Localizing the black M2-M5 intersection

Vasilis Niarchos

Crete Center for Theoretical Physics, University of Crete
based on recent work with K. Siampos

12xx.xxxx, ``The M2-M5 ring intersection spins'',
1206.2935, ``Entropy of the self-dual string soliton'',
1205.1535, ``M2-M5 blackfold funnels'',
in progress

and older work with R. Emparan, T. Harmark and N. A. Obers ➞ blackfold theory

1106.4428, ``Blackfolds in Supergravity and String Theory'',
0912.2352, ``New Horizons for Black Holes and Branes'',
0910.1601, ``Essentials of Blackfold Dynamics'',
0902.0427, ``World-Volume Effective Theory for Higher-Dimensional Black Holes'',
0708.2181, ``The Phase Structure of Higher-Dimensional Black Rings and Black Holes''

+ M.J. Rodriguez
We learn new things about the fundamentals of string/M-theory by studying the low-energy theories on D-branes and M-branes.

Most notably in M-theory, recent progress has clarified the low-energy QFT on $N$ M2-brane and the $N^{3/2}$ dof that it exhibits.

Our understanding of the M5-brane theory is more rudimentary, but efforts to identify the analogous properties of M5-branes, e.g. the $N^3$ scaling of the massless dof, is underway.

References:
- ABJM ’08
- Drukker-Marino-Putrov ’10
- Douglas ’10
- Lambert,Papageorgakis, Schmidt-Sommerfeld ’10
- Hosomichi-Seong-Terashima ’12
- Kim-Kim ’12
- Kallen-Minahan-Nedelin-Zabzine ’12
- ...
It is believed that the M5 theory is a theory of strings.

M2-branes can end on M5-branes just like F-strings can end on D-branes in string theory.

The above intersection is 1/4-BPS (preserves 8 supercharges).
The IR dynamics is controlled by a (1+1)-dim \((4,4)\) SCFT that lives in the intersection.

OUR GOAL: to identify this theory, or key features of this theory e.g. how does the central charge of this CFT scale with \(N_2, N_5\)?

We can approach this question in two different ways:

- from a microscopic analysis of the M5/M2 brane physics

- from a supergravity analysis of the corresponding black brane intersection (e.g. near-extremal black brane thermodynamics gives
  
  for M2-branes \( S \sim N^{3/2} T^2 \)
  
  for M5-branes \( S \sim N^3 T^5 \))

  \(\text{Klebanov-Tseytlin '96}\)
Both approaches are technically complicated (explains why our understanding of this system is still rather rudimentary).

I will describe progress in the SUGRA approach.

Ultimately we are interested in the development of the microscopic theory that lives at the intersection and its implications for the M5-brane theory.
M5 point of view: the Howe-Lambert-West solution

Descriptions of the intersection are possible either from the M2 or M5-brane point of view.

I will not say much about the M2 point of view, as it will be less relevant for what follows.

From the M5 point of view the string intersection appears as a solitonic solution of the M5 brane worldvolume theory.
The Howe-Lambert-West solution: $N_5=1, N_2>0$.

The abelian worldvolume theory on a single M5 brane is known.

It is a theory of a self-dual 3-form field strength and 5 transverse scalars (plus their fermion partners).

Key point: this theory is the leading term in a long-wavelength derivative expansion (analogous to the DBI theory for D-branes).

Hold on to this point...
The 1/4-BPS self-dual string soliton solution
(M-theory analog of Blon solution)

\[ z(\sigma) = \frac{2Q_{sd}}{\sigma^2}, \quad z := x^6 \]

\[ H_3 = *_6 H_3 = *_4 dz \]

An \( S^3 \) spike describes \( N_2 \) M2-branes ending on \( N_5=1 \) M5-branes.
The solution (and the associated derivative expansion) breaks down at some radius, but a **miracle** happens:

at the tip of the spike one recovers the tension of the orthogonal M2 branes. (We will re-encounter and extend this feature below).

*The leading order solution works much better than naively expected.*

Technical issue: we do not know the non-abelian M5-brane $wv$ theory. This obstructs a similar analysis at generic $N_2$, $N_5$. 
Known supergravity solutions

Supergravity allows us to examine the system in the limit $N_2, N_5 \gg 1$.

Brane intersections in supergravity is a subject with a long history and impressive achievements.

Nevertheless, it is technically challenging in many cases to find solutions that describe fully localized intersections.

