Fermi surface, hyperscaling violation and unified frame in effective holographic theories

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Based on 1005.4690 with C. Charmousis, B. Goutraux, E. Kiritsis, R. Meyer
1210.0540 & 1202.6062

Quantum phases of matter at low T (3 slides)

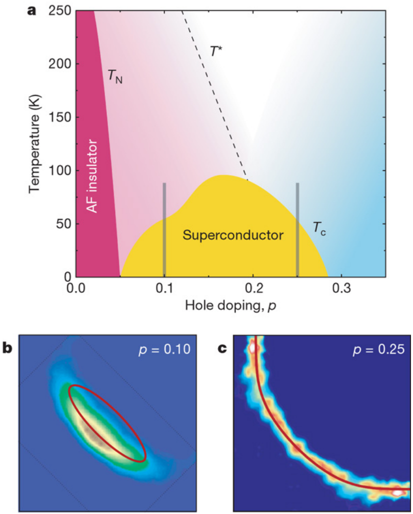
Hyperscaling violation! (3 slides)

Entanglement Entropy (EE) (4 slides)

Hyperscaling violation as a unification tool (3 slides)

Quantum phases of matter at low T

Compressible matter
Fermi surfaces



Doiron-Leyraud *et al.*, Nature 447, 565 (2007)

Dopong dependence and Fermi surfaces!

Several physical properties of interesting quantum phases of matter do not fit into the standard Fermi liquid paradigm. So it is called non-Fermi liquid.
- $\rho \sim T$ (T^2), $S \sim T^{1/z}$ $(T^{d/z})$, $\cdots$

For the materials such as high T_c cuprates, HF, organic insulators $\cdots$ at $T = 0$, the "density" of the ground states can be dialed by a quantum tuning parameter, such as doping, chemical potential, pressure $\cdots$: "compressible"

"Compressible" quantum matter can be described by a modified hamiltonian

$$\mathcal{H}' = \mathcal{H} - \mu\mathcal{Q} \,, \quad [\mathcal{Q}, \mathcal{H}] = 0 \,,$$

- $\mathcal{Q}$: conserved $U(1)$ charge.
- ground state is compressible if $\mathrm{d}\langle\mathcal{Q}\rangle/\mathrm{d}\mu \neq 0$ at $T = 0$.
- requires gapless modes to change the density of the ground states.
- scaling argument implies $\mathrm{d}\langle\mathcal{Q}\rangle/\mathrm{d}\mu \sim T^{d-1} \quad \rightarrow \quad d = 1$.
- hard to get the compressible matter except $d = 1$, Luttinger liquid.

realized in **2+1 dimensional low energy effective theories with emergent gauge bosons and fractionalized gauge-charged fermions and bosons.**

$$c_\sigma = \psi_\sigma \cdot h \ , \quad \sigma = \uparrow, \downarrow \ ,$$

where UV gauge invariant electron split into spinon ψ_σ and holon h.
- an emergent gauge symmetry : $\psi_\sigma(x) \to e^{i\theta(x)} \psi_\sigma(x)$, $h(x) \to e^{-i\theta(x)} h(x)$.

There exist universal, compressible non-Fermi liquid states with Fermi surface (different from FL, but same k_F of free electrons).

$$\mathcal{L} = \psi^\dagger (\partial_t - iA_t - \mu)\psi - 1/2m \ \psi^\dagger (\nabla - i\vec{A})^2 \psi + 1/4g^2 \ F^2 \ ,$$

- Fermi surface is *hidden* (not gauge invariant, not a physical observable) and characterized by singular, non-quasi particle low energy excitations.
- propagator can be computed in a fixed gauge, z is different from UV.
- compressible matter for $d = 2$ and $\theta = 1$, thus $d_{eff} = 1$. Luttinger liquid.

Hyperscaling violation exponent θ

Hyperscaling violation exponent
story of the gravity side

Hyperscaling (HS)?

Hyperscaling (HS) is a property of the physical quantities based on their naive scaling dimensions (power counting). - *e.g.* $S \sim T^{d/z}$.

