Also PDEs...
- Localized soln. in $11d$ for coincident massless flavors $AdS_4 \times \mathcal{M}_7$, with $\mathcal{M}_7$ hyperkähler tri-Sasakian manifold, $\mathcal{N} = 3$, with $U(N_f)$. Reduce to 10d: becomes a mess. [Gauntlett-Gibbons-Papadopoulos-Townsend,...]
- Conformality is kept intact with flavor!

Background action

- The full (bosonic) action in Einstein frame:

$$S^E = S^E_{IIA} + S^E_{sources}$$

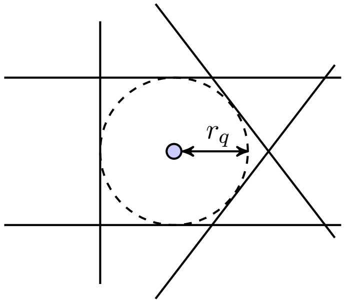
$$S^E_{IIA} = \frac{1}{2\kappa_{10}^2}\left[\int \sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) - \frac{1}{2}\int\left(e^{3\phi/2} * F_2 \wedge F_2 + e^{\phi/2} * F_4 \wedge F_4\right)\right]$$

$$S^E_{sources} = -T_{D6}\int\left(e^{3\phi/4}\mathcal{K} - C_7\right)\wedge\Omega$$

- Key observation: start from the known (unsourced) ABJM solution and try to generalize from there.

Smearing technique

[Bigazzi-Casero-Cotrone-Kiritsis-Paredes'05,. . . ,Nunez-Paredes-Ramallo'10]



- no δ-function sources
- still can preserve (less) SUSY
- much simpler (analytic) solutions
- flavor symmetry: $U(1)^{N_f}$

Backreaction with smearing

- Write $\mathbb{CP}^3$ as an $\mathbb{S}^2$-bundle over $\mathbb{S}^4$:

[Conde-Ramallo]

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\left(ds^2_{\mathbb{S}^4} + \left(dx^i + \epsilon^{ijk}A^j x^k\right)^2\right) \quad , \quad \sum_i (x^i)^2 = 1$$

- The RR two-form F_2 can be written as

$$F_2 = \frac{k}{2}\left(E^1 \wedge E^2 - \left(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\right)\right) \quad , \quad \int_{\mathbb{CP}^1} F_2 = 2\pi k$$

- A^i are $SU(2)$ instantons on $\mathbb{S}^4$, $\mathcal{S}^i$ are (rotated) basis one-forms along $\mathbb{S}^4$, E^i are one-forms along the $\mathbb{S}^2$ fiber; $\left(dx^i + \epsilon^{ijk}A^j x^k\right)^2 = (E^1)^2 + (E^2)^2$
- Idea: some Killing spinors are constant in this basis $\rightarrow$ deform to preserve them

Backreaction with smearing

- Prescription: squash F_2 (induces a violation of Bianchi identity) and the metric:

$$F_2 = \frac{k}{2}\left(E^1 \wedge E^2 - \eta(r)\left(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\right)\right)$$

$$F_4 = K(r) d^3x \wedge dr$$

$$x\frac{dr}{dx} = e^g \quad , \quad q(x) = e^{2f-2g} \ \ (\textit{squashing of } \mathbb{CP}^3)$$

$$ds^2_{10} = h^{-1/2} dx^2_{1,2} + h^{1/2}\left[e^{2g}\frac{dx^2}{x^2} + e^{2f} ds^2_{\mathbb{S}^4} + e^{2g}\left((E^1)^2 + (E^2)^2\right)\right]$$

- Flux quantization $\int *F_4 \sim \textit{integer}$ and $d * F_4 = 0$ imply $K = 3\pi^2 N h^{-2} e^{-4f-2g}$.

Master equation

- Instead to trying to solve 2nd-order Einstein DE, make use of SUSY: 1st order DE.
- It turns out that the BPS equations can be reduced to one second-order differential equation

$$W'' + 4\eta' + (W' + 4\eta)\left[\frac{W' + 10\eta}{3W} - \frac{W' + 4\eta + 6}{x(W' + 4\eta)}\right] = 0$$

$$W(x) \equiv \frac{4}{k} h^{1/4} e^{2f-g-\phi}$$

- Given η all the other functions h, g, f, ϕ, q can be constructed from the master function W!

