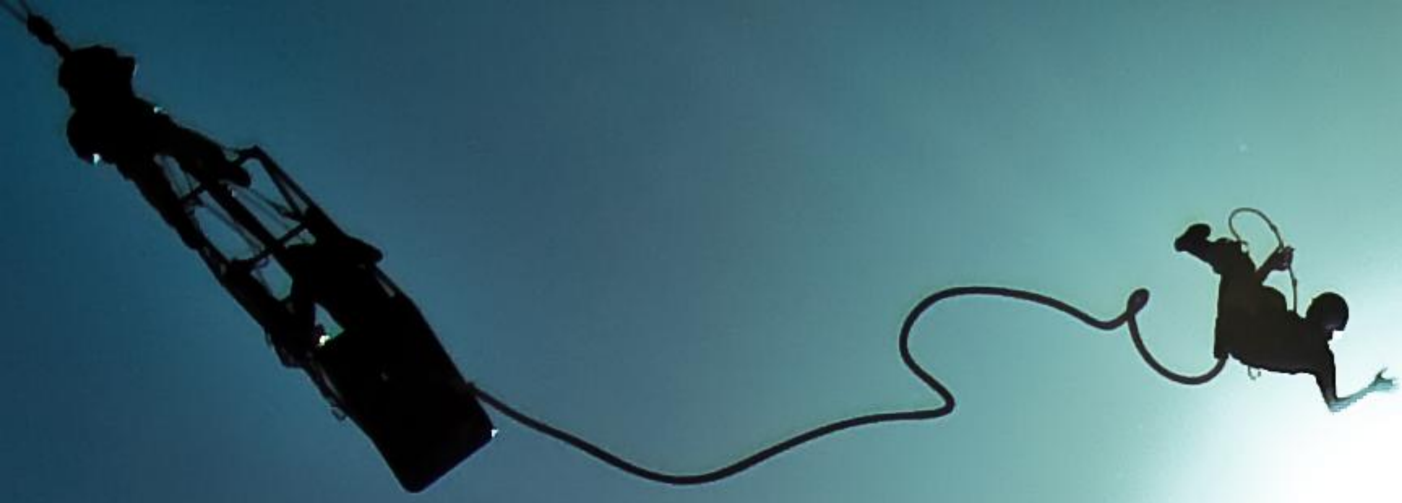

The BD Ghost in massive Gravity

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Work with

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Crete Center for Theoretical Physics

April, 15th 2011

Why Massive Gravity ?

- Phenomenology
- Self-acceleration
- C.C. Problem

Why Massive Gravity ?

- Phenomenology

  what are the theoretical and observational bounds on gravity in the IR ?

  mass of the photon is bounded to $m_\gamma < 10^{-25}$ GeV,
  how about the graviton?

Can we construct a consistent theory for
a massive spin-2 field ?

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- Self-acceleration

  Could dark energy be due to an IR modification of gravity?

  with no ghosts ... ?

  Deffayet, Dvali, Gabadadze, '01
  Koyama, '05

- C.C. Problem

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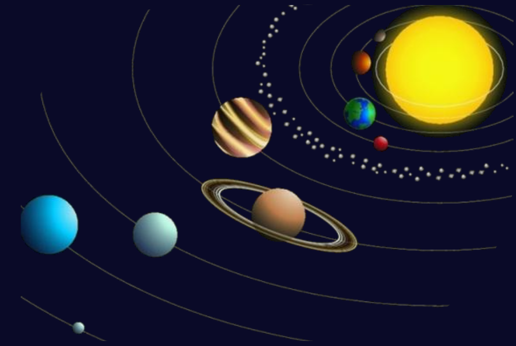
- C.C. Problem

  Is the cosmological constant small ?
  OR
  does it have a small effect on the geometry ?

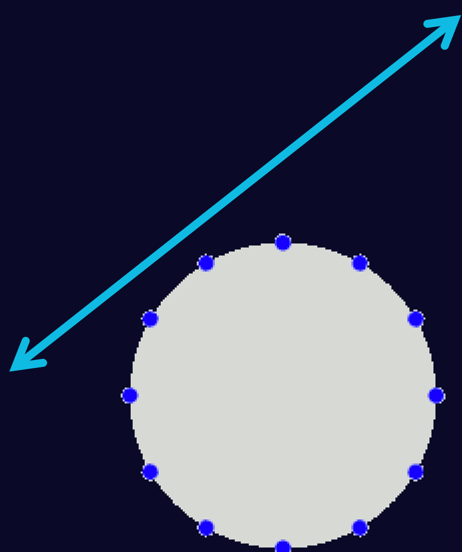
Arkani-Hamed, Dimopoulos, Dvali &Gabadadze, '02
Dvali, Hofmann & Khoury, '07