Finding a solution becomes an even greater challenge as we reduce the amount of supersymmetry, or if we have no supersymmetry at all, e.g. for non-extremal solutions.
In the case of the 1/4-BPS orthogonal M2-M5 intersection

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{M2 :} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\text{M5 :} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

we are looking for a solution with SO(1,1) x SO(4) x SO(4) symmetry.

A partially localized solution with metric element

\[
ds^2 = H_2^{1/3} H_5^{2/3} \left[ (H_2 H_5)^{-1} (-dt^2 + (dx^1)^2) + H_5^{-1} ((dx^2)^2 + \ldots + (dx^5)^2) + H_2^{-1} (dx^6)^2 + (dx^7)^2 + \ldots + (dx^{10})^2 \right]
\]

\[
\nabla_{(789(10))}^2 H_5 = 0 , \quad \left( H_5 \nabla_{(2345)}^2 + \nabla_{(789(10))}^2 \right) H_2 = 0
\]

is known (delocalized along the 6-direction).
Progress towards a fully localized solution has been achieved more recently by Lunin, who reduces the problem to a set of PDEs.

I will now describe a novel treatment of this system in SUGRA that

- works in a long-wavelength DBI-like regime
  (and thus compares more directly with the non-gravity M5 wv description)
- gives immediate intuitive information, and
- easily provides more complicated (less symmetric) configurations that are well beyond the reach of current exact solution generating techniques.
Blackfold theory basics

Blackfolds provide a general effective (long-wavelength) worldvolume description of black brane dynamics

They describe how a black p-brane fluctuates, spins and bends
The **fluid/gravity correspondence** illustrates nicely the general idea.

- a spin-off of the AdS/CFT correspondence

- it describes temperature and velocity fluctuations of AdS black branes in the long-wavelength approximation in terms of a **relativistic conformal fluid**.

\[ \lambda \gg \frac{1}{T} \sim \frac{L_{\text{AdS}}^2}{r_0} \]

- the fluid lives on a time-like surface in the asymptotic region of the black hole spacetime (AdS boundary)

- there is a constructive perturbative procedure that maps uniquely the solutions of the fluid equations to regular bulk spacetimes

\[ \nabla_\mu T^{\mu \nu} = 0 \]
Blackfolds add co-dimension to the fluid-gravity correspondence

- a mix of fluid dynamics + `DBI’.

for neutral AF black branes (in D=n+p+3 dimensions)

\[ ds^2_{p-brane} = \left( \eta_{ab} + \frac{r^0_n}{r_n} u_a u_b \right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r^0_n}{r_n}} + r^2 d\Omega_{n+1}^2 \]

- temperature, velocity and worldvolume bending fluctuations in the long-wavelength approximation \( \lambda \gg \frac{1}{T} \sim r_0 \)
in terms of a relativistic non-conformal fluid that lives on a dynamical hypersurface.

\[ \nabla_\mu T^{\mu\nu} = 0 \]

- the fluid lives on a time-like surface in the asymptotic region of the black hole spacetime

- there is a constructive perturbative procedure that maps the solutions of the fluid equations to regular bulk spacetimes (here focus on the leading order of the expansion)
M2-M5 blackfold funnels

We want to describe a spiky deformation of the planar M5 black brane (with dissolved M2 brane charge)

how the planar black M2-M5 bound state deforms

\[ ds^2_{11} = (HD)^{-1/3} \left[ -f dt^2 + (dx^1)^2 + (dx^2)^2 + D ((dx^3)^2 + (dx^4)^2 + (dx^5)^2) 
+ H \left( f^{-1} dr^2 + r^2 d\Omega_4^2 \right) \right] , \]

\[ C_3 = -\sin \theta (H^{-1} - 1) \coth \alpha dt \wedge dx^1 \wedge dx^2 + \tan \theta DH^{-1} dx^3 \wedge dx^4 \wedge dx^5 , \]

\[ C_6 = \cos \theta D (H^{-1} - 1) \coth \alpha dt \wedge dx^1 \wedge \cdots \wedge dx^5 , \]

\[ H = 1 + \frac{r_0^3 \sinh^2 \alpha}{r^3} , \quad f = 1 - \frac{r_0^3}{r^3} , \quad D^{-1} = \cos^2 \theta + \sin^2 \theta H^{-1} . \]
The parameters of the planar M2-M5 bound state are promoted to slowly-varying functions of an effective 5-brane worldvolume

\[ r_0(\hat{\sigma}^a), \alpha(\hat{\sigma}^a), \theta(\hat{\sigma}^a), u^a(\hat{\sigma}^a), X^{\perp}(\hat{\sigma}^a), \hat{V}(3) \]

local temperature \hspace{1cm} M2, M5 charges \hspace{1cm} local boosts \hspace{1cm} transverse scalars \hspace{1cm} unit 3-volume form controls embedding of M2 charge in 5-brane wv

