String and Supergravity Motivated Cosmology

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Stanford

Crete, September 7, 2004

Outline

1. **Cosmological “Concordance Model”**

2. String Theory- Cosmology: **KKLT model of de Sitter space**

3. **Early Universe Inflation in String Theory**

   a) Racetrack Inflation

   b) KKLMMT model of D3-anti-D3 Brane Inflation

   c) Hybrid Inflation in D3/D7 Brane System

Fundamental Physics

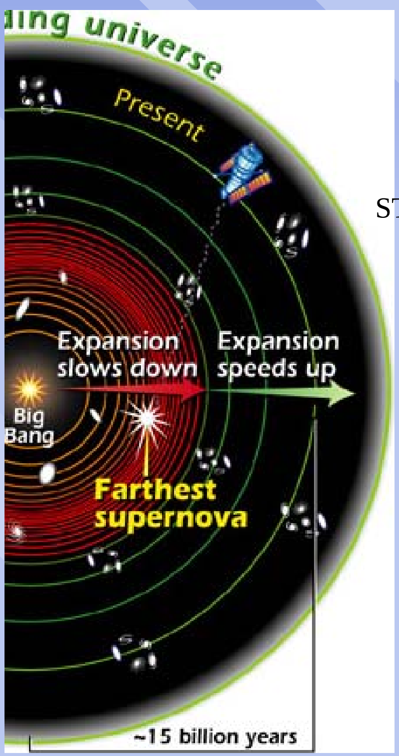
Astrophysics → Cosmology → Field Theory

$a(t)$ → Equation of state $w(z)$ → $V(\phi)$

SN
CMB
LSS

$V(\phi(a(t)))$

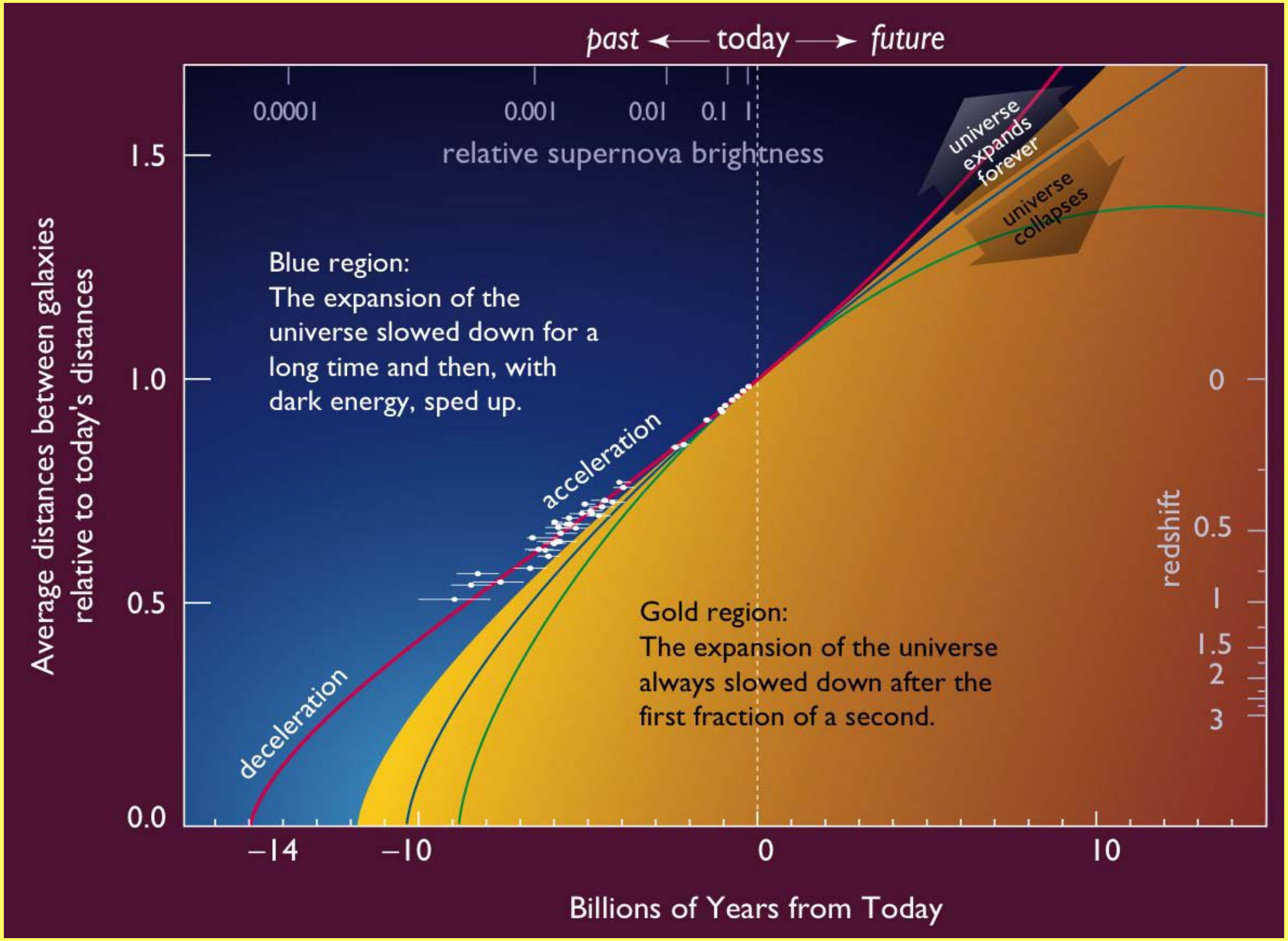
The subtle slowing and growth of scales with time – $a(t)$ – map out the cosmic history like tree rings map out the Earth's climate history.