Massive Gravity

- A massless spin-2 field in 4d, has 2 dof

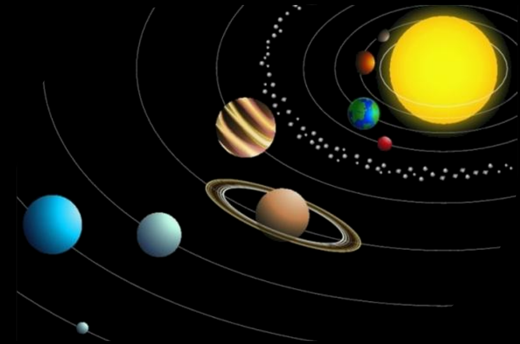
- A massive spin-2 field, has 5 dof



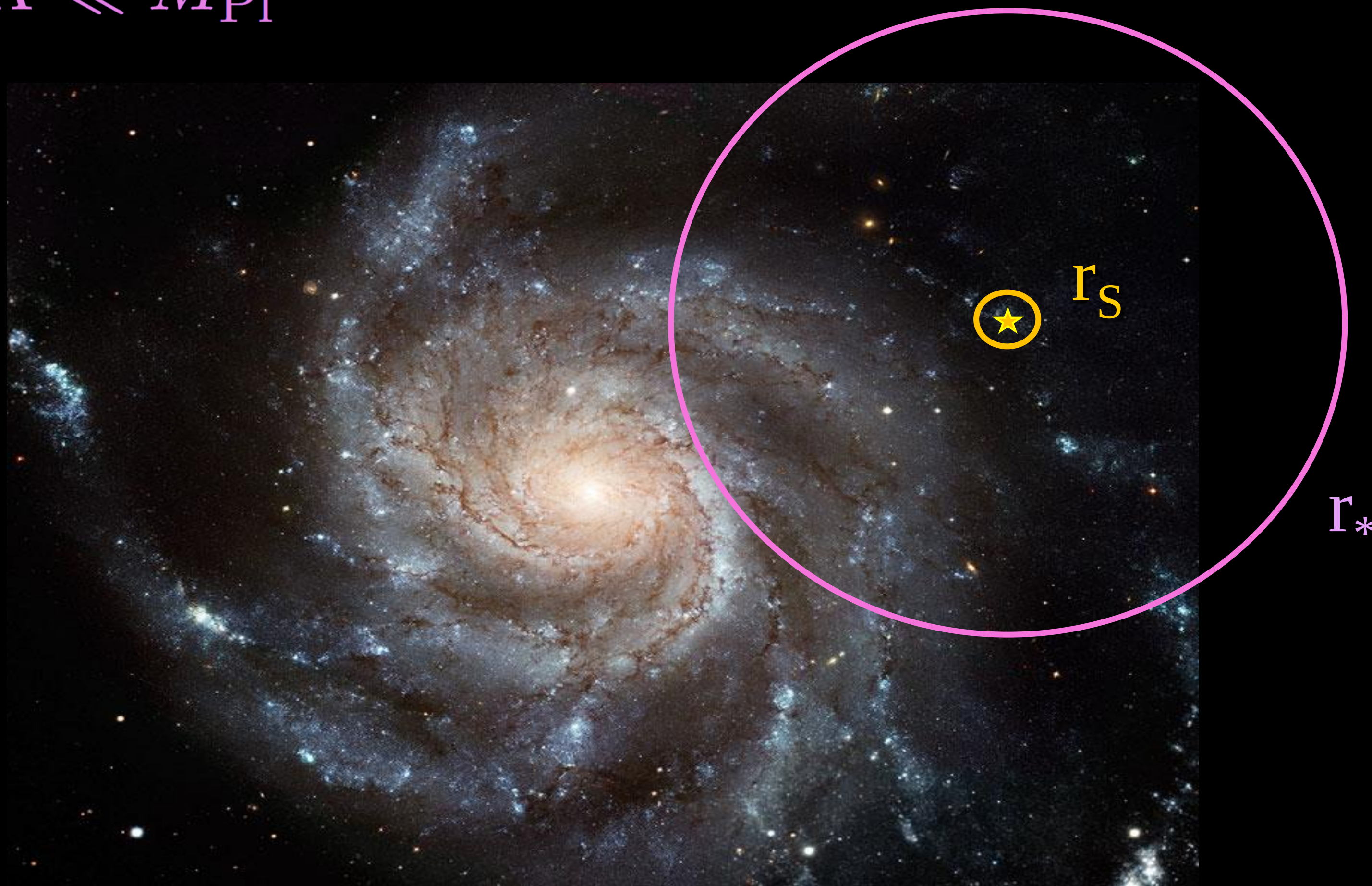
$$2 \oplus 1 \oplus 2$$

$$h_{\mu\nu} = h'_{\mu\nu} + \pi \eta_{\mu\nu}$$

Strong Coupling



- 5^{th} force constraints in the solar system imply that the extra degrees of freedom must be strongly coupled at a scale $\Lambda \ll M_{\rm Pl}$



Boulware-Deser Ghost

- Non-linearities are fundamental for the survival of the theory.

