Universal Hydrodynamics for Quantum Critical Points with Lifshitz scaling

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Based on 1304.7481 and 1309.6794 with Carlos Hoyos and Yaron Oz

Review on theories with Lifshitz scaling (5 slides)

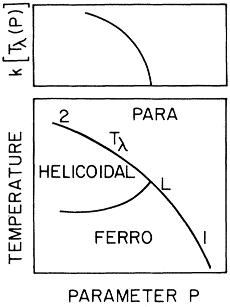
- Motivation
- Lifshitz algebra
- Field theory side
- Gravity theory side

Quantum Critical Point (4 slides)

Universal Hydrodynamics of Lifshitz theories (7 slides)

Drude model of strange metal (3 slides)

Lifshitz critical point



Lifshitz critical point can be realized as a special point in a line of critical points.

Free energy described by a scalar order parameter : Hornreich et. al. PRL 35 (1975) 1678

$$\mathcal{F}(M) = a_2 M^2 + a_4 M^4 + a_6 M^6 + \cdots + c_1 (\nabla M)^2 + c_2 (\nabla^2 M)^2 + \cdots ,$$

- ordinary critical point : a_2, a_4 and c_1.
- tri-critical point : need a_6 if $a_4 = 0$.
- Lifshitz multi-critical point : need c_2 for $c_1 = 0$.

Lifshitz algebra

- Scaling transformation: $D = -(zt\partial_t + x^i\partial_i)$, $\quad (\Leftarrow \ t \to \Omega^{-z} t \ , x^i \to \Omega^{-1} x^i)$
- Rotations: $M_{ij} = -x^i\partial_j + x^j\partial_i$, $\qquad i,j = 1, \cdots, d$
- Time and spatial translations: $H = -\partial_t$, $P_i = -\partial_i$,

$$[M_{ij}, P_k] = \delta_{ik} P_j - \delta_{jk} P_i \ ,$$
$$[D, P_i] = P_i \ , \quad [D, H] = zH \ .$$

- There CAN be various extensions, especially for $z = 1$ and $\underline{z = 2}$.

- Galilean boost: $K_i = -t\partial_i$,

$$[K_i, M_{kl}] = -\delta_{ik} K_j + \delta_{il} P_k \ ,$$
$$[K_i, H] = P_i \ , \quad [D, K_i] = (1 - z) K_i \ ,$$
$$[K_i, P_j] = \delta_{ij} N \ ,$$

- N: a central extension for particle number conservation (Jacobi identity)

$$[N, D] = (z - 2) N \ ,$$

- Special conformal transformation: $C = -t^2\partial_t - tx^i\partial_i$ for $z = 2$

$$[C, D] = 2C \ , \quad [C, H] = D \ , \quad [C, P_i] = K_i \ .$$

Free field theories with Lifshitz scaling

Hoyos, BSK, Oz, 1309.6794

$$2\mathcal{L}=(\partial_t\phi)^2-\lambda(\nabla^z\phi)^2\ , \qquad T^{\mu}_{\ \nu}=\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\nu\phi-\delta^\mu_\nu\mathcal{L}\ ,$$

In momentum space

$$S=\int d^{d+1}x\mathcal{L}=\frac{1}{2}\int\frac{d\omega}{2\pi}\frac{d^dk}{(2\pi)^d}\left[\omega^2-\lambda|k|^{2z}\right]\phi(\omega,k)\phi(-\omega,-k)\ .$$

- classical solutions have a the dispersion relation $\omega^2=\lambda|k|^{2z}$,
- canonical energy-momentum tensor

$$2T^0_{\ 0}=(\omega^2+\lambda|k|^{2z})\phi^2_{\omega,k}\ , \qquad T^i_{\ 0}=-\lambda\omega k^i|k|^{2(z-1)}\phi^2_{\omega,k}\ ,$$
$$T^0_{\ i}=\omega k_i\phi^2_{\omega,k}\ , \qquad T^i_{\ j}=-\lambda[k^ik_j|k|^{2(z-1)}-\alpha\delta^i_j/2(\omega^2-\lambda|k|^{2z})]\phi^2_{\omega,k}\ ,$$

- note that energy-momentum tensor is not symmetric: $T^i_{\ 0}\neq T^0_{\ i}$.
- for non-integer z, the energy-momentum tensor is not analytic.

