Yet another talk about the Holographic Model of the Quantum Hall Effect

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References

D3-D7’

- O. Bergman, G. Lifschytz, N. Jokela, MSL
  Quantum Hall effect in a holographic model
  JHEP **1010** (2010) 063; arXiv:1003.4965 [hep-th]

- G. Lifschytz, N. Jokela, MSL
  Magneto-roton excitation in a holographic quantum Hall fluid
  JHEP **1102** (2011) 104; arXiv: 1012.1230 [hep-th]

- O. Bergman, G. Lifschytz, N. Jokela, MSL
  Striped Instability of a holographic Fermi-like liquid
  JHEP 10 (2011) 034; arXiv: 1106.3883 [hep-th]

D2-D8’

- N. Jokela, M. Järvinen, MSL
  A holographic quantum hall model at integer filling
  JHEP 1105 (2011) 101; arXiv: 1101.3329 [hep-th]

- N. Jokela, M. Järvinen, MSL
  Fluctuations of a holographic integer quantum Hall fluid
  arXiv: 1107.3836 [hep-th]

Outline

1. Introduction
    - Motivation
    - QHE review

2. D2-D8’ Model Construction

3. Embeddings and Conductivity

4. Phase Diagram

5. Fluctuations
    - Normal Modes - Rotons
    - Quasi-Normal Modes - Instability

6. Summary and Open Questions

Holographic fermions

Many phenomena involve strongly-coupled fermions:

- Chiral Symmetry Breaking
- Quantum Critical Points
- Fractional quantum Hall effect (FQHE)
- …

But interesting = difficult

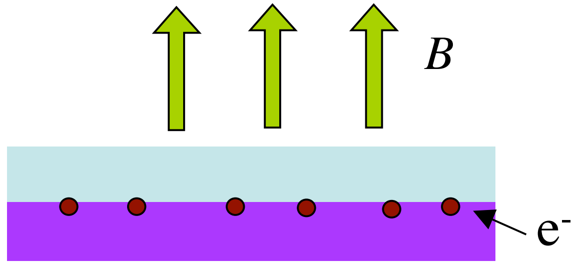
Top-Down Holographic Approach:

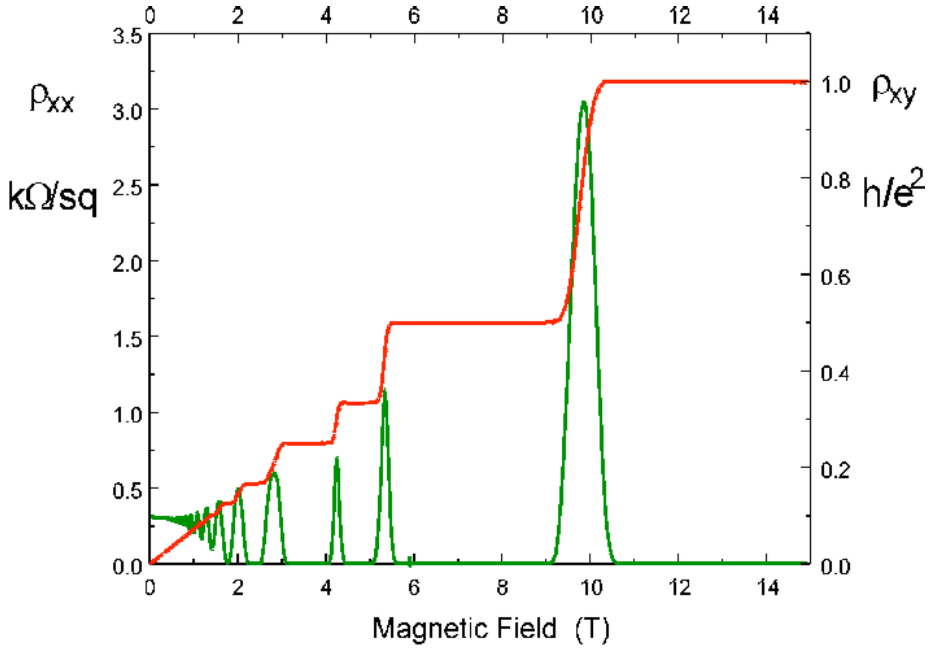
- study concrete string models
- known field theory duals
- gives new effective theories
- look for universal features

Quantum Hall Effect (QHE)

Experimental Setup:

- e^- in 2+1 d
- high magnetic field *B*
- low temperature *T*





Conductivity

Longitudinal: $\sigma_{xx} = \frac{j_x}{E_x} = 0$

Hall: $\sigma_{xy} = \frac{j_y}{E_x} = \frac{e^2}{2\pi\hbar}\nu$

Filling fraction

Filling Fraction

$$\nu \equiv \frac{2\pi\hbar}{e}\frac{D}{B} \sim \frac{\text{\# electrons}}{\text{\# flux quanta}}$$

QH states for particular values of ν

$\nu \in \mathbf{Z}$ ➡ Integer QHE

$\nu \notin \mathbf{Z}$ ➡ Fractional QHE

Open questions:

- microscopic description
- allowed ν's
- Transitions between

Dp-Dq Models

Brane intersections with #ND=6

- fundamental fermions at intersection
- Dq probe in Dp background
- ~~SUSY~~ ➡ stability?