- But non-linearly, the theory seems to contain a ghost, which has been shown explicitly

1. In the ADM formalism,
   counting constraints in Hamiltonian

2. In the Stückelberg language,
   - In the decoupling limit (ghost scale ~ Λ)
   - At higher scales

1. ADM in GR

- The ghost of massive gravity was originally pointed out by Boulware and Deser, using the ADM decomposition

$$ds^2 = -N_0^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

- In GR, both the lapse and shifts play the role of Lagrange multipliers, propagating 4 constraints

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- In GR, the lapse and shifts play the role of Lagrange mult.

$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma)$$

symmetry

$$6 \times 2 - 4 - 4 = 4 = 2 \times 2 \text{ dof in field space}$$

constraints

1. BD Ghost in ADM

- In massive gravity, both the lapse and shifts enter non-linearly

$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma) + m^2\, \mathcal{U}(N_0, N_i, \gamma, P_\gamma)$$

- The Fierz-Pauli combination, ensures that the lapse remains linear at the quadratic order,

$$\begin{aligned} \mathcal{U}(h) &= h_{\mu\nu}^2 - h^2 \\ &= \cancel{\delta N}^2 + \delta N h_{ii} + \cdots \end{aligned}$$

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- The Fierz-Pauli combination, ensures that the lapse remains linear at the quadratic order, but not beyond…

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- There is no possible mass term for which the lapse remains a Lagrange multiplier

$$\begin{aligned} \mathcal{U}(h) &= h_{\mu\nu}^2 - h^2 + (\alpha h_{\mu\nu}^3 + \beta h h_{\mu\nu}^2 + \gamma h^3) + (\sigma h_{\mu\nu}^4 + \cdots) \\ &\supset \delta N h_{ii} + \delta N^2 \left(h_{ii} + h_{ii}^2 + h_{ij}^2 + N_i^2\right) + \delta N^3 h_{ii} + \delta N^4 + \cdots \end{aligned}$$

Boulware & Deser,1972
Creminelli et. al. hep-th/0505147

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constraints

5+1 dof ⟶  ghost ...

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Is that really the right criteria ???

1. BD Ghost in ADM

- Whether or not there is a constraint,

$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma) + m^2\, \mathcal{U}(N_0, N_i, \gamma, P_\gamma)$$

simply depends on the Hessian, $L_{\mu\nu} = \dfrac{\partial^2 \mathcal{U}}{\partial N^\mu \partial N^\nu}$

$$\text{constraint} \quad \Longleftrightarrow \quad \det L = 0$$

Toy Model

As an instructive toy example, we can take

$$\mathcal{H} = N_0 R_0 + N^i R_i - m^2 \underbrace{\sqrt{(1+N_0)^2 - N_i^2}}_{=1 - N_i^2 + N_0^2 + \cdots}$$

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Despite being non-linear in the lapse, there is a constraint:

$$\det L_{\mu\nu} = \#\det \left(\begin{array}{c|c} 1 - N_i^2 & (1+N_0)N_i \\ \hline (1+N_0)N_j & -((1+N_0)^2 - N_i^2)\delta_{ij} - N_i N_j \end{array} \right) = 0$$

Toy Model

As an instructive toy example, we can take

$$\mathcal{H} = N_0 R_0 + N^i R_i - m^2 \underbrace{\sqrt{(1+N_0)^2 - N_i^2}}_{= 1 - N_i^2 + N_0^2 + \cdots}$$

We could have simply redefined the shift to make the constraint transparent: $N_i = n_i(1+N_0)$

$$\mathcal{H} = N_0 R_0 + (1+N_0)\left(n^i R_i - m^2\sqrt{1-n_i^2}\right)$$

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1. In the ADM formalism,
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2. In the Stückelberg language,
   - In the decoupling limit (ghost scale ~ Λ)
   - At higher scales

2. Stückelberg language

- To give the graviton a mass, include the interactions

$$\mathcal{L} = M_{\mathrm{Pl}}^2 \left(R - \frac{m^2}{4} \mathcal{U}(h_{\mu\nu}) \right)$$

- Mass for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

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$$\phi^a = x^a - A^a \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\phi^a = x^a - \partial^a \pi - \cancel{V^a} \qquad g_{\mu\nu} = \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b + H_{\mu\nu}$$

$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi - \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

$$\longrightarrow H^n \sim (\partial\partial\pi)^n \longrightarrow$$  ghost ...