STScI

Map the expansion history of the universe

Discovery! Acceleration



data from Supernova Cosmology Project (LBL)

graphic by Barnett, Linder, Perlmutter & Smoot (for OSTP)

Exploding stars – supernovae – are bright beacons that allow us to measure precisely the expansion over the last 10 billion years.

What Do We Know

“ ‘Most embarrassing observation in physics’ – that’s the only quick thing I can say about dark energy that’s also true.” -- Edward Witten

Dark energy causes acceleration -- “negative gravity” -- through its strongly negative pressure.

Define equation of state ratio by

w(z)=pressure/(energy density)

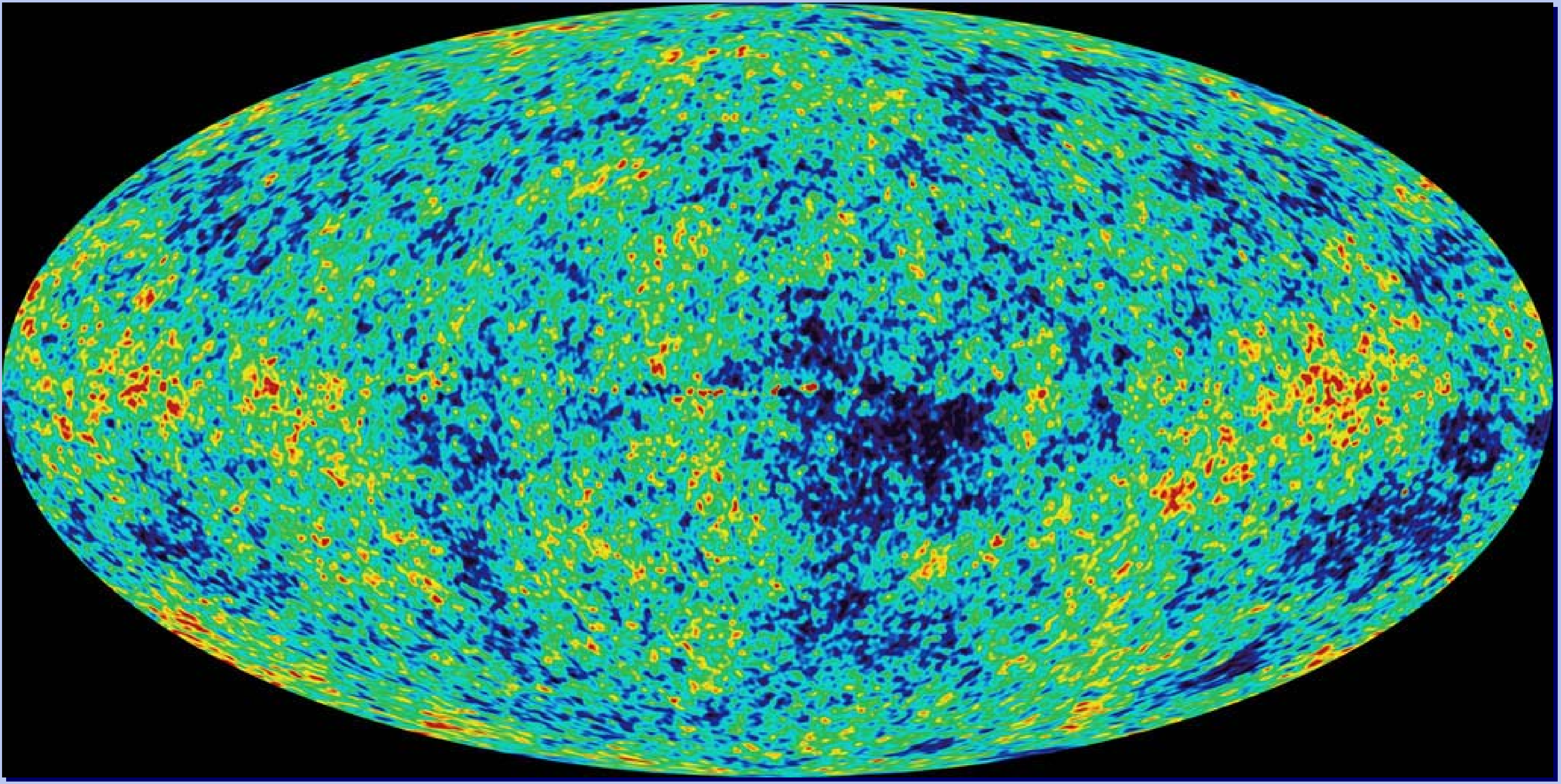
Today’s state of the art:

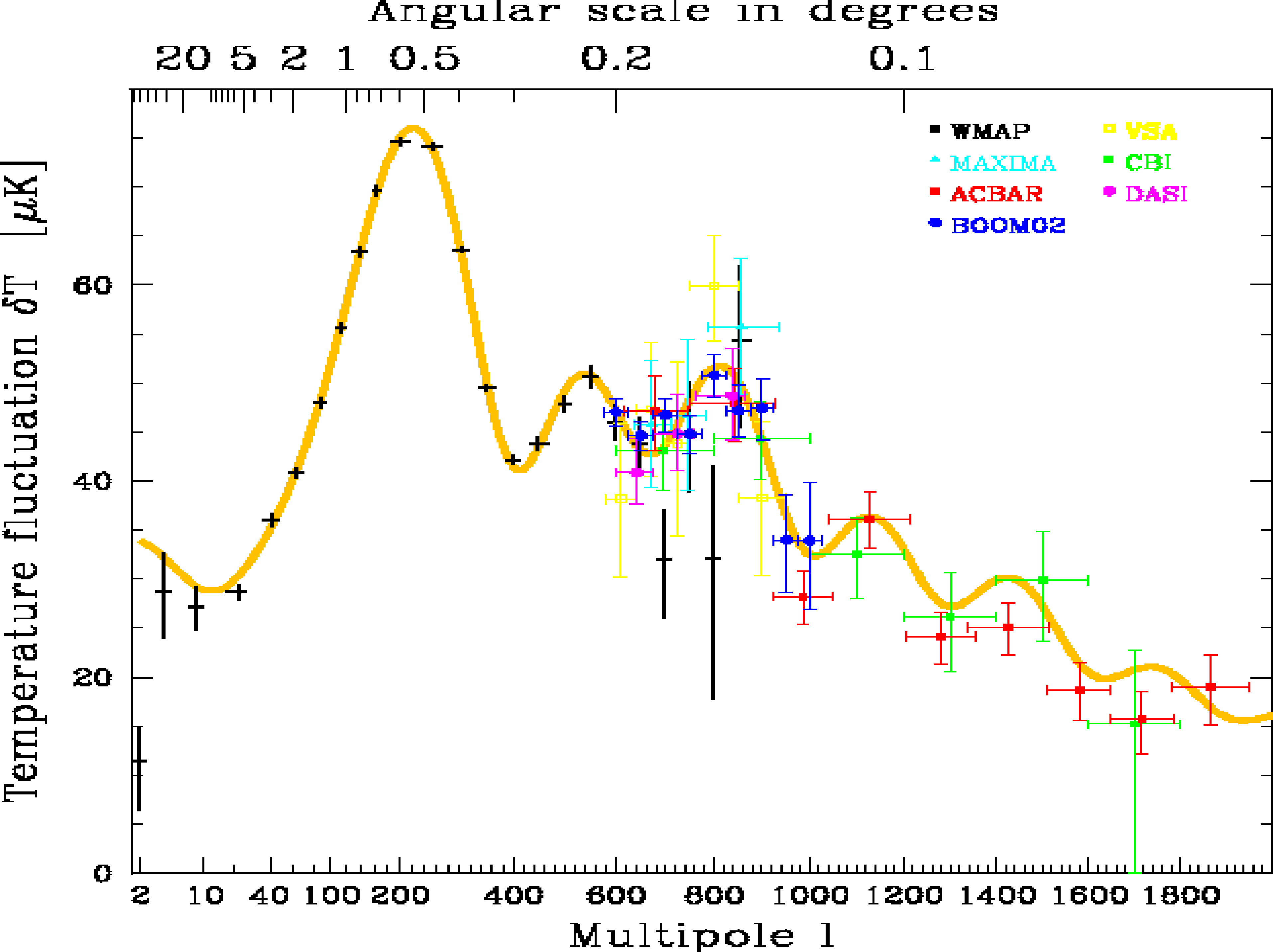
$w_{const} = -1.05^{+0.15}_{-0.20} \pm 0.09$ (Knop et al. 2003) [SN+LSS+CMB]

$w_{const} = -1.08^{+0.18}_{-0.20} \pm ?$ (Riess et al. 2004) [SN+LSS+CMB]

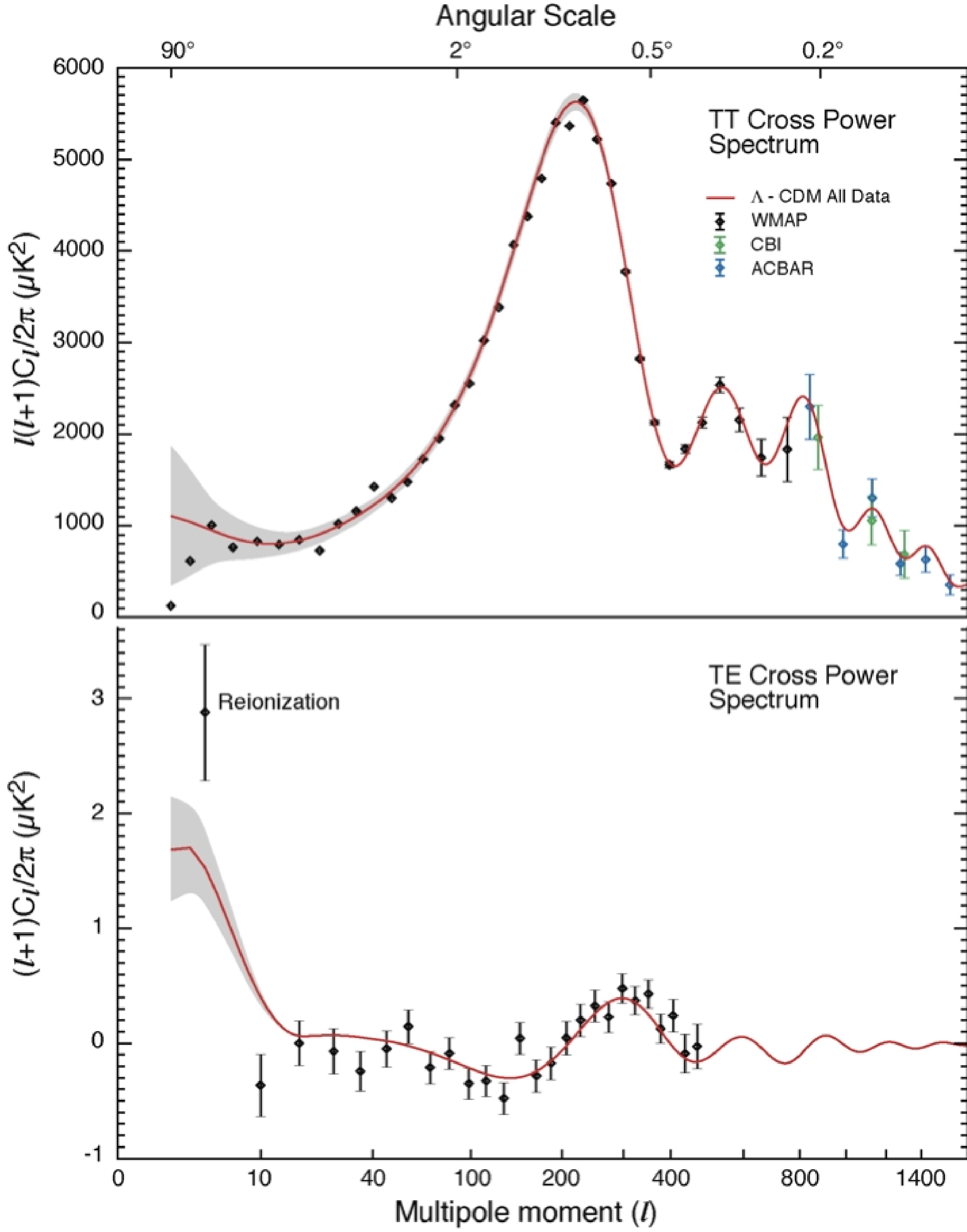
But what about dynamics? Generically expect time variation w'