Example:

Sakai Sugimoto model: D4-D8-$\overline{\text{D8}}$

Rey
Kraus et al
Myers et al
Hong & Yee
…

Two other examples:

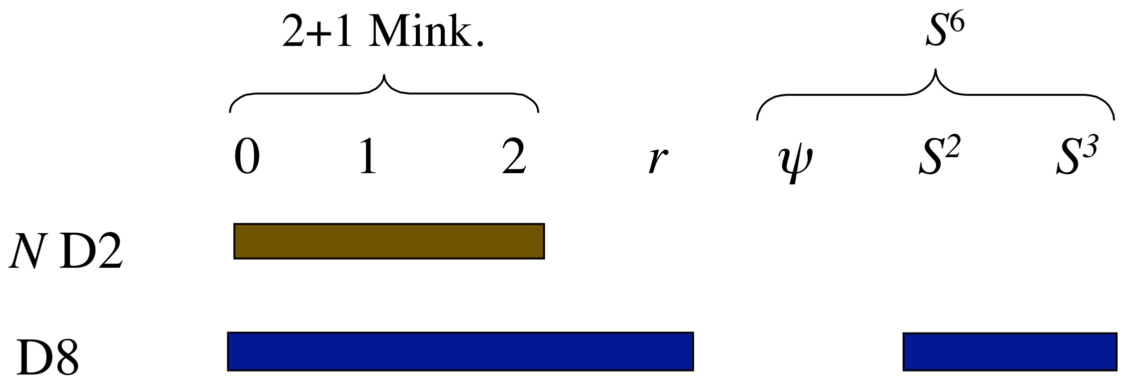
- D3-D7' Model

  2+1-dim fermions, 3+1-dim gauge
  FQHE, ν = irrational, set by internal flux

- D2-D8' Model

  2+1-dim gauge + fermions
  IQHE, $\nu = 1 \ \forall$ internal flux $\neq 0$
  Focus of this talk

D2-D8′ system



$$d\Omega_6^2 = d\psi^2 + \cos^2\psi\ d\Omega_2^2 + \sin^2\psi\ d\Omega_3^2$$

D8-brane embedding:

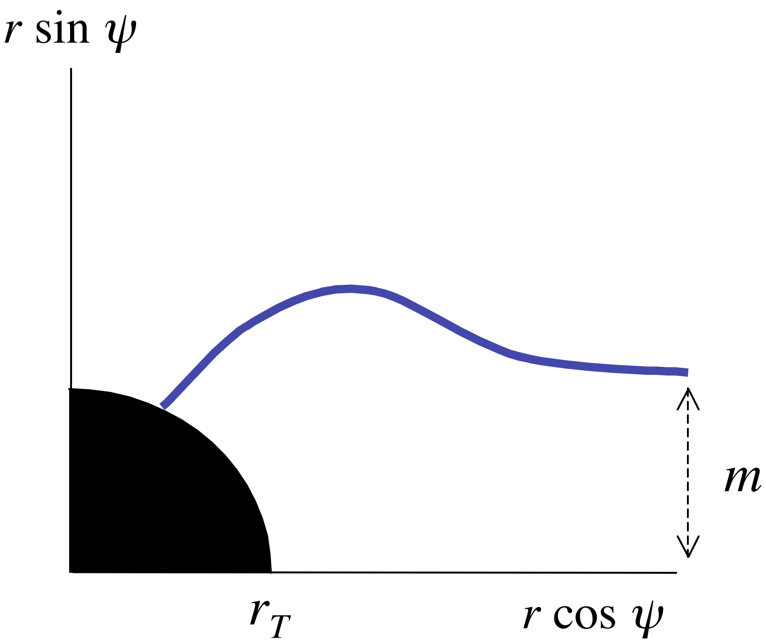
- wraps $S^2 \times S^3 \subset S^6$
- solve for $\psi(r)$

Stabilization

- lowest mode for ψ tachyonic (slipping mode)
- wrap magnetic flux on internal S^2

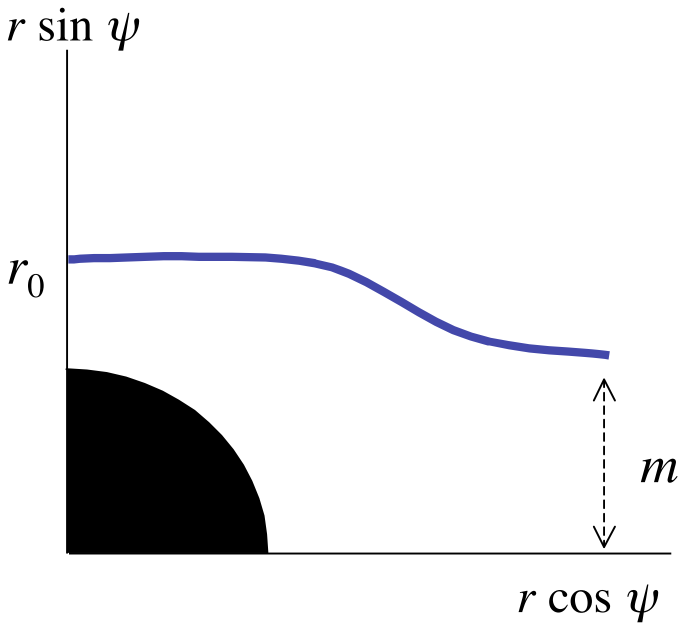
Embeddings

Black Hole (BH)