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$$H_{\mu\nu} = \frac{\hat{h}_{\mu\nu}}{M_{\rm Pl}} + 2\frac{\hat{\Pi}_{\mu\nu}}{M_{\rm Pl} m^2} - \frac{\hat{\Pi}^2_{\mu\nu}}{M_{\rm Pl}^2 m^4}$$

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

Decoupling limit

$$H_{\mu\nu} = \frac{\hat{h}_{\mu\nu}}{M_{\rm Pl}} + 2\frac{\hat{\Pi}_{\mu\nu}}{M_{\rm Pl} m^2} - \frac{\hat{\Pi}^2_{\mu\nu}}{M^2_{\rm Pl} m^4}$$

- In the decoupling limit, $M_{\rm Pl} \to \infty, \quad m \to 0$ with $\Lambda_3^3 = M_{\rm Pl} m^2$ fixed,

$$\mathcal{U}(h_{\mu\nu}, \pi) = \mathcal{U}|_{h_{\mu\nu}=0} + \frac{1}{M_{\rm Pl}} \hat{h}_{\mu\nu} X^{\mu\nu}(\pi) + \frac{1}{M^2_{\rm Pl}} \hat{h}^2_{\mu\nu} \cdots$$

The ghost can be avoided in that limit, if $\mathcal{U}|_{h_{\mu\nu}=0}$ is a total derivative

Decoupling limit

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- $\mathcal{U}|_{h_{\mu\nu}=0}$ is a total derivative, for instance if

$$\mathcal{U} = (\partial_\alpha \partial_\beta \pi)^2 - (\Box \pi)^2$$

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- $\mathcal{U}|_{h_{\mu\nu}=0}$ is a total derivative, for instance if

$$\boxed{\mathcal{U} = \mathcal{K}^2_{\alpha\beta} - (\mathcal{K}^\alpha_\alpha)^2}$$

- In the decoupling limit, $H_{\mu\nu}|_{\rm dec} = 2\Pi_{\mu\nu} - \Pi^2_{\mu\nu}$

or $\Pi_{\mu\nu} = \mathcal{K}_{\mu\nu}|_{\rm dec}$ with $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$

Ghost-free theory

- The mass term

$$\mathcal{U}(H_{\mu\nu}) = \mathcal{K}^\mu_\nu \mathcal{K}^\nu_\mu - \mathcal{K}^2$$

with $\partial_\mu \partial_\nu \pi = \mathcal{K}_{\mu\nu}\Big|_{\rm dec}$ $\qquad$ $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$

- Has no ghosts in the decoupling limit:

$$\mathcal{U}(H_{\mu\nu}) \sim \left((\partial_\mu \partial_\nu \pi)^2 - (\Box \pi)^2\right) + \frac{\hat{h}_{\mu\nu}}{M_{\rm Pl}}(\partial\partial\pi) + \cdots$$

CdR, Gabadadze, Tolley, 1011.1232

Ghost-free decoupling limit

- In the *decoupling limit* (keeping $\Lambda_3^3 = M_{\rm Pl} m^2$ fixed)

$$\mathcal{L} = -\frac{1}{2}\hat{h}^{\mu\nu}(\mathcal{E}\hat{h})_{\mu\nu} - \hat{h}^{\mu\nu}\left(X^{(1)}_{\mu\nu} + \frac{1}{\Lambda_3^3}X^{(2)}_{\mu\nu} + \cdots\right)$$

with

$$X^{(1)}_{\mu\nu} = \partial_\mu\partial_\nu\hat{\pi} - \Box\hat{\pi}\eta_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} \sim (\partial_\mu\partial_\nu\hat{\pi})^2 + \cdots$$

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- The Bianchi identity requires $\partial^\mu X^{(i)}_{\mu\nu} = 0$
- The decoupling limit stops at 2nd order.
- $X^{(i)}_{\mu\nu}$ are at most 2nd order in derivative

→ NO GHOSTS in the decoupling limit

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- The Bianchi identity requires $\partial^\mu X^{(i)}_{\mu\nu} = 0$
- The decoupling limit stops at 2nd order.
- $X^{(i)}_{\mu\nu}$ are at most 2nd order in derivative
- These mixings can be removed by a local field redefinition

$$\hat{h}_{\mu\nu} = \bar{h}_{\mu\nu} + \hat{\pi}\eta_{\mu\nu} + \frac{1}{\Lambda_3^3}\partial_\mu\hat{\pi}\partial_\nu\hat{\pi}$$

Galileon in disguise

Galileon in disguise

- For a stable theory of massive gravity, the decoupling limit is

- The interactions have 3 special features:

$$\mathcal{L} = -\frac{1}{2}\bar{h}^{\mu\nu}(\mathcal{E}\bar{h})_{\mu\nu} + (\partial\hat{\pi})^2\left(1+\frac{\Box\pi}{\Lambda_3^3}+\frac{(\Box\pi)^2+\cdots}{\Lambda_3^6}\right)$$

1. They are local
2. They possess a Shift $\pi \longrightarrow \pi + c$
   and a Galileon symmetry $\pi \longrightarrow \pi + c_\mu x^\mu$
3. They have a well-defined Cauchy problem
   (eom remain 2^{nd} order)

- Corresponds to the Galileon family of interactions

Coupling to matter $\left(\bar{h}_{\mu\nu} + \hat{\pi}\eta_{\mu\nu} + \frac{1}{\Lambda_3^3}\partial_\mu\hat{\pi}\partial_\nu\hat{\pi}\right) T^{\mu\nu}$