Leading order blackfold equations

\[ K_{ab}^i T^{ab} = 0 \] \hspace{1cm} 'DBI' part extrinsic equations
\[ D_a T^{ab} = 0 \] \hspace{1cm} 'hydrodynamic' part intrinsic equations
\[ d \ast J_3 = 0 \hspace{0.5cm} J_3 = Q_2 \hat{V}(3) \]

local stress-energy tensor, constitutive relations,...

\[ T^{ab} = \mathcal{T} s \left( u_a u_b - \frac{1}{3} \gamma_{ab} \right) - \sum_{q=2,5} \Phi_q Q_q h^{(q)}_{ab} \] ....
We are looking for a static solution with a single `excited’ transverse scalar

\[ x^6 = z(\sigma) , \quad \sigma^2 = (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2 \]

For stationary configurations the intrinsic eqs can be solved generically, and we end up with DBI-like eqs for the transverse scalars

For extremal \( T=0 \) configurations we solve the eom of the Dirac action

\[
I \sim \int d\sigma \sigma^3 \sqrt{1 + \frac{\kappa^2}{\sigma^6}} \sqrt{1 + z'^2} , \quad \kappa = 4\pi \frac{N_2}{N_5} \ell_P^3
\]

We recover the extremal (1/4-BPS) 3-sphere spike solution

\[
z(\sigma) = 2\pi \frac{N_2}{N_5} \ell_P^3 \frac{1}{\sigma^2}
\]
The blackfold derivative expansion breaks down when the characteristic scale of the solution becomes comparable with the transverse integrated-out characteristic scale

Breakdown scale: \( \sigma_c = \left( \frac{\pi N_5}{\sqrt{2}} \right)^{\frac{1}{3}} \left( 1 + \sqrt{1 + 4/\lambda^2} \right)^{\frac{1}{6}} \ell_P , \quad \frac{1}{\lambda} := \frac{4N_2}{N_5^2} \)

Derivative corrections are controlled by the ratio:

\[
\frac{1}{\lambda} = \frac{4N_2}{N_5^2} \ll 1
\]

\( \Rightarrow \) we work in the large-\( N \) limit

\( N_2, N_5 \gg 1 \), \( N_2 \ll N_5^2 \)
Despite the breakdown of the effective theory the usual **miracle** happens.

The *leading-order* solution reproduces correctly the tension of M2-branes at the tip of the spike (at any $\lambda$)

$$\left. \frac{1}{L_t L_{x^1}} \frac{dM}{dz} \right|_{\sigma=0} = Q_2 = N_2 T_{M2}$$

(We have also observed this miracle for **non-SUSY extremal** configurations)
Thermalizing the spike

Spikes at finite temperature can be obtained by solving the eom of the action

\[ I \simeq \int d\sigma \sqrt{1 + z'^2} \, F(\sigma; \beta), \quad \beta = \frac{3}{4\pi T} \]

\[ F(\sigma) = \sigma^3 \left( \frac{1 + \frac{\kappa^2}{\sigma^6}}{1 + \sqrt{1 - \frac{4q_5^2}{\beta^6} (1 + \frac{\kappa^2}{\sigma^6})}} \right)^{3/2} \left( -2 + \frac{3\beta^6}{2q_5^2} \frac{1 + \sqrt{1 - \frac{4q_5^2}{\beta^6} (1 + \frac{\kappa^2}{\sigma^6})}}{1 + \frac{\kappa^2}{\sigma^6}} \right) q_5 = \frac{16\pi G}{3\Omega(4)} Q_5 \]

(analogous formula in non-gravity description not obvious, exact non-extremal SUGRA solution also hard)

Boundary conditions:

\[ \lim_{\sigma \to +\infty} z(\sigma) = 0, \quad \lim_{\sigma \to \sigma_0^+} z'(\sigma) = -\infty \]

\[ \sigma_0 = \sigma_0(T) \ll \sigma_c \text{ lies in the breakdown region. What determines it?} \]
With these boundary conditions the general solution of the leading order equations is

\[ z(\sigma) = \int_{\sigma}^{+\infty} ds \left( \frac{F(s)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \]
Black M2 matching conditions and a second set of miracles

The leading order solution `ends' at the tip $\sigma_0$.