D8 enters horizon

Minkowski (MN)



D8 ends where S^3 shrinks

Add charges and magnetic field

Charge density

$$2\pi\alpha' F_{r0} = a_0'(r)$$

Magnetic field

$$2\pi\alpha' F_{xy} = B$$

Chern-Simons

$$S_{CS} \sim \int C_5 \wedge F \wedge F$$

$$\sim B \int dr\, c(r) a_0'(r) \quad \text{where} \quad c(r) \sim \int_{S^2 \times S^3} C_5$$

→ C_5 flux and B induce charge

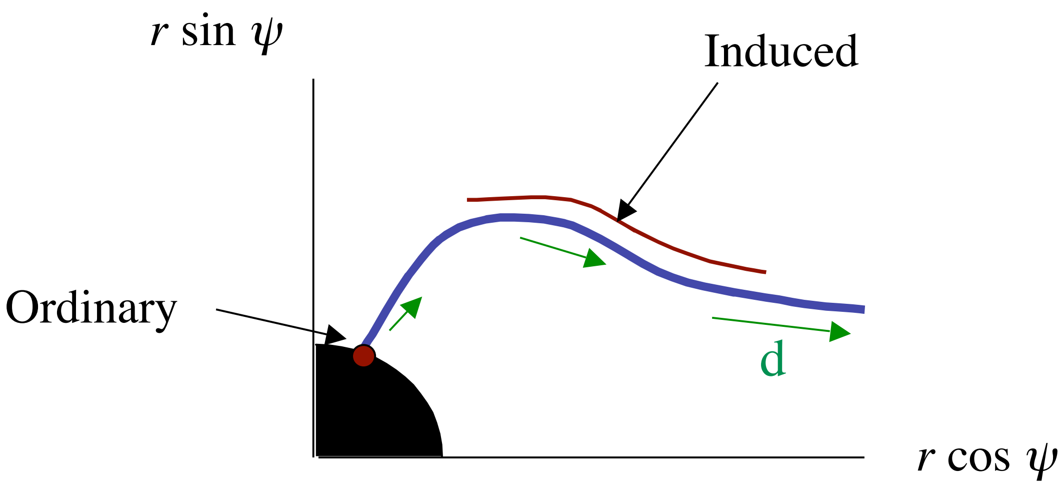
Where's the charge?

Charge density D ⟺ radial displacement flux $d(r)$

Total charge density: $D = d_\infty$

Split between two types:

1. Induced charge: $Bc(\mathrm{r}_{\min})$
2. Ordinary charge: D - $Bc(r_{\min})$



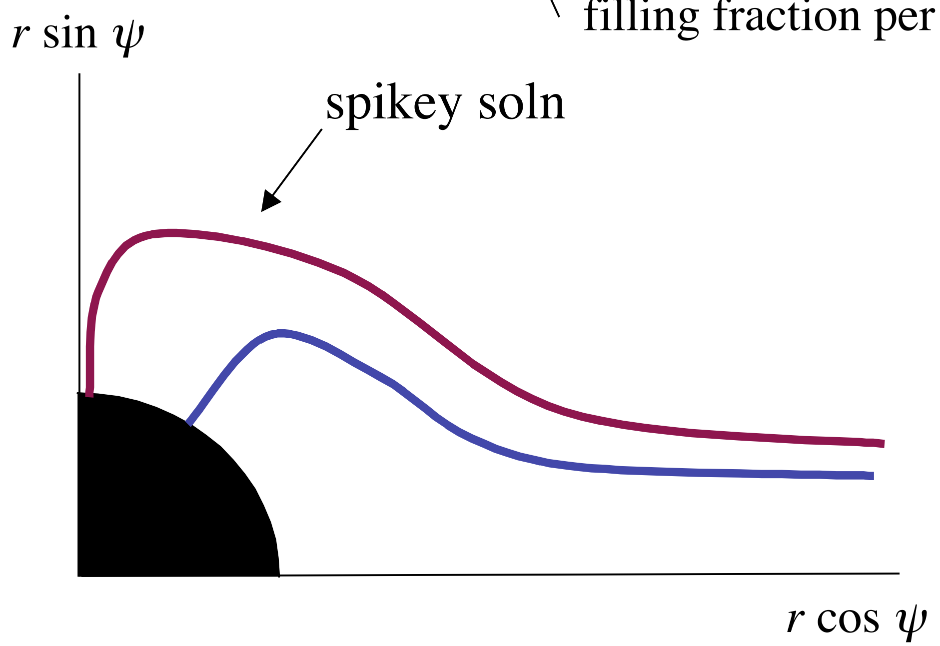
Black Hole Embeddings

Metallic state

- gapless charged excitations
- conductivity (via Karch-O'Bannon)
  - $\sigma_{xx} \neq 0$
  - $\sigma_{xy} \neq 0$ even for $B = 0$ ⟹ AHE