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$\mu \, x^{\mu}$

3. They h
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The BD ghost can be pushed beyond the scale Λ_3

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Ghost-free theory

- There exist actually a 2-parameter family of theories:

$$\mathcal{U}(H_{\mu\nu}) = \mathcal{K}^\mu_\nu \mathcal{K}^2_{\mu\nu} - \mathcal{K}^2 + a_1 \left(\mathcal{K}^3 - 3\mathcal{K}^2_{\mu\nu}\mathcal{K} + 2\mathcal{K}^3_{\mu\nu}\right) + a_2 \left(\mathcal{K}^4 + \cdots\right)$$

with $\partial_\mu \partial_\nu \pi = \mathcal{K}_{\mu\nu}\Big|_{\rm dec}$ $\qquad \mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$

- Leading to the entire family of ghost-free Galileon interactions in the decoupling limit.

CdR, Gabadadze, Tolley, 1011.1232

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- Leading to the entire family of ghost-free Galileon interactions in the decoupling limit.

Is that enough ???

Beyond de decoupling limit

Consider a 2d toy-model,

$$\mathcal{U}(H_{\mu\nu}) = \mathcal{K}^{\mu}_{\nu}\mathcal{K}^{\nu}_{\mu} - \mathcal{K}^2 \qquad g_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + H_{\mu\nu}$$

for simplicity we work in the LIF,

$$\mathcal{U} = \sqrt{(\partial_0\phi^0 + \partial_1\phi^0 + \partial_0\phi^1 + \partial_1\phi^1)(\partial_0\phi^0 - \partial_1\phi^0 - \partial_0\phi^1 + \partial_1\phi^1)}$$

Both ϕ^0 and ϕ^1 propagate dynamical equations…

However they are not independent

$$(\partial_0\phi^0 + \partial_1\phi^1)\frac{\delta\mathcal{U}}{\delta\phi^0} + (\partial_1\phi^0 + \partial_0\phi^1)\frac{\delta\mathcal{U}}{\delta\phi^1} \equiv C$$

There is still a constraint !

Back to the BD ghost…

- We now set unitary gauge, $\phi^a = x^a$ $(\pi = 0)$. In ADM split,

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij} \left(\mathrm{d}x^i + N^i \mathrm{d}t\right) \left(\mathrm{d}x^j + N^j \mathrm{d}t\right)$$

with $N = 1 + \delta N$

- The lapse enters quadratically in the Hamiltonian,

$$\mathcal{H} \supset N_i R^i + m^2 N_i^2 \left[-2 + \delta N - \frac{1}{8} N_k^2 - \frac{1}{2} \delta N^2 \right]$$

Does it really mean that the constraint is lost ?

Boulware & Deser,1972
Creminelli et. al. hep-th/0505147

Back to the BD ghost…

$$\mathcal{H} \supset N_i R^i + m^2 N_i^2 \Big[-2 + \delta N - \frac{1}{8} N_k^2 - \frac{1}{2} \delta N^2 \Big]$$

- The constraint is manifest after integrating over the shift

$$\mathcal{H} \supset \frac{R_i^2}{8m^2} \left(1 + \frac{1}{2}\delta N - \mathcal{O}(R_i^2/m^4) \right) + (1 + \delta N) R_0$$

- This can be shown
  - at least up to 4th order in perturbations
  - completely non-linearly in simplified cases
    - in 2d
    - for conformally flat spatial metric

Summary of BD ghost

We can construct an explicit theory of massive gravity which:

1. Exhibits the Galileon interactions in the decoupling limit (→ has no ghost in the decoupling limit)

2. Propagates a constraint perturbations (does not excite the 6th BD mode to that order)

   at least up to 4th order in and indicates that the same remains true to all orders

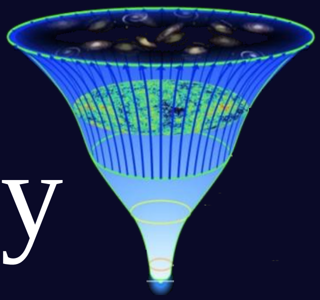
   to all orders for a conformally flat spatial metric

3. Whether or not the constraint propagates is yet unknown. secondary constraint ?

4. Symmetry ???

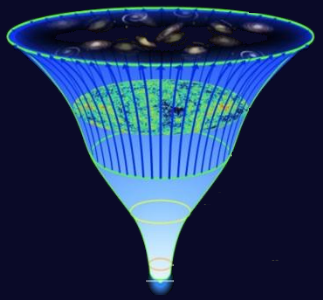
CdR, Gabadadze, Tolley, in progress…

Consequences for Cosmology



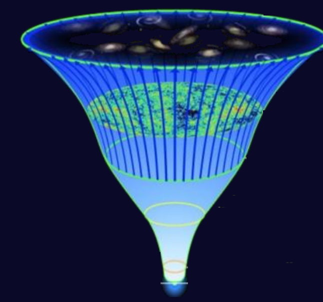
1. For late time acceleration

2. Inflation ?



C.C. Problem

$$\mathcal{L} = -\frac{1}{2}\hat{h}\Box\hat{h} + \frac{1}{2}\hat{h}^{\mu\nu}\,T^{(\mathrm{eff})}_{\mu\nu}(\partial\partial\pi) + \frac{1}{2}\hat{h}^{\mu\nu}\,T^{(\mathrm{source})}_{\mu\nu}$$