For $z=1$ and $\lambda=1$, we recover the relativistic case.
- in particular, the energy-momentum tensor is symmetric, $T^i_{\ 0}=T^0_{\ i}$.
- symmetries can extend to Lorentz boost and conformal symmetries.

One can check the conservation equations and trace condition

$$\omega T^0{}_0 + k_j T^j{}_0 = 0\ , \qquad \omega T^0{}_i + k_j T^j{}_i = 0\ , \qquad T^0{}_0 + \delta^i_j T^j{}_i = 0\ ,$$

which can be corrected to be

$$z\, T^0{}_0 + \delta^i_j \tilde{T}^j{}_i = 0\ ,$$

- $\tilde{T}^i{}_j = T^i{}_j + \Delta T^i{}_j$ with improvement $\Delta T^i{}_j = -\frac{z-1}{d-1}(k^2\delta^i_j - k^i k_j)\lambda |k|^{2(z-1)}\phi^2_{\omega,k}$,
- note that the conservation equations remain intact.

At finite temperature and $w, k \to 0$, one loop diagrams give

$$\langle T^{ij}\rangle = \frac{z}{d}\delta^{ij}\langle \tilde{T}^{00}\rangle \sim T^{\frac{d+z}{z}}\ , \quad \langle T^{0i}\rangle \sim T^{\frac{d+1}{z}}\ , \quad \langle T^{i0}\rangle \sim T^{\frac{d+2z-1}{z}}\ ,$$

$$\langle T_{0i}T_{0j}\rangle \propto T^{\frac{(d+2-z)}{z}}\delta_{ij}\ , \quad \langle T_{0i}T_{j0}\rangle \propto T^{\frac{d+z}{z}}\delta_{ij}\ , \quad \langle T_{i0}T_{j0}\rangle \propto T^{\frac{d-2+3z}{z}}\delta_{ij}\ .$$

- consistent with the recent calculation done in holographic setup and using scaling symmetry 1304.7776 by Korovin, Skenderis and Taylor.
- we expect these asymmetric energy-momentum tensors in hydrodynamics.

** These are checked for covariant formulation of Lifshitz field theories using vierbein. Hoyos, BSK, Oz, 1309.6794

Gravity theories with Lifshitz scaling

A. Hořava-Lifshitz theories : Hořava 0901.3775

B. Einstein gravity with Lifshitz symmetry Kachru, Liu and Mulligan 0808.1725

The gravity system for general z and d is described by metric

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dx^i dx^i + dr^2}{r^2} \,,$$

- invariant under $D: \; t \to \Omega^{-z} t \,, \quad x^i \to \Omega^{-1} x^i \,, \quad r \to \Omega^{-1} r \,,$

$$P^i : \; x^i \to x^i + a^i \,, \quad H : \; t \to t + a \,.$$

- supported by Λ & gauge fields (A_m, B_{mn}) in $d = 2$. Kachru, Liu and Mulligan 0808.1725
- also supported by Λ and massive $U(1)$ gauge field. Taylor 0812.0530
- various numerical and analytic black hole solutions have been constructed.
- shear viscosity to entropy ratio : $\eta/s = 1/4\pi$.

$$\vdots$$

- large amount of works have been devoted for this model!

Review on theories with Lifshitz scaling (5 slides)

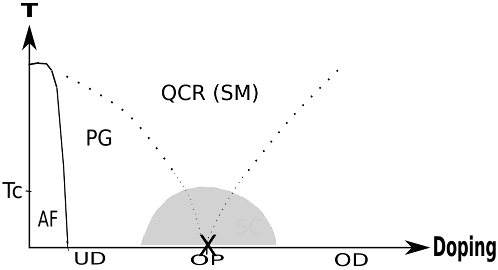
Quantum Critical Point (4 slides)

- Quantum critical point
- Hydrodynamics of quantum critical point?
- Bits and pieces of Lifshitz hydrodynamics
- Goal

Universal Hydrodynamics of Lifshitz theories (7 slides)

Drude model of strange metal (3 slides)

High Tc cuprates Phase Diagram



- HF, high T_c cuprates, organic SC share a similar phase diagram.
- Compressible ground state: properties change by quantum tuning parameter.
- Strange metal phase ($\rho \sim T$) can't be understood from Fermi liquid theory.

