Scalar Bremsstrahlung in Gravity-Mediated Ultrarelativistic Collisions

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Outline

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- Black holes can be formed in colliders for $M > M_*$ and $b \leq R_s$ (Argyres et. al. 1998).
- Various models have been proposed to compute the gravitational bremstrahlung(D'Eath and Payne, Eardley and Giddings) (Colliding waves model, ACV model). Alternative methods are desirable.

Introduction

- Based on work with Dmitry Gal′tsov, Pavel Spirin and Theodore Tomaras
- Gravitational bremstrahlung two massive particles $\rightarrow$ complicated.
- Emission of scalar radiation of gravitationally interacting particles $\rightarrow$ intermediate step.

Introduction

- Based on work with Dmitry Gal′tsov, Pavel Spirin and Theodore Tomaras
- Gravitational bremstrahlung two massive particles $\rightarrow$ complicated.
- Emission of scalar radiation of gravitationally interacting particles $\rightarrow$ intermediate step.
- Two massive particles, one of which is coupled to a scalar field. The interaction is purely gravitational.
- Non-linear problem, because of the graviton - graviton - scalar interaction

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- Two classical massive particles with mass m and m' that interact only gravitationally.
- A massless scalar field Φ that interacts with m but not m'.
- The action will be:

$$S = S_g + S_\Phi + S_m + S_{m'}$$

-

$$S = \int d^D x \sqrt{|g|}\left[-\frac{R}{\varkappa_D^2} + \frac{1}{2} g^{MN} \partial_M \Phi \, \partial_N \Phi\right] - \frac{1}{2}\int \left[e \, g_{MN} \dot{z}^M \dot{z}^N + \frac{(m+f\Phi)^2}{e}\right] d\tau - \frac{1}{2}\int \left[e' g_{MN} \dot{z}'^M \dot{z}'^N + \frac{m'^2}{e'}\right] d\tau'$$

Equations of motion

- The equations of motion for the two particles are produced by varying the action:

$$\frac{d}{d\tau}\left(e g'_{MN}\dot{z}^N\right) = \frac{e}{2} g'_{LR,M}\dot{z}^L\dot{z}^R, \qquad \frac{d}{d\tau}\left(e' g_{MN}\dot{z}'^N\right) = \frac{e'}{2} g_{LR,M}\dot{z}'^L\dot{z}'^R,$$

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- while variation with respect to the einbeins gives

$$e^{-2} = \frac{g'_{MN}\dot{z}^M\dot{z}^N}{(m+f\Phi)^2} \qquad e'^{-2} = \frac{g_{MN}\dot{z}'^M\dot{z}'^N}{m'^2}$$

- Plugging these back into the action and varying with respect to Φ, we obtain the scalar field EOM:

$$\Box_D\Phi = -\frac{\varkappa_D}{2}h'\Box_D\Phi + \varkappa_D h'_{MN}\Phi^{,MN} + f\int\left(g'_{MN}\dot{z}^M\dot{z}^N\right)^{1/2}\delta^D(x-z(\tau))\,d\tau,$$

Equations of motion

- It is sufficient to restrict ourselves to linearized gravity and the EOM are

$$\Box_D h^{MN} = -\varkappa_D \left(T^{MN} - \eta^{MN} \frac{T}{D-2} \right), \; T^{MN} = \int e \dot{z}^M \dot{z}^N \frac{\delta^D(x - z(\tau))}{\sqrt{-g'}} d\tau,$$

- Similarly,

$$\Box_D h'^{MN} = -\varkappa_D \left(T'^{MN} - \eta^{MN} \frac{T'}{D-2} \right), \; T'^{MN} = \int e' \dot{z}'^M \dot{z}'^N \frac{\delta^D(x - z'(\tau))}{\sqrt{-g}} d\tau.$$

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- To solve the EOM, we will apply perturbation theory, with respect to f, the scalar charge and with respect to the gravitational coupling constant.

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- First order: deviation from straight trajectory caused by the gravitational field produced by each particle in the zeroth order.
- Leading contribution to the radiation due to the acceleration of the charged particle, m.
- Note: our solution is valid only if the deviations from straight trajectories are small and the iterative solution is convergent.