Degravitation

$$\mathcal{L} = -\frac{1}{2}\hat{h}\Box\hat{h} + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{eff})}_{\mu\nu}(\partial\partial\pi) + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{source})}_{\mu\nu}$$

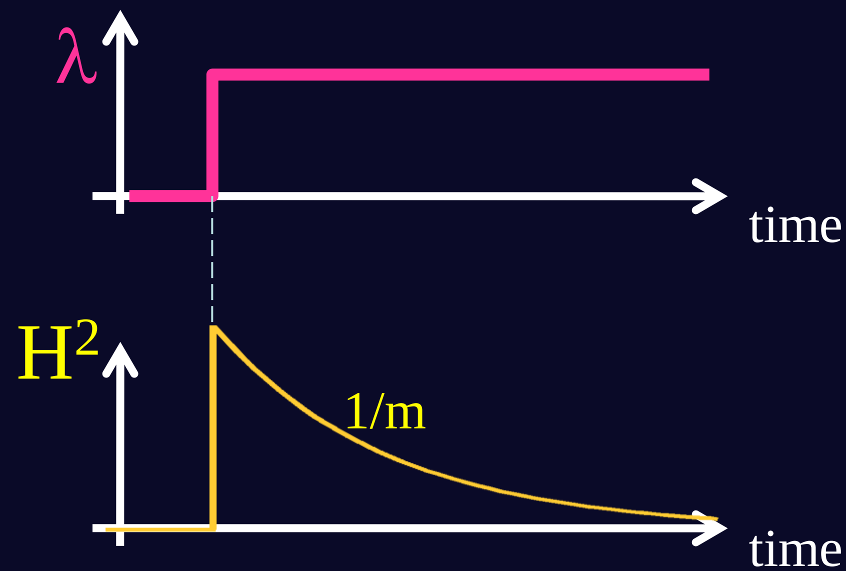
Screening the CC

$$T^{(\mathrm{source})}_{\mu\nu} = -\lambda g_{\mu\nu}$$

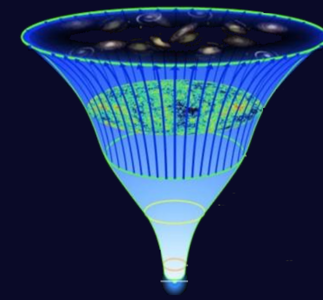
$$\lambda \sim M_{\mathrm{Pl}}^4$$

$$T^{(\mathrm{eff})}_{\mu\nu} = -T^{(\mathrm{source})}_{\mu\nu}$$

Could relax towards a flat geometry even with a large CC



$$H \longrightarrow 0$$



Dark Energy

$$\mathcal{L} = -\frac{1}{2}\hat{h}\Box\hat{h} + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{eff})}_{\mu\nu}(\partial\partial\pi) + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{source})}_{\mu\nu}$$

Screening the CC

$$T^{(\mathrm{source})}_{\mu\nu} = -\lambda g_{\mu\nu}$$

$$\lambda \sim M^4_{\mathrm{Pl}}$$

$$T^{(\mathrm{eff})}_{\mu\nu} = -T^{(\mathrm{source})}_{\mu\nu}$$

Could relax towards a flat geometry even with a large CC

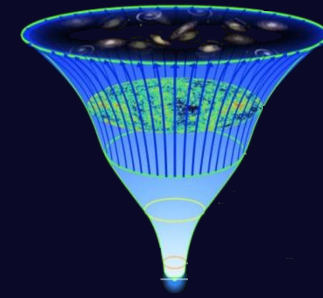
Self-acceleration

$$T^{(\mathrm{source})}_{\mu\nu} = 0$$

$$T^{(\mathrm{eff})}_{\mu\nu} \sim -\Lambda^3_\star\, g_{\mu\nu}$$

Source the late time acceleration

$$H \sim m$$



Dark Energy

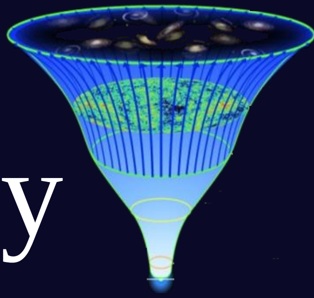
$$\mathcal{L} = -\frac{1}{2}\hat{h}\Box\hat{h} + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{eff})}_{\mu\nu}(\partial\partial\pi) + \frac{1}{2}\hat{h}^{\mu\nu}\, T^{(\mathrm{source})}_{\mu\nu}$$

Screening the CC | **Self-acceleration**

- Which branch is possible depends on parameters
- Branches are stable and ghost-free (unlike self-accelerating branch of DGP)
- In the screening case, solar system tests involve a max CC to be screened.