- QCP at $T=0$ might be responsible for the SM phase.
  Coleman, Schofield, Nature 433, 226 (2005), Sachdev, Keimer, Phys. Today 64N2, 29 (2011).
- Away from QCP ($T>0$) : a correlation length ξ become finite:
  still effects of QCP dominate.
- $T \uparrow$, thermal effects $\uparrow$. QCR is realized over a fan which widens as $T \uparrow$.
- QCP: no wave or particle excitations ($\Leftarrow$ scale invariance & strong coupling).

Hydrodynamics of quantum critical point?

Hydrodynamic description for the QCP with Lifshitz scaling symmetry

$$t \to \Omega^{-z} t\,, \qquad x^i \to \Omega^{-1} x^i\,,$$

would be valid if

$$\xi \gg L \gg \ell_T\,,$$

- $\ell_T \sim 1/T^{1/z}$: characteristic length scale of thermal fluctuations.
- L: typical size of the system,
- ξ: correlation length of quantum fluctuations,

• Well known universal hydrodynamic description for ordinary CP:
  - T dependence of transport coefficient is determined by scaling at CP.

Hohenberg and Halperin, RMP 49 (1977) 43.

• Quantum critical system with $z = 1, 2$ has hydrodynamic description:

Kovtun, Son and Starinets, hep-th/0405231, Rangamani, Ross, Son and Thompson, 0811.2049.

  - there are boost invariance for both cases and well understood(?!).

Bits and pieces of Lifshitz hydrodynamics

- Computing η & ζ is challenging in FT even in weak coupling.
  S. Jeon, hep-ph/9409250
  - Holography gives analytic ways to compute them in strong coupling.

I. Shear viscosity is computed by specific modes of metric fluctuation.
- For known gravity duals: $\eta/s = 1/4\pi$. Policastro, Son and Starinets hep-th/0104066
- Shear viscosity to entropy ratio for Lifshitz: $\eta/s = 1/4\pi$. Pang 0911.2777

II. Bulk viscosity can be obtained by horizon focusing equation. Eling, Oz, 1103.1657
- For conformal case : 0.
- Bulk viscosity to shear viscosity ratio for Lifshitz: $\zeta/\eta = 2(z-1)/d$??
  Hoyos, BSK, Oz, in progress

III. For charged case, conductivities can be computed in several different ways.
- several different ways to compute conductivities, also with magnetic fields.

- Are these $\eta, \zeta, \boldsymbol{\sigma}$ all?

Goal

$*$ Generalization of hydrodynamic descriptions for general z???

- not clear to be successful due to the small set of symmetries!
- obviously interesting for hydrodynamic description of QCP!
- even relativistic case have boost breaking effects due to impurities, $\cdots$.

- there exist bits and pieces of hydrodynamic properties.
- Are $\eta, \zeta, \boldsymbol{\sigma}$ a complete set for first derivative order?
- Need general frame for universal hydrodynamics of Lifshitz theories!

• two ways to go :

A. holographic side :

- understanding precise dictionary between FT and gravity is challenging.
- especially, lack of boost invariance puts technical & conceptual difficulties.

Ross and Saremi, 0907.1846.

- managed to setup the ideal order with a gravity model near the horizon.

Hoyos, BSK, Oz, 1309.6794.

B. field theory side :

- stress energy tensor is not symmetric.
- still manageable, concentrate on this case today.