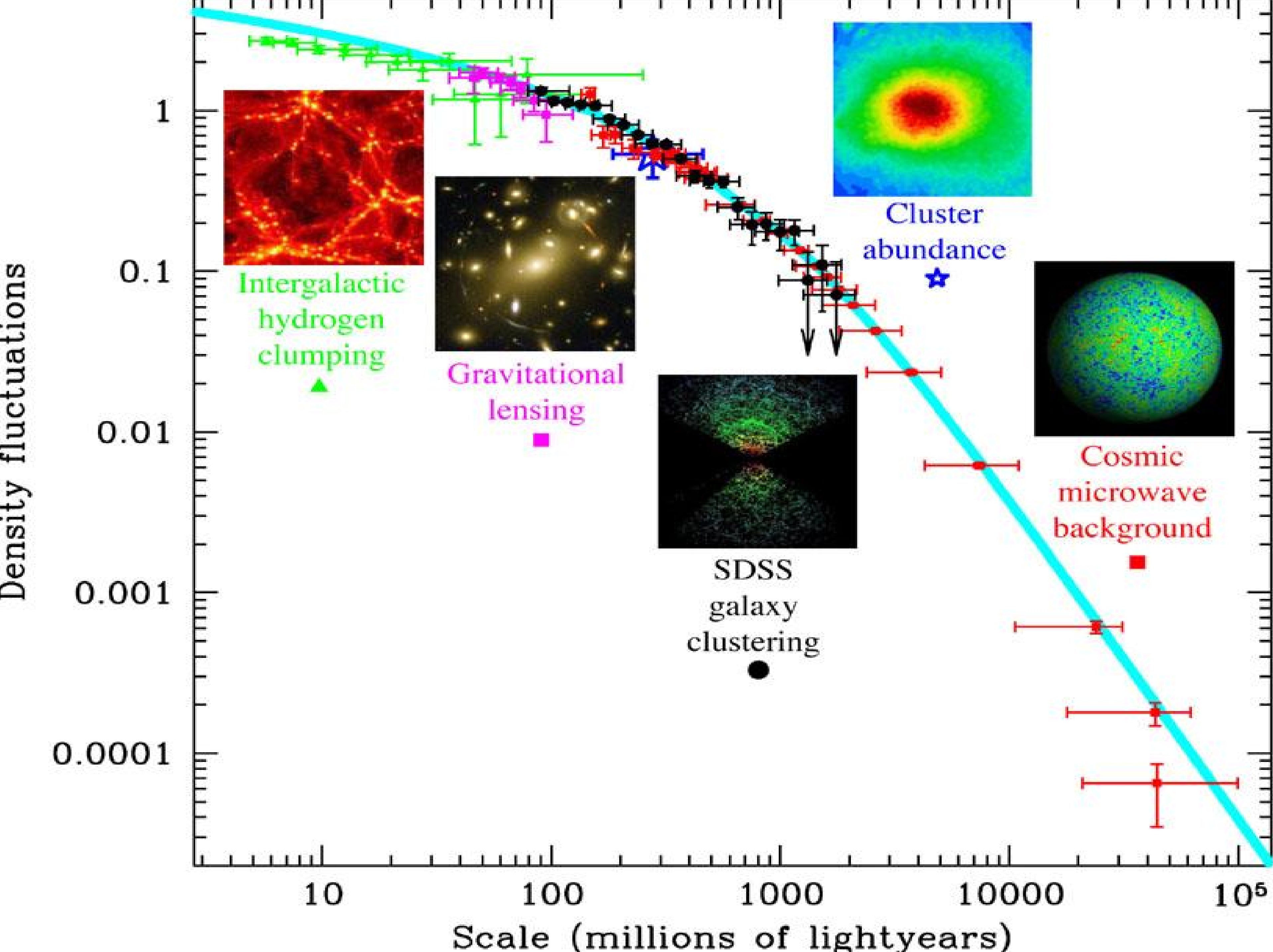
WMAP and the temperature of the sky




Angular scale in degrees
20 5 2 1 0.5 0.2 0.1
WMAP
VSA
MAXIMA
CBI
ACBAR
DASI
BOOM02
Temperature fluctuation δT [μK]
0
20
40
60
2 10 40 100 200 400 600 800 1000 1200 1400 1600 1800
Multipole l