Solutions become spikey as $\frac{2\pi}{N}\frac{D}{B} \to 1$

↖ filling fraction per fermion



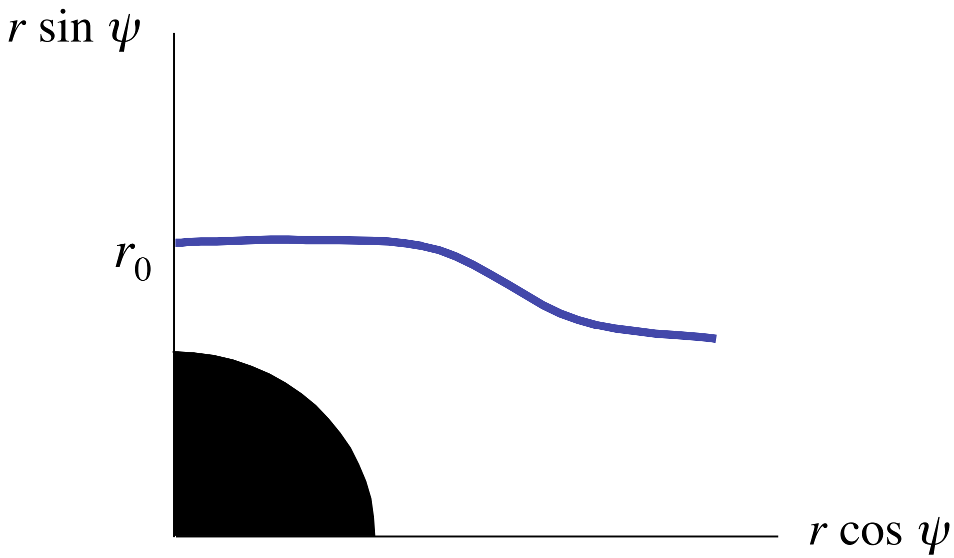
Minkowski Embeddings

QH state

- no sources at tip, all charge induced

$$\Rightarrow \quad D = \frac{N}{2\pi} B$$

- $\nu = 1$ independent of internal flux
- gap for charged excitations $m_g \sim r_0$
- conductivity
  - $\sigma_{xx} = 0$
  - $\sigma_{xy} = \nu/2\pi$