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Universal Hydrodynamics of Lifshitz theories (7 slides)
- Hydrodynamics of relativistic theories : review
- Hydrodynamics of theories with Lifshitz scaling
- Charged hydrodynamics
- Non-relativistic limit

Drude model of strange metal (3 slides)

Glimpse of relativistic Hydrodynamics

• Hydrodynamics is an effective theory, describing the dynamics at large distance and time scales, incorporating dissipative effects.

- hydrodynamic equations: $\partial_\mu T^{\mu\nu}=0, \qquad T^{\mu\nu}\{T(\vec{x}), u^\mu(\vec{x})\}$
- temperature $T(\vec{x})$ and velocity $u^\mu(\vec{x})$ with $u_\mu u^\mu=-1$,
- assuming local thermal equilibrium, order by order in derivative expansion.

• Ideal order : using Landau frame condition $T^{\mu\nu}u_\nu=-\varepsilon u^\mu$

$$T^{\mu\nu}=(\varepsilon+p)u^\mu u^\nu+pg^{\mu\nu} \ ,$$

- $d\varepsilon=Tds, dp=sdT$ and $\varepsilon+p=Ts$.
- local entropy current condition $\partial_\mu T^{\mu\nu}u_\nu=0$ gives $\partial_\mu(su^\mu)=0$.

• First(dissipative) order of derivative expansion, we add $T^{(\mu\nu)}\to T^{(\mu\nu)}+\pi^{(\mu\nu)}$. From the Landau condition and the symmetry,

$$\pi_S^{(\mu\nu)}=-P^{\mu\alpha}P^{\nu\beta}[\eta(\partial_\alpha u_\beta+\partial_\beta u_\alpha)+(\zeta-2\eta)/3\ P_{\alpha\beta}\partial_\lambda u^\lambda] \ ,$$

- η,ζ are shear and bulk viscosities.
- conductivity for charged hydrodynamics.

Hydrodynamics of theories with Lifshitz scaling

- Lifshitz algebra for constant u_μ $(\eta_{\mu\nu} u^\mu u^\nu = -1, \;\; \mu, \nu = 0, \cdots, d)$:

$$P^\parallel = u^\mu \partial_\mu \;, \quad P^\perp_\mu = P_\mu{}^\nu \partial_\nu \;, \quad D = z x^\mu u_\mu P^\parallel - x^\mu P^\perp_\mu \;,$$
$$[D, P^\parallel] = z P^\parallel \;, \quad [D, P^\perp_\mu] = P^\perp_\mu \;, \qquad P_\mu{}^\nu = \delta_\mu{}^\nu + u_\mu u^\nu \;,$$
$$z T^\mu{}_\nu u_\mu u^\nu - T^\mu{}_\nu P_\mu{}^\nu = 0 \;, \qquad \text{Ward identity of } D \Rightarrow z\varepsilon = dp \;.$$

- identical to $z = 1$ case for $u_\mu = (-1, 0, \cdots)$
- confirmed by a gravity models with Lifshitz scaling.
- realized also in the partition function analysis. Hoyos, BSK, Oz, 1309.6794

- Scaling properties under $x^\parallel \to \Omega^{-z} x^\parallel$ and $x^\perp \to \Omega^{-1} x^\perp$:

$$[P^\parallel] = z \;, \quad [P^\perp] = 1 \;, \quad [T] = z \;, \quad [\varepsilon] = [p] = z + d \;.$$

- $T^{\mu\nu} \neq T^{\nu\mu}$ ($\Leftarrow$ absence of boost or rotational invariances).
- Ideal order is fixed by thermodynamics and equation of state.
- Landau frame condition : $T^{\mu\nu} u_\nu = -\varepsilon u^\mu$.

$$\Downarrow \Downarrow \Downarrow$$

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + p\eta^{\mu\nu} + \pi_S^{(\mu\nu)} + \pi_A^{[\mu\nu]} + (u^\mu \pi_A^{[\nu\sigma]} + u^\nu \pi_A^{[\mu\sigma]}) u_\sigma \ .$$

- $\pi_S^{(\mu\nu)}$: symmetric dissipative contributions, $\pi_S^{(\mu\nu)} u_\nu = 0$
- $\pi_A^{[\mu\nu]}$: all anti-symmetric contributions.
- $\pi_A^{[ij]} = 0$ for rotational invariance.