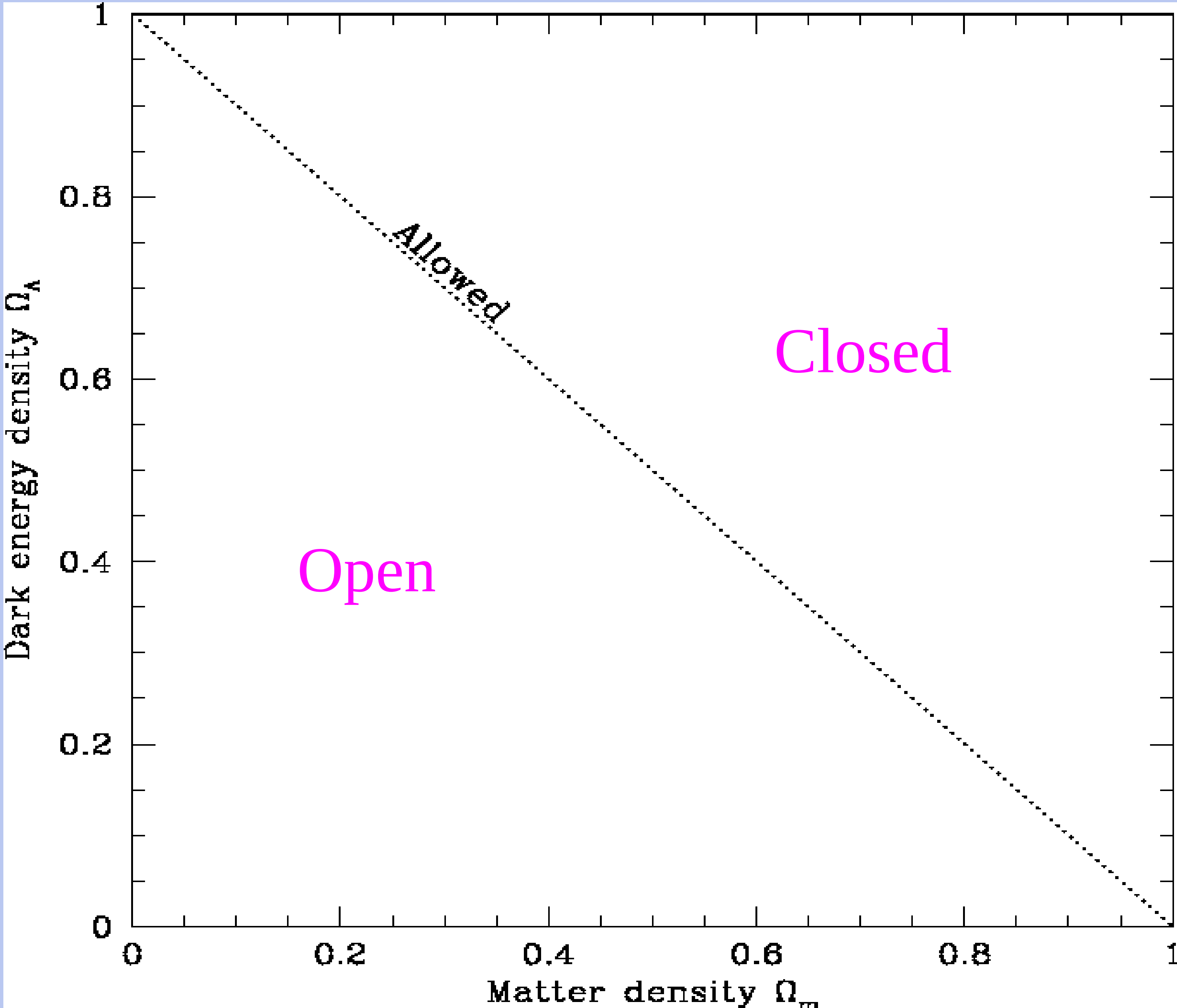
WMAP
and spectrum of the microwave background anisotropy





Density fluctuations
1
0.1
0.01
0.001
0.0001
10
100
1000
10000
10^5
Scale (millions of lightyears)
Intergalactic hydrogen clumping
Gravitational lensing
SDSS galaxy clustering
Cluster abundance
Cosmic microwave background

How much dark energy is there?



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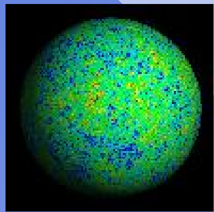
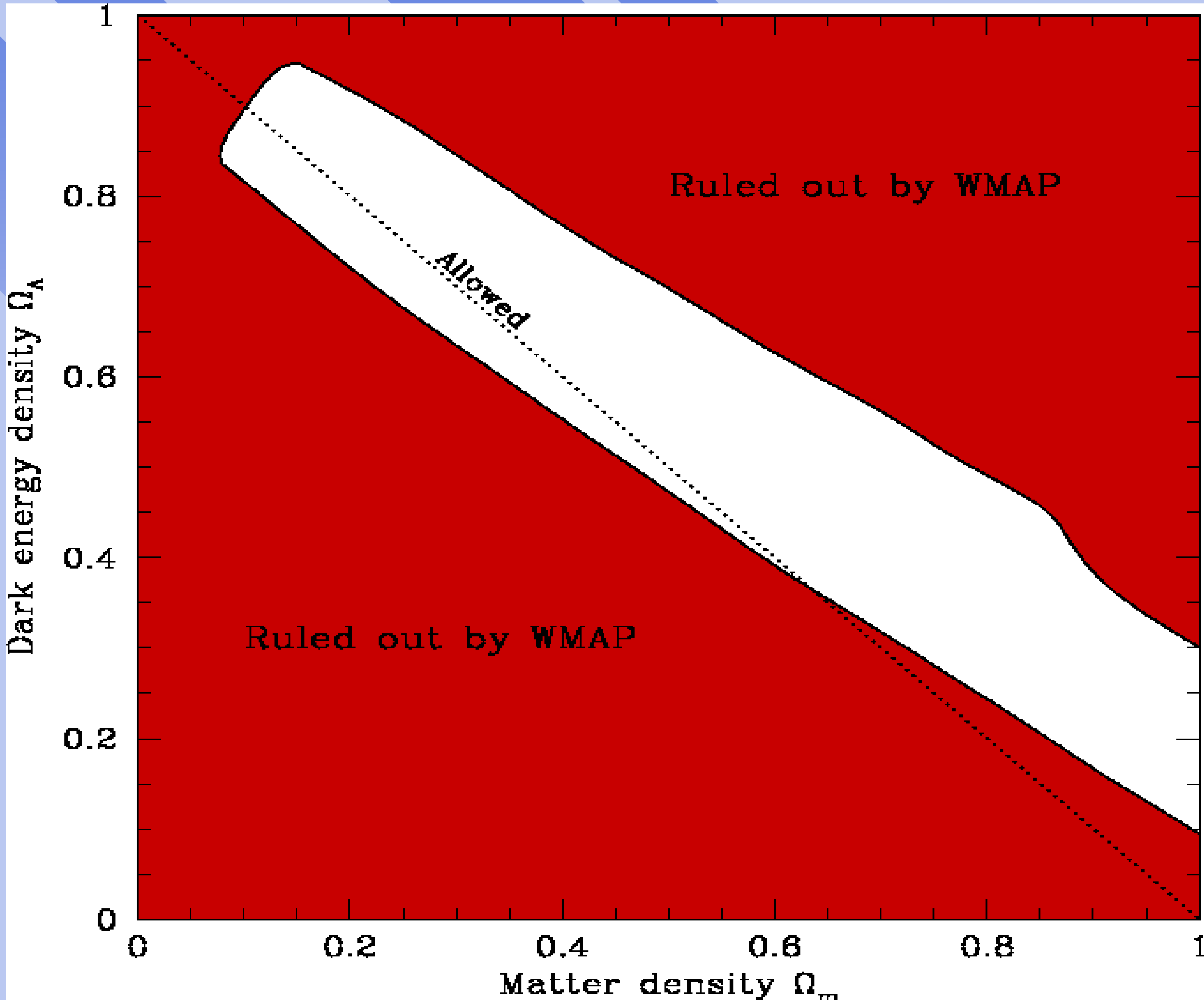
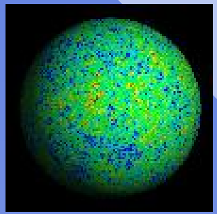