Validity of the approximation(small deviation from flat metric)

With the two masses of the particles taken at the same order and eventually equal, the model is characterized by three classical parameters:

- The classical radius of the charge:

$$r_f = \left(\frac{f^2}{m}\right)^{\frac{1}{d+1}},$$

- the D-dimensional gravitational radius of the mass m at rest

$$r_g = \left(\varkappa_D^2 m\right)^{\frac{1}{d+1}},$$

- the Schwarzschild radius of the black hole, associated with the collision energy $\sqrt{s}$:

$$r_S = \frac{1}{\sqrt{\pi}}\left[\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2}\right]^{\frac{1}{d+1}}\left(\frac{G_D\sqrt{s}}{c^4}\right)^{\frac{1}{d+1}}.$$

Validity of the approximation

In the rest frame, $\sqrt{s} = 2mm'\gamma$, so

$$r_s \sim r_g \gamma^{\nu}, \quad \nu = \frac{1}{2(d+1)}$$

We assume that

$$r_g \sim r_f \ll b\gamma^{-2\nu},$$

or equivalently

$$b \gg r_s \gamma^{\nu}$$

Under this condition the deviation of the metric from unity in the rest frame of m' is small, i.e. $\varkappa_D h_{MN} \dot{z}'^M \dot{z}'^N \ll 1$

The formal expansion

- The world line of the two particles are:

$$z^M = {}^0z^M + {}^1z^M + \dots, \qquad {}^0z^M = u^M\tau + b^M,$$
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- The scalar field is expanded:

$$\Phi = {}^0\Phi + {}^1\Phi + \dots .$$

With EOM at zeroth order

$$\Box_D {}^0\Phi = f \int \delta^D(x - u\tau - b)\, d\tau.$$

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- Finally for the metrics:

$$h_{MN} = {}^0h_{MN} + {}^1h_{MN} + \dots,$$

and similarly for h'_{MN}. The sources:

$${}^0T^{MN} = m\int \delta^D(x - {}^0z(\tau))\, u^M u^N d\tau, \quad {}^0T'^{MN} = m\int \delta^D(x - {}^0z'(\tau))\, u'^M u'^N d\tau$$

First order EOM

By choosing an appropriate gauge, the EOM of the two particles read:

$$\Pi^{MN}\,{}^{1}\ddot{z}_N = -\varkappa_D \Pi^{MN}\left(h'_{NL,R} - \frac{1}{2}h'_{LR,N}\right)u^L u^R, \tag{1}$$

$$\Pi'^{MN}\,{}^{1}\ddot{z}'_N = -\varkappa_D \Pi'^{MN}\left(h_{NL,R} - \frac{1}{2}h_{LR,N}\right)u'^L u'^R,$$

where the projectors onto the space transverse to the world-lines are

$$\Pi^{MN} = \eta^{MN} - u^M u^N, \quad \Pi'^{MN} = \eta^{MN} - u'^M u'^N,$$

Second order equation for Φ radiation

As mentioned before, the radiation comes from the first order in Φ, the first order EOM of Φ is

$$\Box_D\,{}^1\Phi(x,y) = j(x,y) \equiv \rho(x,y) + \sigma(x,y),$$

where the first term is localized on the world-line of the radiating particle m

$$\rho(x,y) = -f \int {}^1z^\mu(\tau)\,\partial_\mu \delta^4(x - u\tau - b)\,\delta^d(\mathbf{y})d\tau,$$

while the second is the non-local current

$$\sigma(x,y) = \varkappa_D\,\partial_M \left(h'^{MN}\,\partial_N\,{}^0\Phi - \frac{1}{2} h'\,\partial^M\,{}^0\Phi \right).$$

Φ radiation

- The momentum radiated due to scalar bremstrahlung:

$$P^{\mu} = \int\limits_V d^d y \int\limits_\Omega \partial_N T^{N\mu} d^4 x = \int\limits_V d^d y \int\limits_\Omega \left(\partial^{\mu}\Phi\right)\ \Box_D \Phi\ d^4 x.$$

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- Once we substitute Φ,

$$P^\mu = \frac{1}{16\pi^3 V} \sum_n \int \frac{d^3 k}{k^0} k^\mu \, |j^n(k)|^2 \Big|_{k^0 = \sqrt{\mathbf{k}^2 + k_T^2}} ,$$

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- Finally the energy emitted is:

$$\frac{dE}{d|\mathbf{k}| d\Omega_2} = \frac{1}{16\pi^3 V} \sum_n \mathbf{k}^2 |j^n(k)|^2, \quad d\Omega_2 = \sin\theta \, d\theta \, d\varphi.$$