Solutions to the master equation

- The master equation has analytic solutions.
- Take $\eta = const.$:

$$W = A_0(\eta)x$$

- Corresponds to the massless smeared flavor solution, which is remarkably simple ($q = const.$). For the particular case $A_0(1) = 2$ one obtains the ABJM solution.

  [Conde-Ramallo]

- Take $\eta = 1$ (ABJM at the IR, G_2 cone at UV), running solution:

$$W = \frac{4(1+4\gamma x)x}{1+\sqrt{1+4\gamma x}} \, .$$

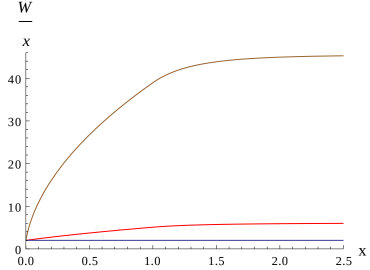
- We are interested in interpolating solutions: ABJM $(\eta = 1)$ at IR and massless smeared ($\eta = const. > 1$) at UV. Need to resort to numerics.

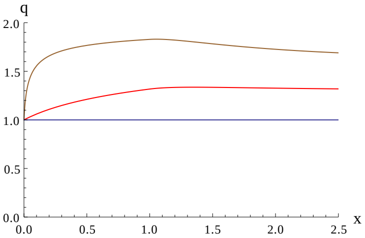
Interpolating solutions

- The profile η, which corresponds to N_f smeared flavor D6-branes ending at $r = r_q$ (can set $x_q = 1$) is determined by kappa symmetry:

$$\eta(x) = 1 + \hat{\epsilon}\left(1 - \frac{1}{x^2}\right)\Theta(x-1) \quad , \quad \hat{\epsilon} = \frac{3N_f}{4k}$$

- Match running solution with numerical at $x = 1$, only one $\gamma(\hat{\epsilon}) \sim 0.4\hat{\epsilon} + 0.3\hat{\epsilon}^2$ works for which $W \sim x, x \to \infty$.





Description of the RG flow

- Only one parameter r_q mass of unquenched quarks. (and $\hat{\epsilon}$)
- Interpolating solution between two asymptotically AdS_4 with $L_{IR} > L_{UV} \sim \frac{N}{N_f}$.
- Along the flow $\mathcal{N}=1$ is preserved and $U(1)^{N_f}$. The flow is generated by changing the quark mass r_q, $r_q \to \infty$: ABJM and $r_q \to 0$ unquenched massless flavors.
- From UV expansions one can infer deviations from conformality, controlled by the quark mass, and where b determines the dimension $\Delta = 3 - b$ of the $q\bar{q}$ bilinear.

$$b=\frac{2q_{UV}}{q_{UV}+1} \; , \; q_{UV}=\frac{3}{2}+3(1+\hat{\epsilon})-\sqrt{9(1+\hat{\epsilon})^2-2(1+\hat{\epsilon})+9}$$

$$e^{g(r)}=\frac{r}{b}\left[1+\tilde{g}_2\left(\frac{r_q}{r}\right)^{2b}+\ldots\right] \; , \; e^{f(r)}=\frac{q_{UV}\,r}{b}\left[1+\tilde{f}_2\left(\frac{r_q}{r}\right)^{2b}+\ldots\right]$$

$$h(r)=\left(\frac{L_{UV}}{r}\right)^4\left[1+\tilde{h}_2\left(\frac{r_q}{r}\right)^{2b}+\ldots\right] \; , \; e^{\phi(r)}=e^{\phi_{UV}}\left[1+\tilde{\phi}_2\left(\frac{r_q}{r}\right)^{2b}+\ldots\right]$$

Pros in comparison to other smeared solutions

- Our solution is very simple and we have a lot of analytic control.
- Our solution has a good UV behavior, no Landau pole. The spacetime has an *AdS*-factor!
- Our solution has a good IR behavior, no IR singularity due to massless flavors. The IR fix point is stable.