CMB

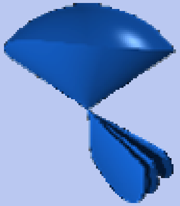


How much dark energy is there?

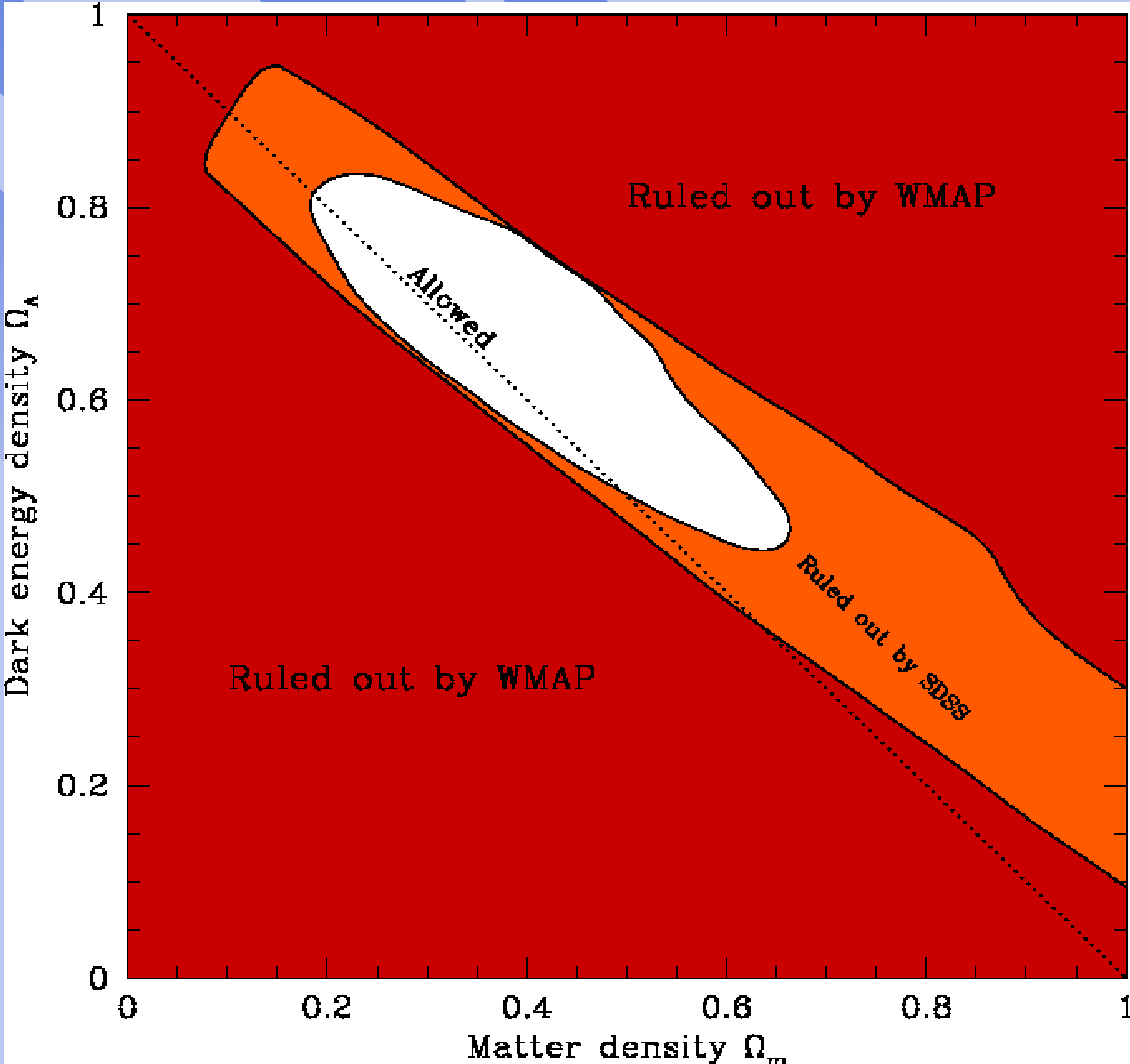


CMB

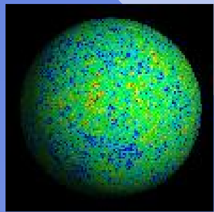
+



LSS

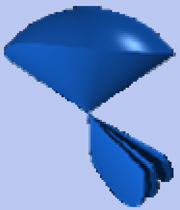


How much dark energy is there?