Does the extremal matching to M2 at the `tip' extend into the near-extremal regime?
Matching the thermodynamic data of the near-extremal spike with those of the emerging M2 we obtain

\[
\left( \frac{1}{L_x} \frac{dM}{dz} \right)_{\sigma=\sigma_0^+}^{M_2-M_5} = \left( \frac{M}{L_x L_z} \right)_{M_2}, \quad \left( \frac{1}{L_x} \frac{dS}{dz} \right)_{\sigma=\sigma_0^+}^{M_2-M_5} = \left( \frac{S}{L_x L_z} \right)_{M_2}
\]

\[
\sigma_0^{(M)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}} \left( c_1^{(M)} + c_2^{(M)} \frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6}) \right), \quad c_1^{(S)} \approx 1.234, \quad c_2^{(M)} \approx -0.068
\]

\[
\sigma_0^{(S)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}} \left( c_1^{(S)} + c_2^{(S)} \frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6}) \right), \quad c_1^{(S)} \approx 1.189, \quad c_2^{(M)} \approx 0.052
\]

\[
\Rightarrow \sigma_0(T) \approx c_1 \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}}, \quad c_1 \approx 1.2, \quad q_2 = \frac{16\pi G}{3\Omega(3)\Omega(4)} Q_2
\]

The matching of the leading order coefficients \((c_1^{(M)}, c_1^{(S)})\) within 4% is impressive.
Entropy of thermal spikes

Given a solution of the blackfold equation the formalism provides specific
formulae for thermodynamic data. For the entropy in this particular
application

\[
\frac{S}{L_{x_1}} = \frac{\Omega_{(3)} \Omega_{(4)} \beta^4}{4G} \int_{\sigma_0}^{+\infty} d\sigma \sigma^3 \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \frac{1}{\cosh^3 \alpha(\sigma)}
\]

Expanding in positive powers of \( T \) we wish to identify the \( O(T) \) contribution
from the \((1+1)\)-dimensional intersection.

The contributions far from the core \((M5)\) and close to the core \((M2)\) are
subtracted.
The leading order contribution to $S$ is indeed $O(T)$ (as expected)

\[
\frac{S}{L_{x^1}} = \frac{8\sqrt{\pi} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{1}{6}\right)}{135 c_1^8} \frac{N_2^2}{N_5} T + O(T^4) \, , \, \, c_1 \simeq 1.2
\]

Comparing to the Cardy formula for 2-dimensional CFT

\[
\frac{S}{L_{x^1}} = \frac{\pi c}{6} T
\]

we find an expression for the central charge $c$

\[
c \simeq 0.6 \frac{N_2^2}{N_5} + \ldots
\]
These results indicate that the $d=2 \ N=(4,4)$ SCFT at the intersection has a strong t’ Hooft like expansion

$$N_2, \ N_5 \gg 1, \ \lambda \sim \frac{N_5^2}{N_2} \gg 1$$

In this limit the leading order contribution to the central charge takes the highly suggestive form

$$c \simeq 0.6 \frac{N_2^2}{N_5} + \ldots = 0.04 \frac{N_5^3}{\lambda^2} + \ldots = 0.3 \frac{N_2^{3/2}}{\sqrt{\lambda}} + \ldots$$

- **What is the field theory interpretation of this result?**
- **What does it teach us about M2 and M5 brane physics?**
- **How does the $\frac{1}{\lambda}$ expansion arise in field theory?**
Further work

Much remains to be done:

- The result relied on a set of interesting `miracles'. Extra checks are under consideration.

- Explore the implications for field theory.

- Probe more complicated configurations of the intersection.
**The M2-M5 ring intersection spins**

Search for a closed M2-M5 string intersection in supergravity.

- A rotating black M2 cylinder ending on a black M5.

- The configuration preserves less symmetry: $\text{SO}(1,1) \times \text{SO}(3) \times \text{SO}(4)$
- The configuration is stationary: the blackfold fluid rotates.
- The black M5 has to be also cylindrical
- The extremal configuration carries a null momentum wave along the intersection.
- Surprisingly, although non-SUSY it exhibits many of the miracles of supersymmetric configurations (e.g. thermo data at the tip of the spike)
- ...