- Local entropy current is required to be 0 (ideal order) or positive!

$$\begin{aligned} 0 &= \partial_\mu T^{\mu\nu} u_\nu \\ &= -T\partial_\mu(s u^\mu) + \cdots - \pi_A^{[\mu\sigma]}(\partial_{[\mu} u_{\sigma]} - u_{[\mu} u^\alpha \partial_\alpha u_{\sigma]}) \ . \end{aligned}$$

- identify the entropy current : $j_S^\mu = s u^\mu$,
- symmetric part $\cdots$ has usual shear η and bulk ζ viscosities :

$$\pi_S^{(\mu\nu)} = -P^{\mu\alpha} P^{\nu\beta} [\eta(\partial_\alpha u_\beta + \partial_\beta u_\alpha) + (\frac{\zeta - 2\eta}{d}) P_{\alpha\beta} \partial_\lambda u^\lambda] \ .$$

- antisymmetric part gives new transport coefficients at the viscous level :

$$\pi_A^{[\mu\nu]} = -\alpha^{\mu\nu\sigma\rho}(\partial_{[\sigma} u_{\rho]} - u_{[\sigma} u^\alpha \partial_\alpha u_{\rho]}), \qquad \tau_{\mu\nu} \alpha^{\mu\nu\sigma\rho} \tau_{\sigma\rho} \geq 0 \ .$$

- rotation invariance $\Rightarrow$ $\quad \pi_{A\,[0i]} = \alpha(\partial_{[0} u_{i]} - u_{[0} u^\alpha \partial_\alpha u_{i]})$.
- theories with Lifshitz scaling gives $[\alpha] = d$ and $\alpha \sim T^{d/z}$.

* There is only one new transport coefficient α at the first derivative level!
* We explicitly write down the Kubo formula of α.

Charged hydrodynamics

Conserved current

$$J^\mu = \rho u^\mu - \nu^\mu \ , \quad \nu^\mu = \sigma T P^{\mu\alpha}\partial_\alpha\left(\frac{\mu}{T}\right) + \nu_A^\mu \ , \qquad \nu^\mu u_\mu = 0 \ .$$

Entropy current $\partial_\mu T^{\mu\nu}u_\nu + \mu\partial_\mu J^\mu = 0$ is now

$$\begin{aligned} T\partial_\mu\left(su^\mu - \frac{\mu}{T}\nu_S^\mu\right) &= -\pi_A^{[\mu\sigma]}(\partial_{[\mu}u_{\sigma]} - u_{[\mu}u^\alpha\partial_\alpha u_{\sigma]}) + \mu\partial_\mu\nu_A^\mu + \cdots \\ &= -u^\mu\partial_\mu u^\alpha V_\alpha + \mu\partial_\mu\nu_A^\mu + \cdots \\ &= T\partial_\mu(u^\alpha\partial_\alpha u^\mu) + \cdots \end{aligned}$$

where

$$\pi_A^{[\mu\nu]} = u^{[\mu}P^{\nu]\alpha}V_\alpha \ , \quad \nu_A^\mu = \frac{T}{\mu}u^\alpha\partial_\alpha u^\mu, \quad V_\alpha = \mu\partial_\alpha\frac{T}{\mu}.$$

Thus we discover a new dissipationless transport coefficient C

$$j_s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu}{T}\nu_A^\mu,$$

where

$$\nu_A^\mu = -C\frac{T}{\mu}u^\alpha\partial_\alpha u^\mu, \quad \pi_A^{[\mu\nu]} = -C\mu u^{[\mu}P^{\nu]\alpha}\partial_\alpha\left(\frac{T}{\mu}\right) \ ,$$

which breaks only boost invariance. $\pi_A^{[\mu\nu]}P_{\mu\alpha}P_{\nu\beta} = 0$