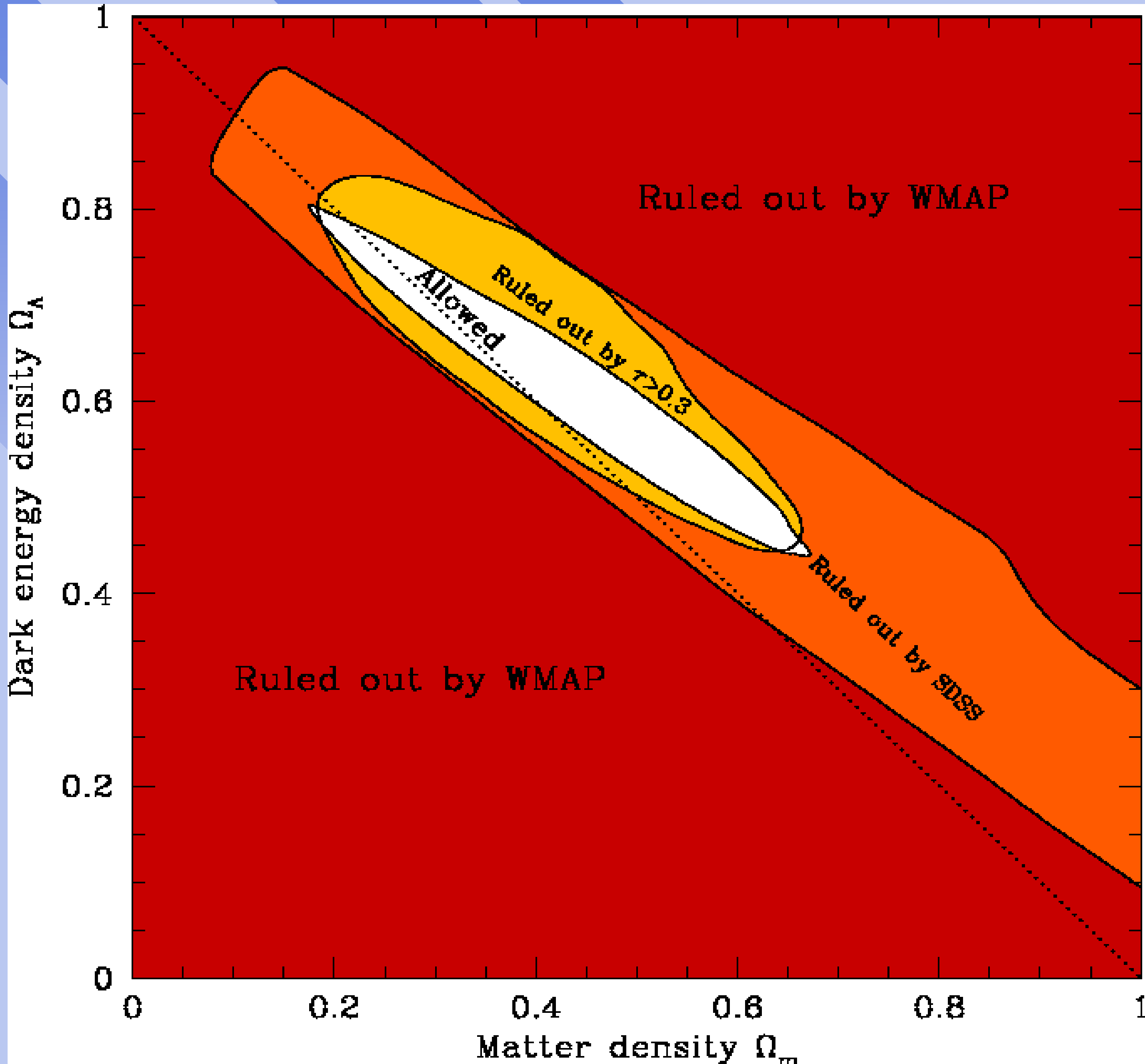


CMB

+



LSS



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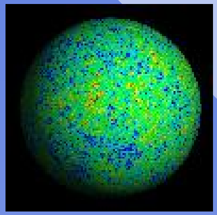
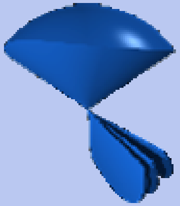
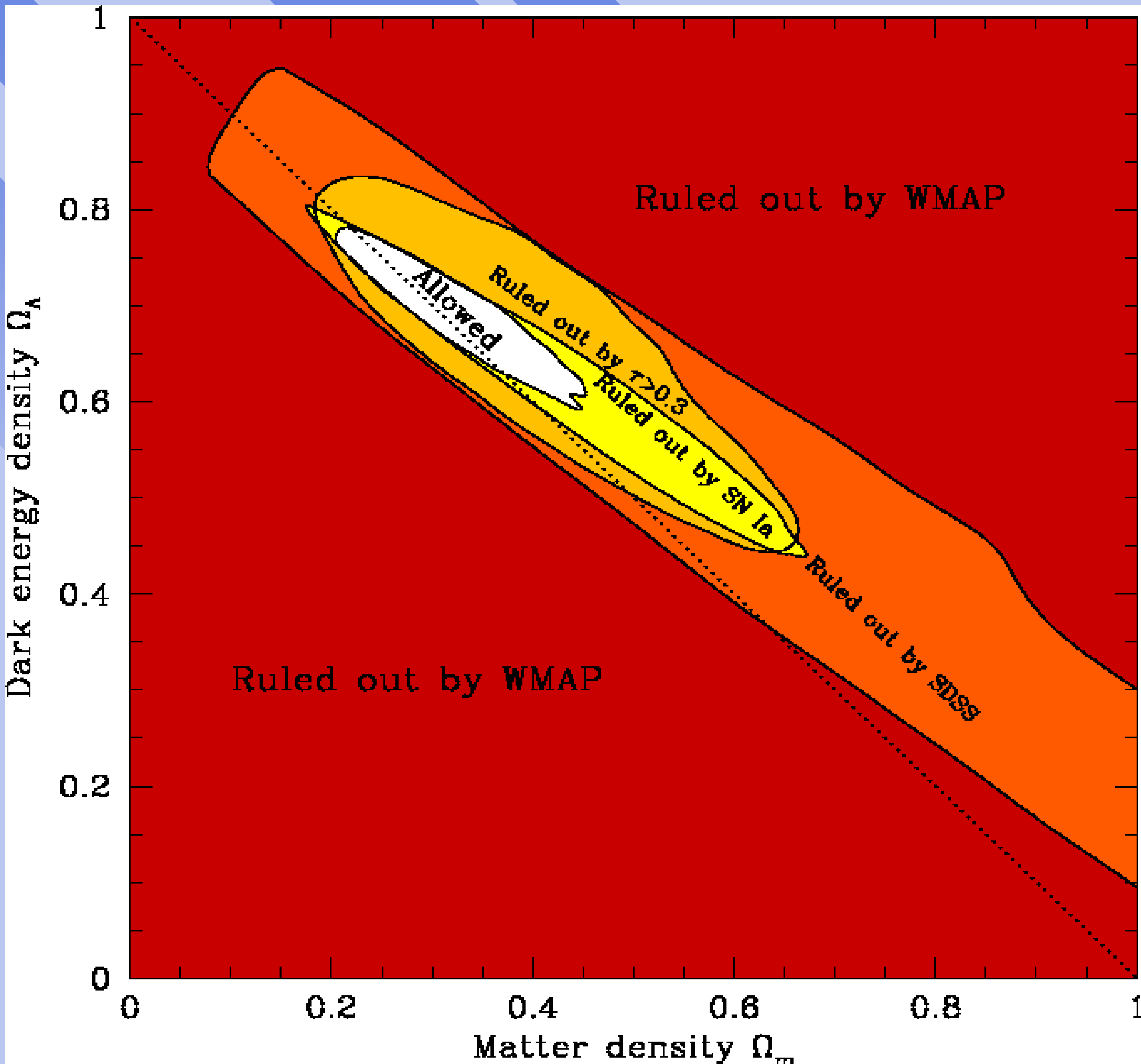


CMB

+



LSS



DARK ENERGY

Total energy in 3d flat FRW universe

$$\Omega_D \sim 0.7$$

$$\Omega_T = \Omega_D + \Omega_M = 1$$

70% of the total energy of the universe is DARK

Cosmological Constant (CC) Problem

The simplest form of dark energy: **CC**

$$V = \Lambda = 3H^2 M_P^2 > 0 \qquad H = 10^{-60} M_P$$

$$\Lambda = 10^{-120} M_P^4$$

New data

Second year WMAP, Fall 2004

Planck, SNAP, LSST ..., 2010-2012

It is likely that 70% of Dark Energy and Early Universe Inflation will be confirmed, but we have to wait

Cosmological Concordance Model

Early Universe Inflation

Near de Sitter space

13.7 billion years ago

During 10^{-35} sec

Current Acceleration

Near de Sitter space

Now

During few billion years

$$\frac{\dot{a}}{a} = H \approx \text{const}$$

$$V \sim H^2 M_P^2$$

$$H_{infl} \leq 10^{-5} M_p$$

$$V \sim H^2 M_P^2$$

$$H_{accel} \sim 10^{-60} M_P$$

$$\frac{\ddot{a}}{a} > 0$$

String Theory and Cosmology