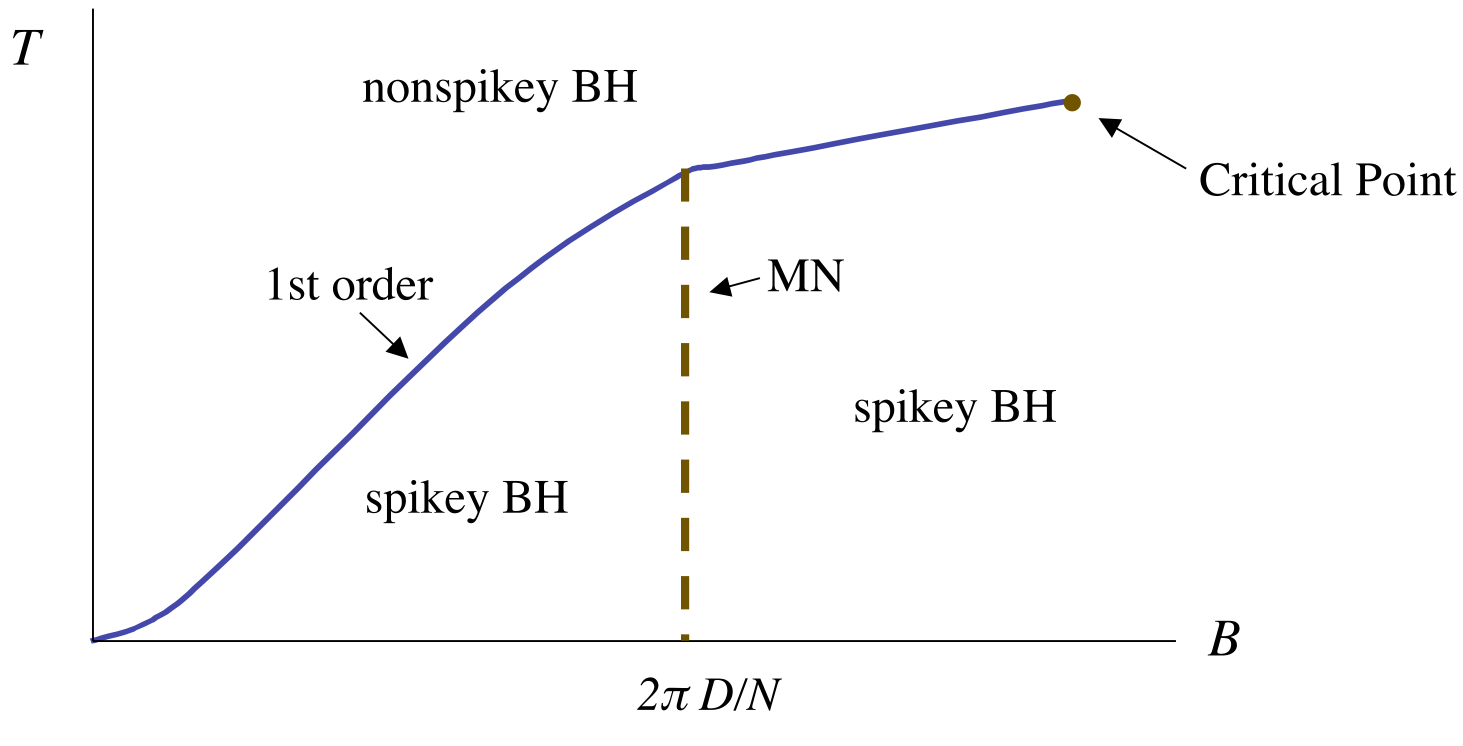


Phase Diagram

- Fixed *D*
- Phase Transition

MN/spikey BH ⟺ nonspikey BH

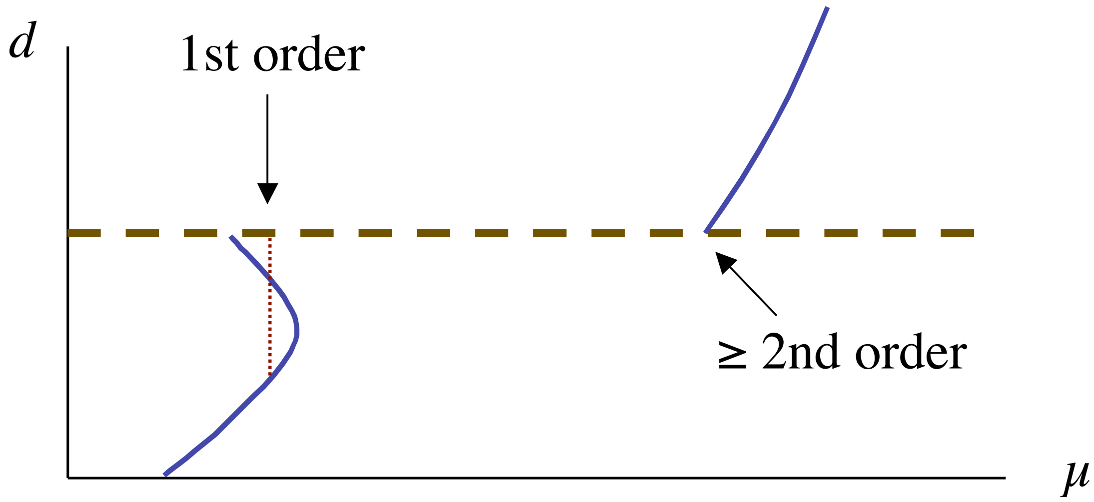
1st order, end in critical point



MN to spikey BH transition

MN solution indep. of μ ⇨ CE ill-defined

Use GCE, where d(μ)



MN ⇔ spikey BH is

- 1st order for B increasing
- At least 2nd order for B decreasing

Fluctuation Analysis

in four easy steps:

1. Choose better coordinates $\rho(R)$