  [Bianchi-Penati-Siani]
- At the massless limit $r_q \to 0$, our solution has a simple $T \neq 0$ generalization by just including the blackening factor in the metric.

  [NJ-Mas-Ramallo-Zoakos]

Observables: entanglement entropy on the disk

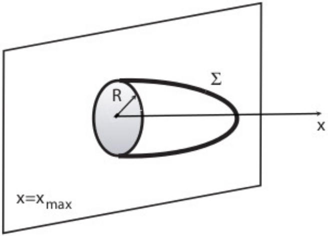
- Minimize the hanging surface ending on a disk of radius R: [Ryu-Takayanagi]

$$S(R) = \frac{1}{4G_{10}} \int_\Sigma d^8\xi \, e^{-2\phi} \sqrt{\det g_8}$$

- The expression is divergent $S_{div} = \frac{F_{UV}(\mathbb{S}^3)}{L_{UV}^2} r_{max} R$ and to extract the finite piece is ambiguous. We use [Liu-Mezei]

$$\mathcal{F} \equiv R\frac{\partial S}{\partial R} - S$$

- For 3d CFT: $S_{CFT} = \alpha R - \gamma$. Notice that S is of this form (as $R \to \infty$). Hence $\mathcal{F} = \gamma$, and at the fixed point $\mathcal{F}$ is constant and equal to free energy on the three-sphere. [Casini-Huerta-Myers]

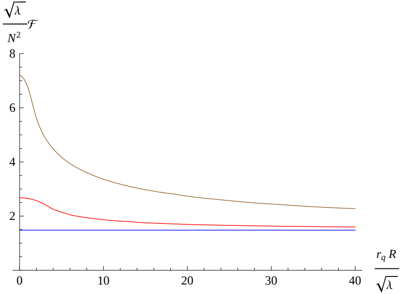


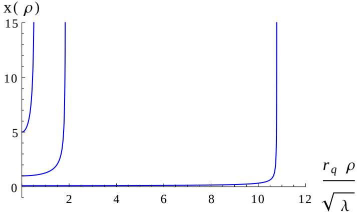
Entanglement entropy on the disk

- The asymptotic values ($\Delta_{UV} = 3 - b$):

$$\mathcal{F}(R) = \begin{cases} F_{UV}(\mathbb{S}^3) + c_{UV}(r_q R)^{2(3-\Delta_{UV})} + \ldots & , r_q R \to 0 \\ F_{IR}(\mathbb{S}^3) + \ldots & , r_q R \to \infty \end{cases}$$

- The $\mathcal{F}$ is finite and monotonic along the flow: F-theorem!
[Myers-Sinha,Klebanov-et al.,Casini-Huerta]

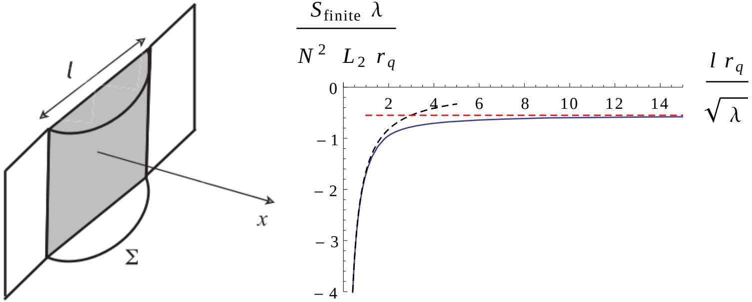




Entanglement entropy on the strip

- Repeat the holographic entanglement entropy for a surface ending on an infinite strip.
- Asymptotically

$$\frac{S_{finite}(\ell)}{r_q L_2} = \begin{cases} -\frac{4\pi^2 F_{UV}(\mathbb{S}^3)}{\Gamma(\frac{1}{4})^4}\frac{1}{r_q\ell} + \dots & , r_q\ell \to 0 \\ S_\infty - \frac{4\pi^2 F_{IR}(\mathbb{S}^3)}{\Gamma(\frac{1}{4})^4}\frac{1}{r_q\ell} + \dots & , r_q\ell \to \infty \end{cases}$$