- Assuming that the impact parameter $b \ll R$, the compactification radius, the sum over the KK modes can be approximated as an integral and can be computed easily:

$$\frac{dE}{d\omega d\Omega_{d+2}} = \frac{\omega^{d+2}}{2(2\pi)^{d+3}} |j(k)|^2.$$

The source

What remains now is to find $\rho^n(k)$ and $\sigma^n(k)$ and substitute in the previous formula. We need to Fourrier transform the expression we saw before:

$$\rho(x, y) = -f \int {}^1 z^\mu(\tau)\, \partial_\mu \delta^4(x - u\tau - b)\, \delta^d(\mathbf{y}) d\tau,$$

$$\sigma(x, y) = \varkappa_D\, \partial_M \left(h'^{MN}\, \partial_N\, {}^0\Phi - \frac{1}{2} h'\, \partial^M\, {}^0\Phi \right),$$

and substitute z(k) and h in them.

The local amplitude

After some calculations the local part of the source can be expressed in terms of the MacDonald functions:

$$\rho^n(k) = -\frac{\varkappa_D^2 m' f}{4\pi vV} e^{i(kb)} \sum_l \left\{ \left[\left(2 - \frac{\gamma_*^2}{v^2\gamma^2}\right)\frac{z'}{z} - \frac{2}{\gamma}\left(\frac{1}{d+2} - \frac{\gamma_*^2}{2v^2\gamma^2}\right)\right] K_0(z_l) - i\frac{\gamma_*^2}{\gamma^2}\frac{(kb)}{v^2\gamma z^2}\hat{K}_1(z_l) \right\} .$$

where:

$$z \equiv \frac{(ku)b}{\gamma v}, \quad z' \equiv \frac{(ku')b}{\gamma v}, \quad z_l \equiv \left(z^2 + p_T^2 b^2\right)^{1/2},$$

the hatted Macdonald functions defined by $\hat{K}_\nu(x) \equiv x^\nu K_\nu(x)$ and $\gamma_*^2 \equiv \gamma^2 - (d+2)^{-1}$

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After taking the ultra-relativistic limit and accounting for the effective number of interaction modes,

$$\rho^n(k) \simeq -\frac{\lambda e^{i(kb)}}{v}\left[\frac{z'}{z}\hat{K}_{d/2}(z) - i\frac{(kb)}{\gamma z^2}\hat{K}_{d/2+1}(z) + \frac{1}{(d+2)\gamma}\left(d - \frac{(d+1)z'}{\gamma z}\right)\hat{K}_{d/2}(z) + \dots\right],$$

where

$$\lambda \equiv \frac{\varkappa_D^2 m' f}{2(2\pi)^{d/2+1} b^d} .$$

The non-local amplitude

We write the non-local amplitude as a sum of two terms, one depending on z and one depending on z′ as follows:

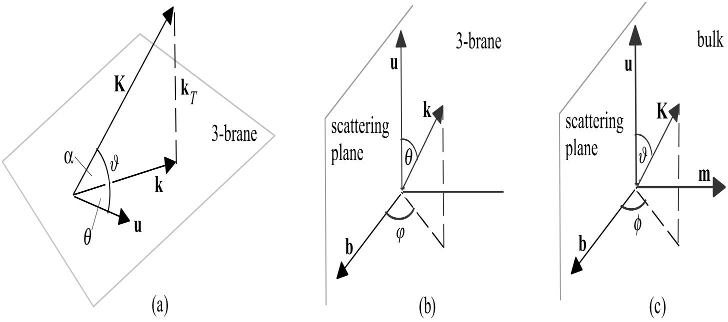
$$\sigma^n(k) \equiv \sigma_0^n(k) + \sigma_1^n(k)\,,$$

$$\sigma_0^n(k) = \lambda\, e^{i(kb)}\, \frac{\gamma v z'^2}{a^2\,\xi^2} \left(\beta \hat{K}_{d/2}(z) - i(kb)\hat{K}_{d/2+1}(z) - \frac{(d+1)\beta}{a^2}\hat{K}_{d/2+1}(z) + \frac{\beta\,\sin^2\phi}{a^2}\hat{K}_{d/2+2}(z) \right)\,,$$

and

$$\sigma_1^n(k) \simeq \lambda \frac{\gamma v z'^2}{a^2\,\xi^2}\left((\xi^2 - \beta)\,\hat{K}_{d/2}(z') + i(kb)\,\hat{K}_{d/2+1}(z') \right)\,,$$