HS is violated by random-field fluctuations, which dominates over thermal ones, near a quantum critical point. Specifically, the free energy $\mathcal{F}$ grows with modified scaling. - *e.g.* $S \sim T^{(d-\theta)/z}$, thus $d_{eff} = d - \theta$. Fisher 1986

Holographically, HS violation is realized as a property of metric

$$\mathrm{d}s^2 = r^{-2+2\theta/d} \left(-r^{-2(z-1)} f(r) \mathrm{d}t^2 + \sum_{i=1}^{d} \mathrm{d}y_i^2 + \frac{\mathrm{d}r^2}{f(r)} \right) ,$$

$$t \to \lambda^z t , \quad x_i \to \lambda x_i , \quad r \to \lambda r , \quad ds \to \lambda^{\theta/d} ds .$$

Pointed out first in Gouteraux-Kiritsis 1107.2116
based on an explicit solution Charmousis-Gouteraux-BSK-Kiritsis-Meyer 1005.4690,
further investigated in Huijse-Sachdev-Swingle 1112.0573.

Holographic realization of HS violation (EMD)

Charmousis-Gouteraux-BSK-Kiritsis-Meyer 1005.4690 Gouteraux-Kiritsis 1107.2116

Explicit solution is given by

$$ds^2 = r^{-2+\theta}\left[-r^{2-2z}f(r)\mathrm{d}t^2 + \sum_{i=1}^{2}\mathrm{d}x_i^2 + \frac{\mathrm{d}r^2}{f(r)}\right] ,$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{Z}{4}F^2 - \frac{1}{2}(\partial\phi)^2 + V\right] , \quad f(r) = 1 - \left(\frac{r}{r_H}\right)^{2+z-\theta}$$

$$e^{\phi} = r^s , \qquad Z = \frac{1}{q^2}e^{\frac{4-\theta}{s}\phi} , \qquad A_t = q\sqrt{\frac{2z-2}{z+2-\theta}}\,r^{-2-z+\theta}f(r) ,$$

$$V = (2+z-\theta)(1+z-\theta)e^{-\frac{\theta}{s}\phi} , \qquad s = \pm\sqrt{4(z-1) - 2z\theta + \theta^2} .$$

- Effective holographic theories(EHT) valid only for certain range of r.
- Generalization of Lifshitz case with θ, or AdS with (z, θ).
- Worked out for general $p + 2$ dimension.
- Thermodynamic and transport properties are analyzed.
- Most general IR scaling asymptotics at finite density with single ϕ, F.
- Embedded in higher-dimensional solutions.

Gravity side	Field theory side
$g_{\mu\nu}$	$T^{\mu\nu}$
A_μ	J^μ
ϕ	$Tr(\Phi\Phi)$, $Tr(\Psi\Psi)$
ψ	$Tr(\Phi\Psi)$,
$\tilde{\phi}(dilaton)$	$Tr(F^2)$

'Elementary fields' F, Φ, Ψ (ψ_σ, h): not gauge invariant, not measurable,
'mesonic' operators dual to $\tilde{\phi}, \phi, \psi$ (c) : measurable.

Identify charge density and chemical potential in 3+1 bulk (2+1 boundary),

$$A_t(r) = \mu + \langle J^t \rangle r + \cdots \ , \qquad F_{tr}|_{r=0} = \langle J^t \rangle.$$

Systems with charge density need electric flux at infinity.

The horizon at finite charge density is identified as deconfined phase.
in fractionalized phase : flux is sourced by the horizon, Sachdev and others
in the mesonic (cohesive) phases : flux is sourced by charged fields in the bulk.

Fermi surface identification? Entanglement Entropy

How to identify Fermi surfaces in holography?
Novel phases for Lifshtiz and Schrödinger spaces

Fermi surface identification? Luttinger v.s. EE

Defining fractionalization using Luttinger count for the compressible matter : **the total charge density is equal to the sum over the momentum space volumes of all Fermi surfaces in the theory, weighted by the charge of the corresponding fermionic operators**.

$$\langle J^t \rangle = \sum_{\ell} q_\ell V_\ell .$$

- Extremal Reissner-Nordström BH : maximally violated,
- 'electron star' geometry : fully satisfied.

* NOT able to check fractionalized Fermi surface explicitly in holography!

Entanglement entropy (EE) is useful for classifying phases of matter.

- FT calculation for fermionic system with fermi surface : log violation of EE

  Wolf, Swingle, Zhang-Grover-Vishwanath
- Useful definition for systems with Fermi surface :

  EE show logarithmic violation of the area law . Ogawa-Takayanagi-Ugajin 1111.1023
- EMD system: concrete holographic description for Fermi surface.