2. Perturb fields, choose wavelike ansatz

$$\rho(t,x,R) \to \rho(R) + \epsilon\ \delta\rho(R) e^{-i\omega t + ikx}$$
$$a_\mu(t,x,R) \to a_\mu(R) + \epsilon\ \delta a_\mu(R) e^{-i\omega t + ikx}$$

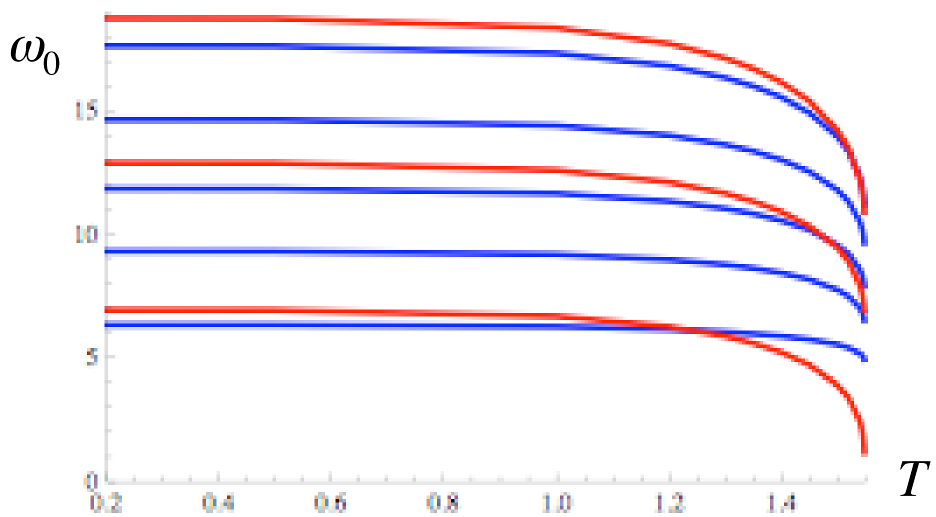
3. Expand ⟹ Coupled 2nd order ODEs
   - Very long and gross
   - Normalizability ⟹ $\delta\rho = \delta a_\mu = 0$ in UV
   - Solve by shooting from IR
   - Use determinant method