CdR, Gabadadze, Heisenberg, Pirtskhalava, 1010.1780

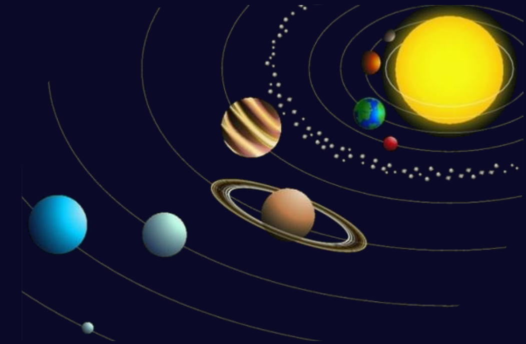
Consequences for Cosmology



1. For late time acceleration

2. Inflation ?

EFT and relevant operators



- Higher derivative interactions are essential for the viability of this class of models.

- Within the solar system, π reaches the scale Λ_*, yet, we are still within the regime of validity of the theory

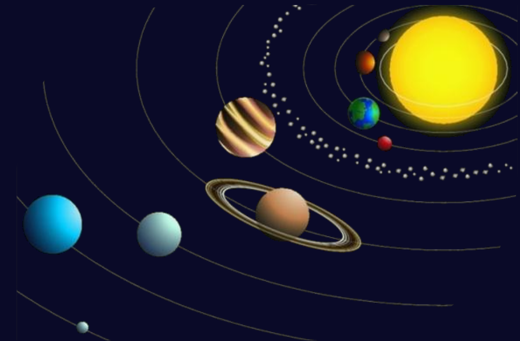
$$3\Box\hat{\pi} + \frac{\alpha}{\Lambda_\star^3}\left((\Box\hat{\pi})^2 + \cdots\right) + \frac{\beta}{\Lambda_\star^6}\left((\Box\hat{\pi})^3 + \cdots\right) + \frac{\gamma}{\Lambda_\star^9}\left((\Box\hat{\pi})^4 + \cdots\right) = -\hat{T}$$

$$+\frac{(\partial^3\pi)^2}{\Lambda_\star^5} + \frac{\Box\pi\Box^2\pi}{\Lambda_\star^5} + \ldots$$

Vainshtein, Phys. Lett. B 39 (1972) 393
Babichev, Deffayet & Ziour, 0901.0393

Luty, Porrati, Rattazzi hep-th/0303116
Nicolis & Rattazzi, hep-th/0404159

EFT and relevant operators



- Higher derivative interactions are essential for the viability of this class of models.

- Within the solar system, π reaches the scale Λ_*, yet, we are still within the regime of validity of the theory

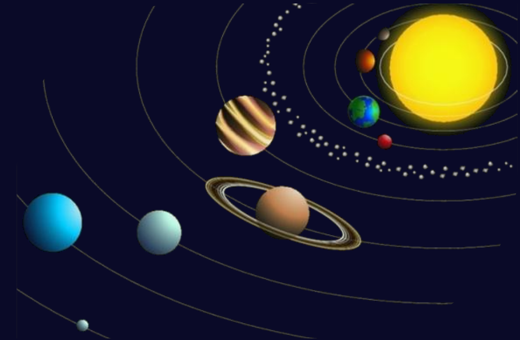
- The breakdown of the EFT is not measured by "$\partial\pi$" but by "∂" itself $\longrightarrow$ gradients should be small

- So we can trust a regime where $\partial\partial\pi \sim \Lambda_\star^3$

  as long as $\partial\partial\partial\pi \ll \Lambda_\star^4$

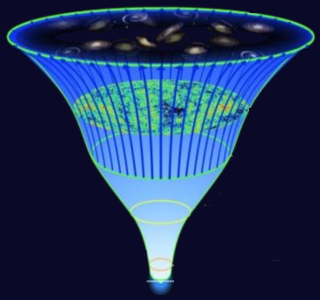
Luty, Porrati, Rattazzi hep-th/0303116
Nicolis & Rattazzi, hep-th/0404159

EFT and relevant operators



Can we use these ideas to build
a radiatively stable
model of inflation ?