Non-relativistic limit

Taking a non-relativistic limit $c \to \infty$ using

$$\varepsilon = c^2\rho - \rho v^2/2 + U \ , \quad u^\mu = (1, \beta^i)/\sqrt{1-\beta^2} \ , \quad \beta^i = v^i/c \ ,$$
$$\partial_0 = \partial_t/c \ , \quad \eta \to c\eta \ , \quad \zeta \to c\zeta \ , \quad \alpha \to c^2\alpha \ .$$

Three different contributions of order of $\mathcal{O}(c), \mathcal{O}(1/c), \mathcal{O}(c^0)$, for $\partial_\mu T^{\mu\nu} = 0$

$$\partial_t\rho + \partial_i(\rho v^i) = 0 \ ,$$
$$\partial_t U + \partial_i(U v^i) + p\partial_i v^i = \frac{\eta}{2}\sigma^{ij}\sigma_{ij} + \frac{\zeta}{d}(\partial_i v^i)^2 + \boldsymbol{\frac{\alpha}{2}\mathbf{V_A^i}^2} \ , \qquad (**)$$
$$\partial_t(\rho v^i) + \partial_j(\rho v^j v^i) + \partial^i p = \partial_j\left(\eta\sigma^{ij} + \frac{\zeta}{d}\delta^{ij}\partial_k v^k\right) + \boldsymbol{\partial_t(\alpha \mathbf{V_A^i}) + \partial_j\left(\frac{\alpha}{2}[\mathbf{v^j V_A^i + v^i V_A^j}]\right)} \ .$$

where

$$\sigma_{ij} = \partial_i v_j + \partial_j v_i - (2/d)\delta_{ij}\partial_k v^k \ ,$$
$$V_A^i = 2D_t v^i + \omega^{ij} v_j \ ,$$

- $D_t v^i$: relative acceleration with material derivative $D_t = \partial_t + v^i\partial_i$.
- $\omega^{ij} v_j$: acceleration due to Coriolis effect with vorticity tensor $\omega^{ij} = 2\partial^{[i}v^{j]}$.

Scaling symmetry at non-relativistic limit

• Splitting as $\varepsilon = c^2\rho - \rho v^2/2 + U$ and reorganizing by power of c breaks the 'relativistic' scaling symmetry for $\rho \neq 0$.

• For scaling invariant theories in non-relativistic limit, modify scaling properties with scaling transformation

$$D = -\xi^t \partial_t - \xi^i \partial_i \ , \qquad \xi^t = zt \ , \ \xi^i = x^i + (z-2) v^i t/2 \ .$$

- $z=2$ is special and can be extended to conformal Schrödinger symmetry.

• Ward identity gives the desired equation of state

$$0 = -z j_\varepsilon^t + T^i{}_i + (z-2) v^i T^t{}_i = -zU + dp \ .$$

• Scaling property of physical quantities :

$$\begin{aligned} [v^i] &= z-1 \ , & [\rho] &= d+2-z \ , \\ [T] &= z \ , & [p] &= [U] = z+d \ , \\ [\eta] &= [\zeta] = d \ , & [\alpha] &= d-2(z-1) \ . \end{aligned}$$

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- Conductivity
- Dissipative effects

Conductivity

- Consider 'relativistic' conservation equations with a current $J^\mu = \rho u^\mu + \nu^\mu$ by introducing E^i and drag term λ

$$\partial_\mu J^\mu = 0 \ , \qquad \partial_\mu T^{\mu 0} = J^i E_i \ , \qquad \partial_\mu T^{\mu i} = J^0 E^i - \lambda c J^i \ .$$

- take non-relativistic limit : left sides of the equations are identical to (**).
- consider incompressible fluid $\partial_i v^i = 0$ for simplicity.
- to be consistent with stationary case, need transverse electric field $\partial_i E^i = 0$.
- these allow a solution with $\partial_\mu \rho = \partial_t v^i = \partial_t p = 0$.
- then $V_A^i = \sigma^{ik} v_k$ and $\sigma^{ik} = \partial^i v^k + \partial^k v^i$.