4. Solutions
   - QH fluid (MN) normal modes
   - Metal (BH) quasi-normal modes

Normal mdes of QH fluid

Neutral Spectrum

- gapped
- k=0, scalars (red) and
  vectors (blue) decouple
- level crossings

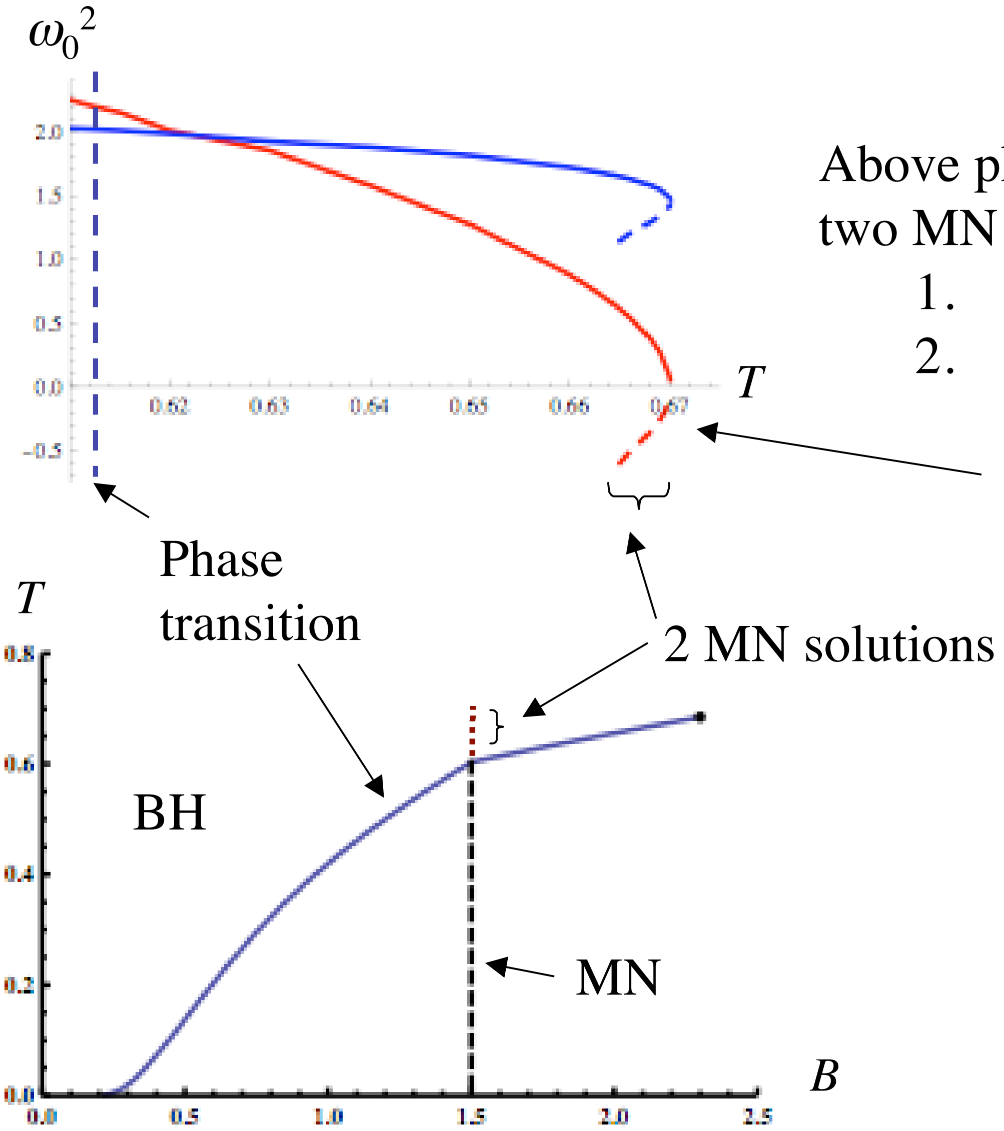


Unstable branch

Above phase transition,
two MN solutions

1. metastable
2. unstable

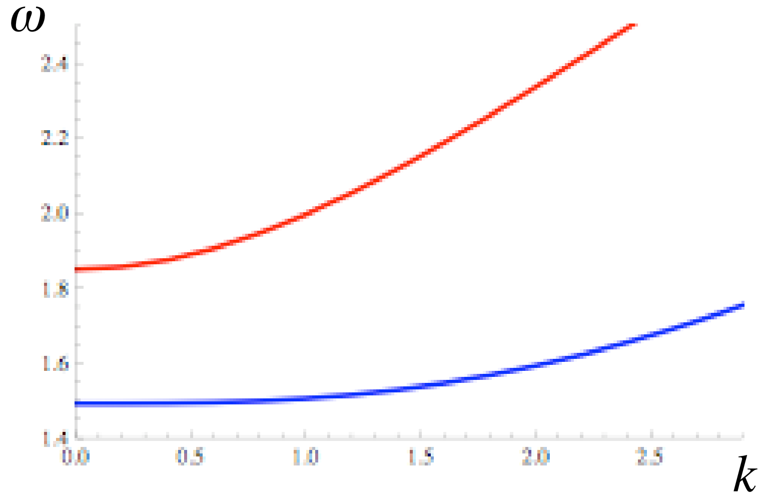
lowest mode tachyonic



Dispersion

Massive dispersion $\omega \approx \sqrt{\omega_0^2 + c_s^2 k^2}$

- generic
- speed of sound c_s indep. of mode #

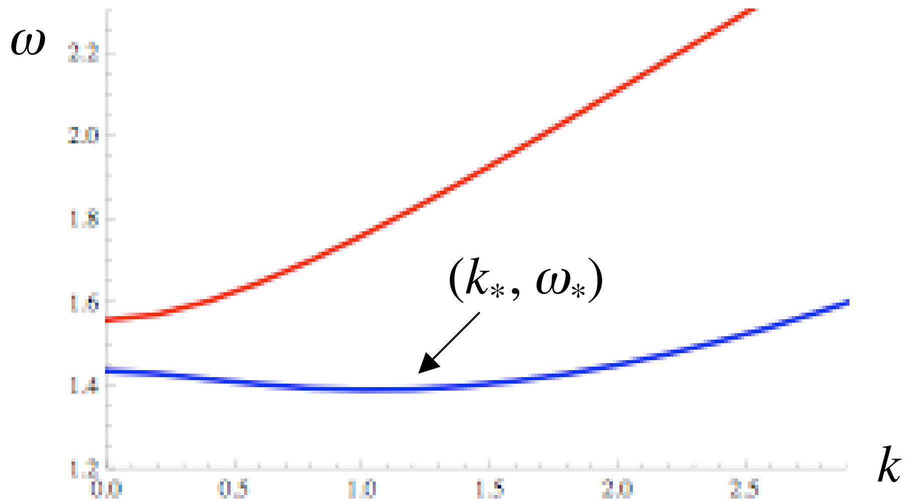


Magneto-Roton Dispersion $\omega \approx \sqrt{\omega_\star^2 + c_s^2 \left(k - k_\star\right)^2}$

- $\omega_* < \omega_0$ at $k_* > 0$
- lower mode near level crossings
- quasiparticle-quasihole dipole
- seen in experiments