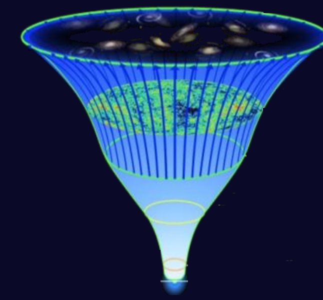
Galileon Inflation



- Model of Inflation grounded on the Galileon Symmetry

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots + \lambda^3\phi$$

Galileon Inflation



- Model of Inflation grounded on the Galileon Symmetry

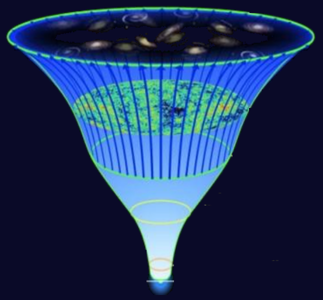
$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + \frac{2\alpha}{\Lambda^3} H \dot{\phi}^3 + \cdots + \lambda^3 \phi$$

- Shift symmetry guarantees the conservation of ζ outside the horizon ⟶ time independent power spectrum

- Non-renormalization theorem allows us to consider these interactions to be large, without breaking the regime of validity of EFT.

Nicolis & Rattazzi, hep-th/0404159

Galileon Inflation



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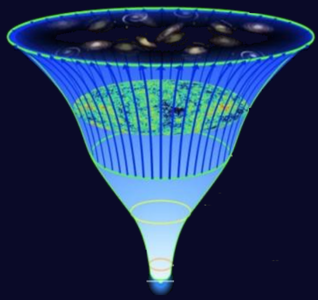
- Shift symmetry guarantees the conservation of ζ outside the horizon $\longrightarrow$ time independent power spectrum

- Non-renormalization theorem allows us to consider these interactions to be large, without breaking the regime of validity of EFT.

- Remains true with a mass term



Strong coupling

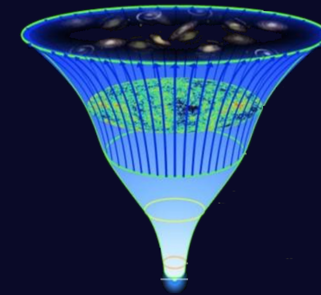
- When the interactions are important, $Z = \frac{H\dot{\phi}}{\Lambda^3} \gtrsim 1$

  the inflationary phase satisfies $H^2\dot{\phi}^2 \sim \Lambda^3\lambda^3$

- And perturbations are given by $t \to t + \xi(x,t)$

$$\mathcal{L} \supseteq a^3 \left[\alpha \left(\dot{\xi}^2 - \frac{c_s^2}{a^2}(\partial_i \xi)^2 \right) + g_1 \dot{\xi}^3 + \frac{g_2}{a^2}\dot{\xi}(\partial_i \xi)^2 + \frac{g_3}{a^4}(\partial_i \xi)^2 \partial_j^2 \xi \right]$$

Strong coupling



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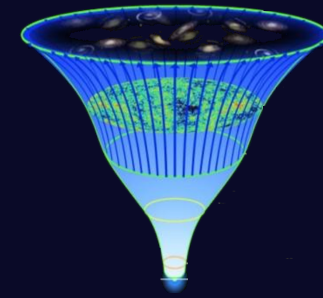
dim - 6   dim - 7

Non-Gaussianities



$$+g_1\dot{\xi}^3 + \frac{g_2}{a^2}\dot{\xi}(\partial_i\xi)^2 + \frac{g_3}{a^4}(\partial_i\xi)^2\partial_j^2\xi$$

$$f_{\rm NL} \sim \quad \frac{H}{\alpha} \qquad \frac{H}{\alpha c_S^2} \qquad \frac{H^2}{\alpha c_S^4}$$

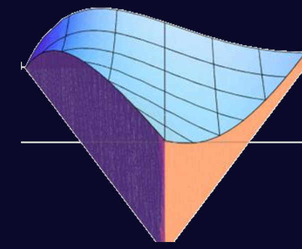
- The operator 7 can be of the same order as the others if the dim-6 operators are suppressed by additional H/Λ.

- This suppression is stable thanks to the Galileon symmetry.

Potentially "large" nG, but with no specific shape.

Burrage, CdR, Seery, Tolley, 1009.2497

New shapes of non-Gaussianity



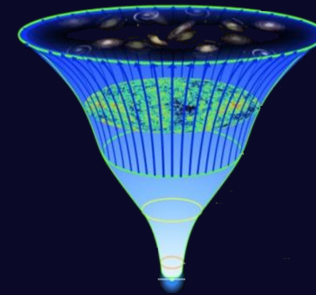
- The situation is different, if the leading interactions are tuned to vanish

- Leading interactions then arise from $(\partial^2\phi)^n$ terms

- nG can rely on dim-9 operators
  specific shapes nG:
  - 2 operators lead to ~ equilateral triangles
  - 1 to flattened isocele triangle.

Creminelli, d'Amicob, Musso, Norena and Trincherini, 1011.3004

Summary



- Galileon interactions arise naturally
  - in braneworlds with induced curvature (soft mass gravity)
  - in hard massive gravity with no ghosts in the dec. limit

- The Galileon can play a crucial role in (stable) models of self-acceleration…

- …or provide a framework for the study of degravitation

- On different scales, it can provide a radiatively stable model of inflation leading to potentially large nG...

- ... Similar in spirit than DBI, but with different signatures