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

- then, fluid is described by Navier-Stokes equation

$$\rho v^k \partial_k v^i + \partial^i p = \rho E^i - \lambda \rho v^i + \eta \nabla^2 v^i + \frac{\alpha}{2} \ \partial_j \Big([v^j \sigma^{ik} + v^i \sigma^{jk}] v_k \Big) \ .$$

- Solve it order by order in derivatives upto $\mathcal{O}(\partial^2)$,
- keeping $\partial^i p = 0$,
- From Ohm's law $J^i = \boldsymbol{\sigma}_{ij} E^j$, we get for a divergenceless electric field $E_x(y)$

$$\begin{aligned}\boldsymbol{\sigma}_{xx}(E_x) &= \frac{\rho}{\lambda}\left[1 + \frac{1}{\rho\lambda E_x}\left(\eta\partial_y^2 E_x + \frac{\alpha}{6\lambda^2}\partial_y^2 E_x^3\right)\right] \\ &\rightarrow \quad \frac{\rho}{\lambda}\left[1 - \frac{\eta}{\lambda\rho L^2} + \frac{\alpha E_0^2}{4\lambda^3 \rho L^2}\right].\end{aligned}$$

- bottom line: evaluated for $E_x = E_0 \cos(y/L)$ and averaged over y.
- enhancement of the conductivity with the electric field.
- η and α can be directly compared.

- ideal order contribution ($\boldsymbol{\sigma}_{xx} = \rho/\lambda$):

$$\boldsymbol{\sigma}_{xx} = T^{\frac{d-2(z-1)}{z}} f\left(T^{\frac{d+2-z}{z}}/\rho\right) \simeq \rho/T \qquad \rightarrow \quad \boldsymbol{\rho}_{xx} \simeq T!$$

- use the dimension $[\lambda] = z$.
- use reasonable assumption $\boldsymbol{\sigma}_{xx} \sim \rho$.
- independent of z and d, thus universal!

Dissipative effects

- constant homogeneous force from electric field or temperature gradient

$$a^i = -\partial^i p/\rho + E^i = (s/\rho)\partial^i T + E^i \ , \qquad \partial_t a^i = 0 \ , \partial_j a^i = 0 \ .$$

- Navier-Stokes equations for homogeneous configurations ($V_A^i = 2\partial_t v^i$)

$$\partial_t v^i - 2(\alpha/\rho)\partial_t^2 v^i + \lambda v^i = a^i \ .$$

  - forces are suddenly switched on at $t = 0$,
  - with the initial conditions $\partial_t v^i_{t=0} = \frac{\rho a^i}{4\alpha\lambda}\left(\sqrt{8\alpha\lambda/\rho+1}-1\right) \quad \rightarrow \quad a^i$ for $\alpha \rightarrow 0$,
  - at large times the velocity stays constant.

- Energy conservation equation ($\rightarrow$ heat production rate):

$$\partial_t U = \lambda\rho v^2 + 2\alpha(\partial_t v)^2 \ .$$

Total heat production is

$$\Delta Q = \int_0^\infty dt \, \left(\partial_t U - \frac{\rho}{\lambda} a^2\right) = -\frac{\rho a^2}{2\lambda^2}\left(\sqrt{8\alpha\lambda/\rho+1}+2\right) \ .$$

  - at late times the heat production rate becomes constant $v^i = a^i/\lambda$.
  - we subtract this contribution for all times.
  - in Lifshitz holography, heat production is considered in Kiritsis, 1207.2325

Summary & future directions

• We show hydrodynamic description near QCP with Lifshitz scaling is well defined by Landau frame condition and local second law of thermodynamics.

• Surprisingly, there exist two new transport coefficients α, C due to the lack of boost invariance, as well as many for broken rotational invariance. The effects of broken boost invariance is expected to be small, but present ubiquitously due to static impurities and other means, even for boost invariant cases.

• In non-relativistic limit, scaling properties are different. The new coefficient α provides relative acceleration and acceleration due to Coriolis effect. We proposed simple ways to verify the effects of α.

• Future directions :
- holographic realizations for derivative orders
- computing α in a specific model
- scaling anomaly analysis
- experimental verifications

$\vdots$