Similarly we will separate the total amplitude in a z and z′ part.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j_z^n in the frequency range $\omega \gg \gamma/b$

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- In the case of $\vartheta \sim 1/\gamma$, in which $z \sim 1$. Adding $\rho^n(k)$ and $\sigma_0^n(k)$ and using the ultra-relativistic expansions we obtain in leading order:

$$j_z^n(k) \simeq \frac{\lambda\,(d+1)e^{i(kb)}}{\gamma\psi}\left[\frac{2\psi-\gamma^{-2}}{d+2}\hat{K}_{d/2}(z) - \frac{\cos^2\alpha}{\psi^2\omega^2 b^2}\left(\left(\sin^2\theta+\tan^2\alpha\right)\hat{K}_{d/2+1}(z) - \frac{\sin^2\theta\sin^2\varphi+\tan^2\alpha}{d+1}\hat{K}_{d/2+2}(z)\right)\right].$$

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- This is a general phenomenon of destructive interference related to the gravitational interaction. The two leading powers in the ultra-relativistic expansion of the direct Φ emission are cancelled by the indirect emission due to $\Phi - \Phi$-h interaction.
- The consequence is that the z-type frequency is highly suppressed in this limit, contrary to what one would expect naively.

j_z^n in the frequency range $\omega \lesssim \gamma/b$

- For $\omega \ll \gamma/b$ and $\vartheta \sim 1/\gamma$ $|\rho^n| \gg |\sigma^n|$ and, therefore,

$$j^n(k)\Big|_{\omega\ll\gamma/b} \simeq \rho^n(k) \simeq -\lambda\left[\frac{1}{\gamma\psi}\hat{K}_{d/2}(z) + i\frac{\sin\vartheta\cos\phi}{\gamma\psi^2\omega b}\hat{K}_{d/2+1}(z)\right].$$

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- For $\vartheta \sim 1$, on the other hand, ρ^n, σ_0^n and σ_1^n are all of the same order, but suppressed compared to the previous case. In addition, the contribution of this regime to the emitted energy is further suppressed by the integration measure.

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- More interesting is the case with $\omega \sim \gamma/b$. If $\vartheta \sim 1$, then $z \sim \gamma$, j_z^n is exponentially suppressed because of the Macdonald functions.
- However, for $\vartheta \sim 1/\gamma$, $\rho^n \sim \gamma$ and $\sigma_0^n \sim \gamma$.

The part $j^n_{z'}(k)$ of the amplitude

- Using the angles ϑ and ϕ the expression can be written as:

$$j^n_{z'} \simeq -\frac{\lambda}{\gamma\psi}\left[\left(\frac{1}{\gamma^2\psi}-1\right)\hat{K}_{d/2}(z') + i\frac{\sin\vartheta\cos\phi}{\gamma z'\psi}\hat{K}_{d/2+1}(z')\right]$$

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- For $(\omega \sim \gamma/b, \vartheta \sim 1)$ $j^n_{z'}$ it is dominated by its real part which is of order $\mathcal{O}(1/\gamma)$.
- For $(\omega \ll \gamma/b, \vartheta \sim 1)$ one obtains $j^n_{z'} \sim \sigma^n_0 \sim 1/\gamma$, however, this region contributes negligibly little to the emitted energy.
- Similarly, for $(\omega \ll \gamma/b, \vartheta \sim 1/\gamma)$ the amplitude is $j^n_{z'} \sim \sigma^n_0 \ll \rho^n \sim \gamma^2$.

The part $j^n_{z'}(k)$ of the amplitude

- Using the angles ϑ and ϕ the expression can be written as:

$$j^n_{z'} \simeq -\frac{\lambda}{\gamma\psi}\left[\left(\frac{1}{\gamma^2\psi}-1\right)\hat{K}_{d/2}(z') + i\frac{\sin\vartheta\cos\phi}{\gamma z'\psi}\hat{K}_{d/2+1}(z')\right]$$

- For $\omega \gg \gamma/b$ one has $z' \gg 1$ and, consequently, $j^n_{z'}$ is exponentially suppressed.
- For $(\omega \sim \gamma/b, \vartheta \sim 1)$ $j^n_{z'}$ it is dominated by its real part which is of order $\mathcal{O}(1/\gamma)$.
- For $(\omega \ll \gamma/b, \vartheta \sim 1)$ one obtains $j^n_{z'} \sim \sigma^n_0 \sim 1/\gamma$, however, this region contributes negligibly little to the emitted energy.
- Similarly, for $(\omega \ll \gamma/b, \vartheta \sim 1/\gamma)$ the amplitude is $j^n_{z'} \sim \sigma^n_0 \ll \rho^n \sim \gamma^2$.
- Finally, based on numerical study and previous results in $D = 4$ one obtains that this expression is valid also in the regime $(\omega \sim \gamma/b, \vartheta \sim 1/\gamma)$ and gives $j^n_{z'} \sim \gamma$.