For a strip geometry, $-l \leq x_1 \leq l\,,\ 0 \leq x_i \leq L\,,\ i = 2, \cdots, d$, located at $r = r_F$ with $l \ll L$, the entanglement entropy is given by

$$S_{EE} = \frac{(RM_{Pl})^d}{4(d-\theta-1)} \left(\left(\frac{L}{\epsilon}\right)^{d-1} \left(\frac{\epsilon}{r_F}\right)^{\theta} - c \left(\frac{L}{l}\right)^{d-1} \left(\frac{l}{r_F}\right)^{\theta} \right) .$$

EE is independent of z and modified by θ. Reduces to the AdS case for $\theta = 0$.
cf. EE of Schrödinger type theories depends on z and θ.

For $\theta = d-1$: $S_{EE} = \frac{(RM_{Pl})^d}{2} \frac{L^{d-1}}{r_F^{d-1}} \log \frac{2l}{\epsilon}$, log-violation of area law!

For $\theta = d$: $S_{EE} = \frac{(RM_{Pl})^d}{2} \frac{L^{d-1} l}{r_F^d}$, entropy is proportional to volume!

Entanglement entropy analysis : novel phases for $d - 1 < \theta < d$.

For finite temperature, EE can not be evaluated analytically.
For $T \to 0$, it approaches to the zero temperature result,
while reproduces the thermal entropy at high temperature limit.

- "Codimension 2" Schrödinger holography : $(D+2)$-dimensional gravity with Schrödinger isometry are equivalent to D-dimensional field theory with the symmetry.

$$ds^2 = r^{-2+2\theta/D}\left(-r^{-2(z-1)}dt^2 - 2dtd\xi + dx_i^2 + dr^2\right) , \qquad D = d+1 .$$

- Several solutions are generated by null Melvin-twist of Schrödinger solutions. Null energy condition is used to constrain 'consistent' parameter space (z,θ).

$$(d+1)(z-1)(d+2z) - (d+1)z\theta + \theta^2 \geq 0 ,$$
$$(z-1)(d+2z-\theta) \geq 0 .$$

- Effects of θ : scaling dimension is shifted by $\theta/2$.

$$\langle \mathcal{O}(x')\mathcal{O}(x)\rangle \sim \frac{\theta(\Delta t)}{|\Delta t|^{\Delta-\theta/2}} e^{iM\frac{|\Delta\vec{x}|^2}{2|\Delta t|}} ,$$

Similar to the stress-energy tensor : vacuum structure might be modified(??)

- EE analysis using ADM form of metric Hubeny-Rangamani-Takayanagi 0705.0016

$$ds^2 = r^{-2+2\theta/(d+1)}\left[-r^{-2(z-1)}\left(dt + r^{2(z-1)}d\xi\right)^2 + r^{2(z-1)}d\xi^2 + \sum_{i=1}^{d} dx_i^2 + dr^2\right].$$

  - impose stationary condition involved with ξ coordinate.
  - using that there is a fixed length scale associated with ξ : $\int d\xi = L_\xi$.
  - demonstrate $(d-1)$-d area law for EE for $(d+3)$-d Schrödinger background

$$\mathcal{S} = \frac{(RM_{Pl})^{(d+1)}}{4(\alpha-1)}\left(\left(\frac{\epsilon}{R_\theta}\right)^{\theta}\frac{L^{d-1}L_\xi}{\epsilon^{d-z+1}} - c_\theta\left(\frac{l}{R_\theta}\right)^{\theta}\frac{L^{d-1}L_\xi}{l^{d-z+1}}\right),$$

- Novel phases :

$$d+1-z < \theta < d+2-z\,, \quad \text{for} \quad \theta \neq 0\,,$$
$$d+1 < z < d+2\,, \quad \text{for} \quad \theta = 0\,.$$

  - The latter is surprising compared to the known Lifshitz case.
  - What are the properties of these novel phases? Not explorered yet ...

Hyperscaling violation exponent as a unification tool

- For a given metric, several solutions with different matter contents exist

Balasubramanian-McGreevy 0909.0263.

- Consider metric only to classify lower dimensional low energy EHT.

Gouteraux-Kiritsis 1212.2625.

- Constrain the parameter space (z, θ) of consistent theories using
  (a) null energy condition(NEC); (b) positive specific heat constraint(SHC).