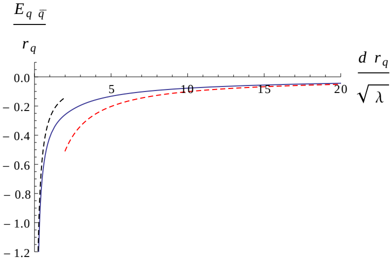


- Compute the regularized Nambu-Goto action for a hanging string. [Maldacena,Rey-Yee]
- Asymptotically:

$$\frac{E_{q\bar{q}}}{r_q} = \begin{cases} -\frac{4\pi^3\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\sigma\frac{1}{r_q d} + \dots & , r_q d \to 0 \\ -\frac{4\pi^3\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{r_q d} + \dots & , r_q d \to \infty \end{cases}$$

- σ characterizes corrections of the static $q\bar{q}$ potential due to screening produced by unquenched flavors.

$$\sigma \to \begin{cases} 1 & , N_f \to 0 \\ \sqrt{k/N_f} & , N_f \text{ large} \end{cases}$$



Two-point function of high dimension operators

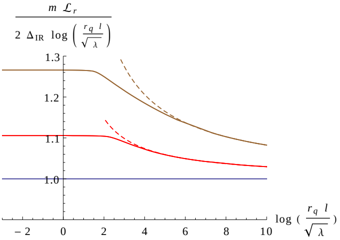
- Semi-classical calculation of geodesics of massive particles in the dual geometry: [Balasubramanian-Ross,Louko-Marolf-Ross,Kraus-Ooguri-Shenker]

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim e^{-m\mathcal{L}_r(x,y)}$$

- Here $\mathcal{L}_r(x,y)$ is the regularized length along the geodesic and m large to make saddle-point approximation applicable.

- Asymptotically (CFT expectations):

$$\langle \mathcal{O}(t,\ell)\mathcal{O}(t,0)\rangle \to \begin{cases} \dfrac{1+c_\Delta\left(\frac{r_q\ell}{\sqrt{\lambda}}\right)^{2b}+\ldots}{\left(r_q\ell/\sqrt{\lambda}\right)^{2\Delta_{UV}}} \\ \dfrac{\mathcal{N}+\ldots}{\left(r_q\ell/\sqrt{\lambda}\right)^{2\Delta_{IR}}} \end{cases}$$



Meson spectrum

- Embed a flavor probe D6-brane and study its fluctuations analytically/numerically.
- Focus on vector mesons:

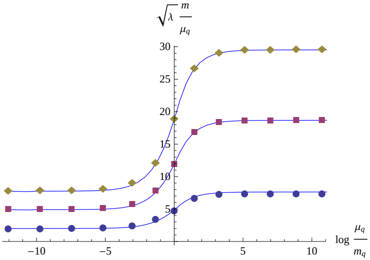
$$A_\mu = \xi_\mu e^{ik_\nu x^\nu} R(x)\ ,\ A_{angular\ \ directions} = 0\ .$$

- WKB results (UV and IR are exact!):

$$m_{WKB} = \frac{\pi}{\sqrt{2}\xi(x_*)}\sqrt{(n+1)(2n+1)}\, ,\ \xi(x_*) = \int_{x_*}^{\infty} dx \frac{e^{g(x)}\sqrt{h(x)}}{\sqrt{x^2 - x_*^2}}$$

- Asymptotically:

$$\frac{m_{WKB}^{UV}}{m_{WKB}^{IR}} = \frac{\Gamma\left(\frac{b+1}{2b}\right)}{\Gamma\left(\frac{2b+1}{2b}\right)}\frac{1}{\sigma}$$



Conclusions and outlook

- We described how to smear massive D6-branes in the ABJM background, while keeping $\mathcal{N} = 1$.
- We obtained a non-trivial holographic RG flow connecting two scale-invariant fixed points: the unflavored ABJM theory at the IR and the massless flavored model at the UV.
- We studied several observables along the RG flow, *e.g.*, we confirmed the F-theorem and showed that infinitely massive flavors can be smoothly decoupled.
- Lots of avenues how to generalize our case: ABJ, Romans mass, include gauge fields,. . .

Thank you!