Summary

The behaviour of the local and non-local currents in all characteristic frequency and angular regimes is summarized in the following Table.

ϑ \ ω_D	$\omega_D \ll \gamma/b$	$\omega_D \sim \gamma/b$	$\omega_D \gg \gamma/b$
γ^{-1}	no destructive interference $j^n \sim \rho^n \gg \sigma_0^n \sim \sigma_1^n$	no destructive interference $j_z^n \sim \rho^n \sim \sigma_0^n \sim \gamma$ $j_{z'}^n \sim \gamma$	destructive interference $j_z^n \sim \rho^n/\gamma^2 \sim 1/\gamma$ $j_{z'}^n \sim \exp(-\gamma)$
1	no destructive interference $j^n \sim \rho^n \sim j_z^n \sim j_{z'}^n$	destructive interference $j_z^n \sim \exp(-\gamma)$ $j_{z'}^n \sim \gamma^{-1}$ (1)	destructive interference $j_z^n \sim \exp(-\gamma)$ $j_{z'}^n \sim \exp(-\gamma)$

The emitted energy

- Now that we have computed and studied j, we have to integrate to get the total energy radiated. We have split j in two parts, j_z^n and $j_{z'}^n$ so it useful to split the energy in three pieces proportional to $|j_z^n(k)|^2$, $|j_{z'}^n(k)|^2$ and $\overline{j_z^n} j_{z'}^n + j_z^n \overline{j_{z'}^n}$

$$dE = dE^z + dE^{z'} + dE^{zz'} ,$$

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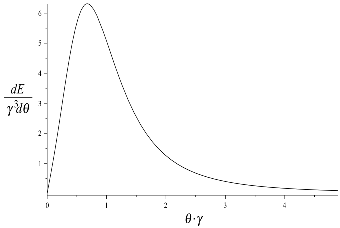
$$dE = dE^z + dE^{z'} + dE^{zz'} ,$$

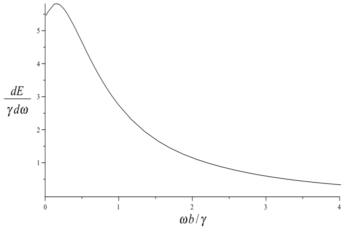
- The general expression of the emitted energy will be

$$E \sim \frac{1}{8(2\pi)^{2d+5}} \frac{\varkappa_D^4 m'^2 f^2}{b^{3d+3}} \gamma^{\#} ,$$

- Φ radiation is emitted in well-defined, relatively narrow windows, with the amplitudes shown in the previous table. Thus it is straightforward to compute the powers of γ, since the range of integration does not introduce further powers of γ.

Frequency and angular distribution for d=0 and $\gamma = 10^5$





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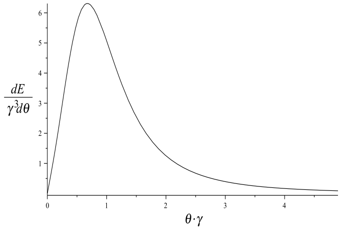

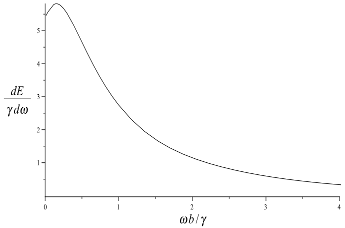

The powers of γ are shown in the following table

ϑ \ ω	$\omega \ll \gamma/b$	$\omega \sim \gamma/b$	$\omega \sim \gamma^2/b$	$\omega \gg \gamma^2/b$
γ^{-1}	negligible (phase space)	$E_d \sim \gamma^3$, from j_z^n and $j_{z'}^n$	$E_d \sim \gamma^{d+2}$, from j_z^n	negligible radiation
1	negligible (phase space)	$E_d \sim \gamma^{d+1}$, from $j_{z'}^n$	negligible radiation	negligible radiation