  e.g. for $\nu = 1/3$

  Hirjibehedin et al.
  cond-mat/0407145



Quasinormal Modes of BH

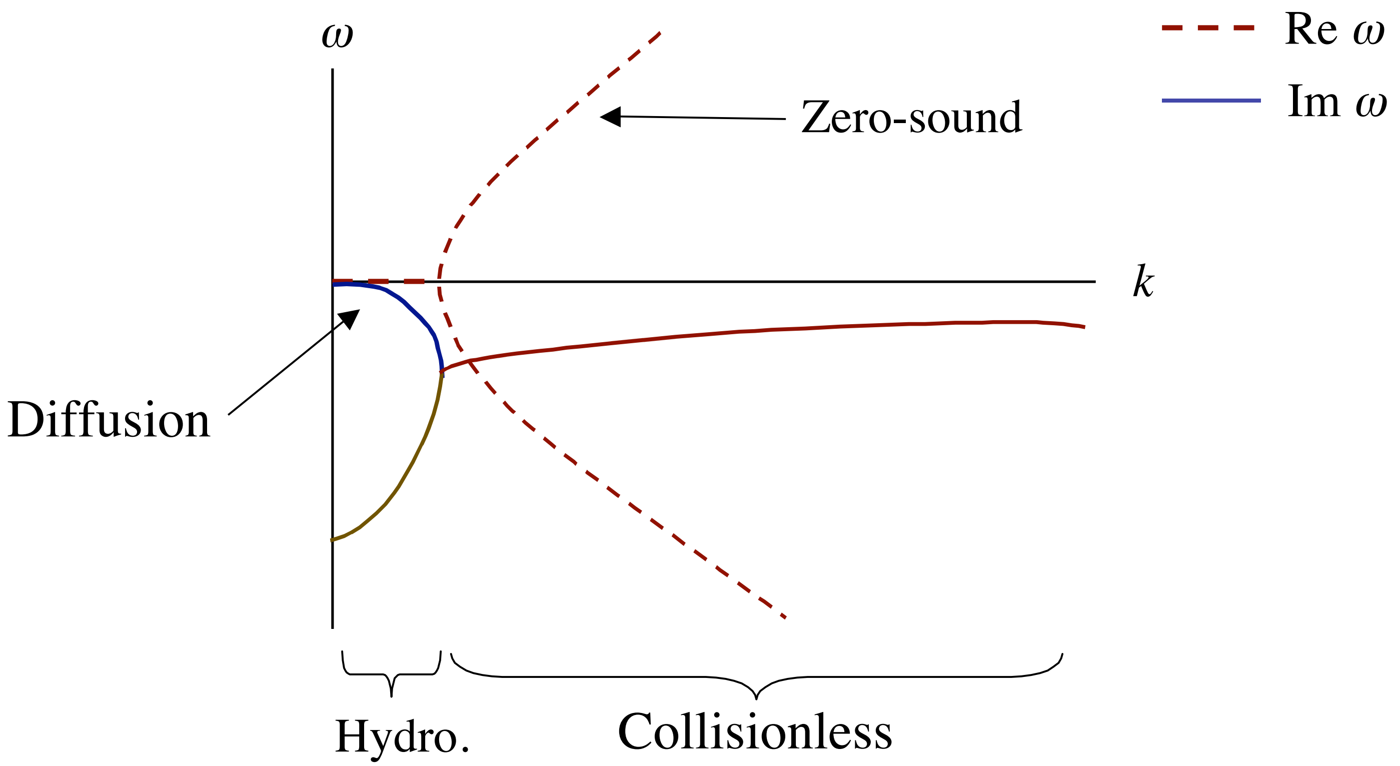
Longest-lived Mode

- Diffusive, hyrodynamical mode at small ω

  $$\omega = -iDk^2$$

- Zero sound (modified for T > 0)

  $$\omega \sim \pm vk$$



NOP Instability

Maxwell-Axion theory in (3+1) dim

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\Phi^2 + \frac{\alpha}{2}\Phi F \wedge F$$

Perturb around background $F_{03} = E$

Linearized EOM + Bianchi give

$$(\partial_\mu \partial^\mu + m^2)\Phi = 4\alpha E F_{12}$$

$$\partial_\mu \partial^\mu F_{12} = 4\alpha E(\partial_1 \partial^1 + \partial_2 \partial^2)\Phi$$

Plane wave ansatz ⇒ dispersion relation

$$\omega^2 = k^2 + \frac{1}{2}m^2 \pm \sqrt{m^4 + 64\alpha^2 E^2 k^2}$$

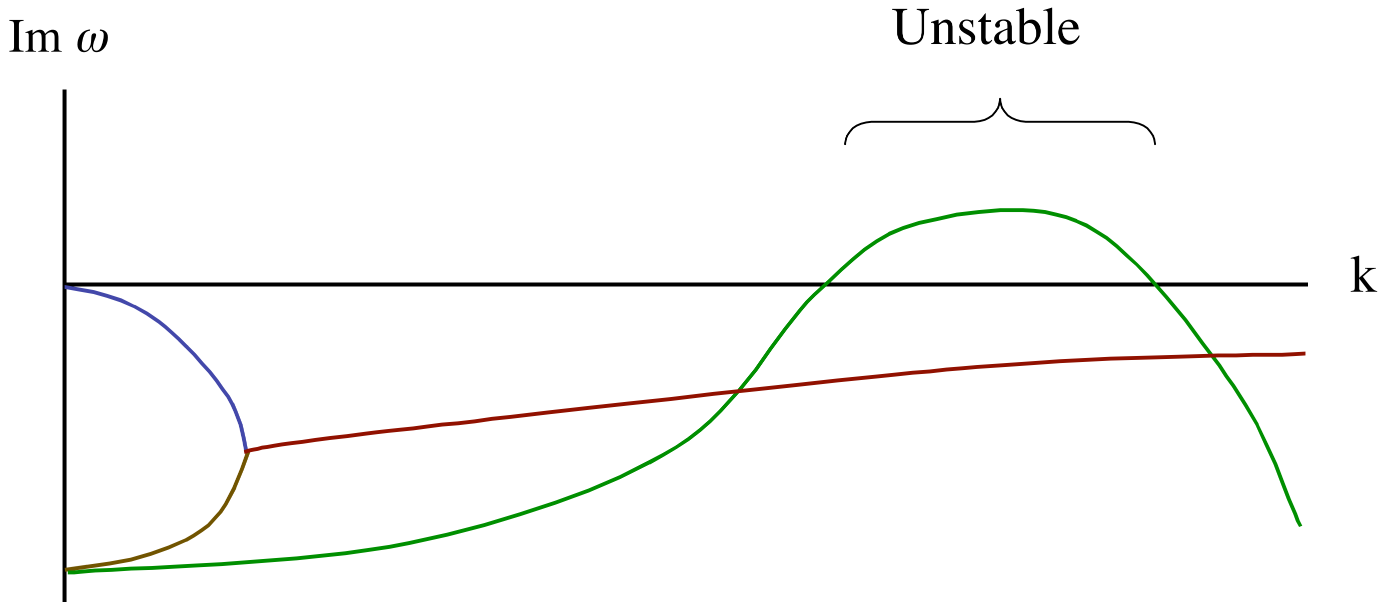
Tachyonic for

$$0 < k < \sqrt{16\alpha^2 E^2 - m^2}$$

NOP for D2-D8

For small enough T,

Instability for $k_{\min} < k < k_{\max}$



True ground state ⇨ Charge/spin density wave?

Summary

Top-down models of QHE

Features:

- Quantized ν
- Mass gap
- Conductivities
- Fluctuations

Bugs:

- Only one QH state per model
- Limited choice of ν

Open Questions

QHE features

- multiple filling fractions & transitions
- impurities, plateaux
- boundaries, edge states
- connect to bottom-up models

  e.g. Lee et al. arXiv/1008.1917

Modulated instability

- dependence on *B*
- striped ground state