- Classifications are tied with the number of scales in microscopic theories.
  - checked with various string theory solutions with sphere reductions

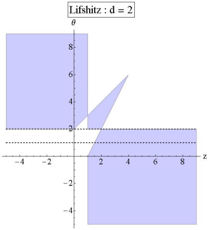
$$\mathrm{d}s^2 = r^{-2+2\frac{\theta}{d}} \left[-r^{2-2z}\mathrm{d}t^2 + \mathrm{d}r^2 + \mathrm{d}x_i^2 + r^{2-2\sharp_j}\mathrm{d}x_j^2 \right] ,$$

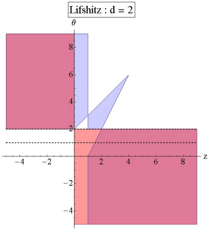
  where $i = 1, \cdots, c$ and $j = c+1, \cdots, d$
  - Conformal cases (D3, M2, M5 branes) : $\theta = 0$, $\sharp_j = 1$.
  - Lifshitz theories with a scale (Dp branes) : fixed by θ, z & $\sharp_j = 1$.
  - Intersecting M2-M5/D1-D5 require spatial anisotropic exponents : $\sharp_j = 0$.

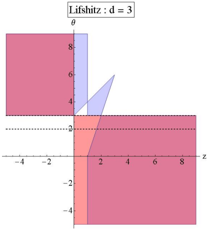
$$\mathrm{d}s^2_{D1D5} = \frac{\rho^2}{\rho_1\rho_5}[-f\mathrm{d}t^2 + \mathrm{d}x_1^2] + \frac{\rho_1\rho_5}{\rho^2}\frac{\mathrm{d}\rho^2}{f} + \frac{\rho_1}{\rho_5}\mathrm{d}s^2_{M_4} + \rho_1\rho_5\mathrm{d}\Omega_3^2 ,$$

  which is $AdS_3 \times M_4 \times S^3$.

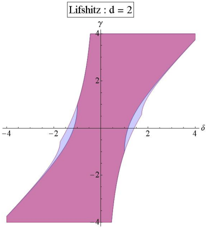
NEC + SHC Dong-Harrison-Kachru-Torroda-Wang 1201.1905, BSK 1210.0540

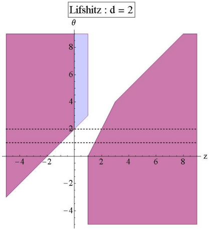


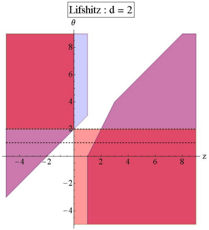




Gubser + Fluctuation + SHC Charmousis-Gouteraux-BSK-Kiritsis-Meyer 1005.4690







Short summary for Schrödinger geometry

- Classifications for Schrödinger metrics can be carried out similarly for $T = 0$.

$$\mathrm{d}s^2 = r^{-2+2\theta/(d+1)}\left[-r^{2-2z}\mathrm{d}t^2 - 2\mathrm{d}t\mathrm{d}\xi + \mathrm{d}r^2 + \mathrm{d}x_i^2 + r^{2-2\sharp_j}\mathrm{d}x_j^2\right] .$$

  Need care for finite T due to a new thermodynamic variable b.

- Null Melvin twist of NS5A brane solution shows log-violation of EE area law, providing an example of candidate dual Fermi surface ($d = 4, \theta = 2, z = 3$).

- Null Melvin twist of KK monopole solution

$$\mathrm{d}s_E^2 \sim r\left[-r^4\mathrm{d}t^2 - 2\mathrm{d}t\mathrm{d}\xi + \mathrm{d}x_i^2 + \mathrm{d}r^2\right] , \quad i = 1, \cdots, 5 ,$$

  reveals $d = 5, \theta = 9$ and $z = -1$, possessing negative dynamical exponent.
  - Space and time scales in opposite way.
  - There exist other examples in Schrödinger case, *e.g.* Bobev-Kundu-Pilch 0905.0673.
  - EE reveals imaginary part.
  - $\cdots$
  - Can we make sense of the system with $z < 0$?
  - Euclidean version of the geometry studied in Cvetic-Lu-Pope hep-th/9810123.

Future Questions

• What are those novel phases? physical properties? realized in nature?

• Does the gravity backgrounds with $\theta = 1$ describe the systems with dual Fermi surfaces? Is there other physical properties we can investigate to pin down this question?

• Should we throw away Schrödinger geometry with negative dynamical exponent? How about Lifshitz geometry with negative dynamical exponent?

• Can one say something for the string landscape picture by following the holographic effective approach, possibly by incorporating hyperscaling violation and by considering metric only?