Let us now look at the energy emitted from the important cells of the table:

- According to Table II, the z-type radiation (due to $|j_z^n|^2$) is always beamed inside $\vartheta \sim 1/\gamma$. Furthermore, for $d \geqslant 2$ it is dominant with characteristic frequency $\omega \sim \gamma^2/b$. The cases $d = 0$ and $d = 1$ will be treated separately.

$$E^z = C_d \frac{\varkappa_D^4 m'^2 f^2}{b^{3d+3}} \gamma^{d+2}$$

with $C_2 = 1.42 \times 10^{-6}$, $C_3 = 6.02 \times 10^{-7}$, $C_4 = 3.45 \times 10^{-7}$, $C_5 = 2.67 \times 10^{-7}$ and $C_6 = 2.76 \times 10^{-7}$.

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- Wide angle radiation ($\vartheta \sim 1$) is mainly z'-type (due to $|j_{z'}^n|^2$) in all dimensions and has characteristic frequency $\omega \sim \gamma/b$. Also, for $d \geqslant 3$ radiation with $\omega \sim \gamma/b$ is predominantly emitted in wide angles. For $d \geqslant 3$ the emitted energy is given by

$$E^{z'} = C'_d \frac{\kappa_D^4 m'^2 f^2}{b^{3d+3}} \gamma^{d+1}, \qquad C'_d = \frac{2^{d-8}\Gamma\left(\frac{3d+3}{2}\right)\Gamma^2\left(\frac{2d+3}{2}\right)\Gamma\left(\frac{d+3}{2}\right)\Gamma\left(\frac{d-2}{2}\right)}{\pi^{3d/2+4}\Gamma(2d+3)\Gamma(d)}$$

While for $d = 2$ one obtains

$$E^{z'} = \frac{105\varkappa_6^4 m'^2 f^2}{2^{16}(2\pi)^7 b^9}\,\gamma^3 \ln\gamma\,.$$

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- According to Table II the emitted energy in 4D is concentrated in the region $\omega \sim \gamma/b$, $\theta \sim \gamma^{-1}$. The total emitted energy is

$$E_0 = C_0 \frac{\varkappa_4^4 m'^2 f^2}{b^3}\,\gamma^3\,, \qquad C_0 \approx 8.3 \times 10^{-5}\,.$$

and

$$E_1 = C_1 \frac{\varkappa_5^4 m'^2 f^2}{b^6}\gamma^3 \ln\gamma\,, \qquad C_1 \approx 1.64 \times 10^{-5}\,.$$

- The total radiated energy E_d depends on the phase-space in an intricate way, so that the resulting radiation does not have a simple universal expression. Specifically, it was found that in the absence of extra dimensions one obtains

$$E_0 = C_0\, m \left(\frac{r_g}{b}\right)^2 \left(\frac{r_f}{b}\right) \gamma^3, \qquad C_0 \approx 8.3 \times 10^{-5},$$

with the ``basic'' relativistic enhancement factor γ^3. For one extra dimension one has

$$E_1 = C_1\, m \left(\frac{r_g}{b}\right)^4 \left(\frac{r_f}{b}\right)^2 \gamma^3 \ln\gamma, \qquad C_1 \approx 1.64 \times 10^{-5}\,,$$

with almost the same (up to the logarithm) enhancement factor. For $d \geqslant 2$ one finds

$$E_d = C_d\, m \left(\frac{r_g}{b}\right)^{2(d+1)} \left(\frac{r_f}{b}\right)^{d+1} \gamma^{d+2}\,,$$

Future work

- Extend to vector radiation

Future work

- Extend to vector radiation
- Extend to full gravitational problem

Future work

- Extend to vector radiation
- Extend to full gravitational problem
- Include back - reaction

THANK YOU!

Transplanckian gravity is classical

$$\lambda_B = \frac{\hbar c}{\sqrt{s}}, \qquad r_S = \frac{1}{\sqrt{\pi}}\left[\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2}\right]^{\frac{1}{d+1}}\left(\frac{G_D\sqrt{s}}{c^4}\right)^{\frac{1}{d+1}}. \tag{2}$$

$$l_* = \left(\hbar G_D/c^3\right)^{1/(d+2)} = \hbar/M_* c \tag{3}$$

The system is classical if:

$$\hbar \to 0, \qquad \lambda_B \ll l_* \ll r_s$$

Or equivalently if $\sqrt{s} \gg M_*$