All observations fit 4d Einstein GR: how to get this picture from the compactified fundamental 10d string theory or 11d M-theory and supergravity

How to get de Sitter or near de Sitter 4d space?

$$H_{infl} \leq 10^{-5} M_p \qquad H_{accel} \sim 10^{-60} M_P$$

Towards cosmology in type IIB string theory

Dilaton stabilization   Giddings, Kachru and Polchinski 2001

Volume stabilization, KKLT   Kachru, R. K, Linde, Trivedi 2003

Kachru, R. K., Maldacena, McAllister, Linde and Trivedi 2003

The KKLMMT model

Can string theory afford runaway moduli: a dilaton and the total volume?

No-scale supergravity has **non-canonical kinetic terms**

$$K = -\ln(-i(\tau - \bar{\tau})) - 3\ln(-i(\rho - \bar{\rho}))$$
$$V = e^K V_0$$

$$-\frac{\partial\tau\partial\bar{\tau}}{(\tau-\bar{\tau})^2} - 3\frac{\partial\rho\partial\bar{\rho}}{(\rho-\bar{\rho})^2} \qquad \tau = a + ie^{\phi} \qquad \rho = \alpha + i\sigma$$

To compare with observations one should switch to canonical kinetic terms for the dilaton and the volume, ignore axions for simplicity

$$-\frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{2}(\partial\tilde{\sigma})^2 - e^{-\sqrt{2}\,\tilde{\phi}-\sqrt{6}\,\tilde{\sigma}}V_0$$

Compare with observations

$$-\frac{1}{2}(\partial\varphi)^2 - e^{-\lambda\varphi}$$

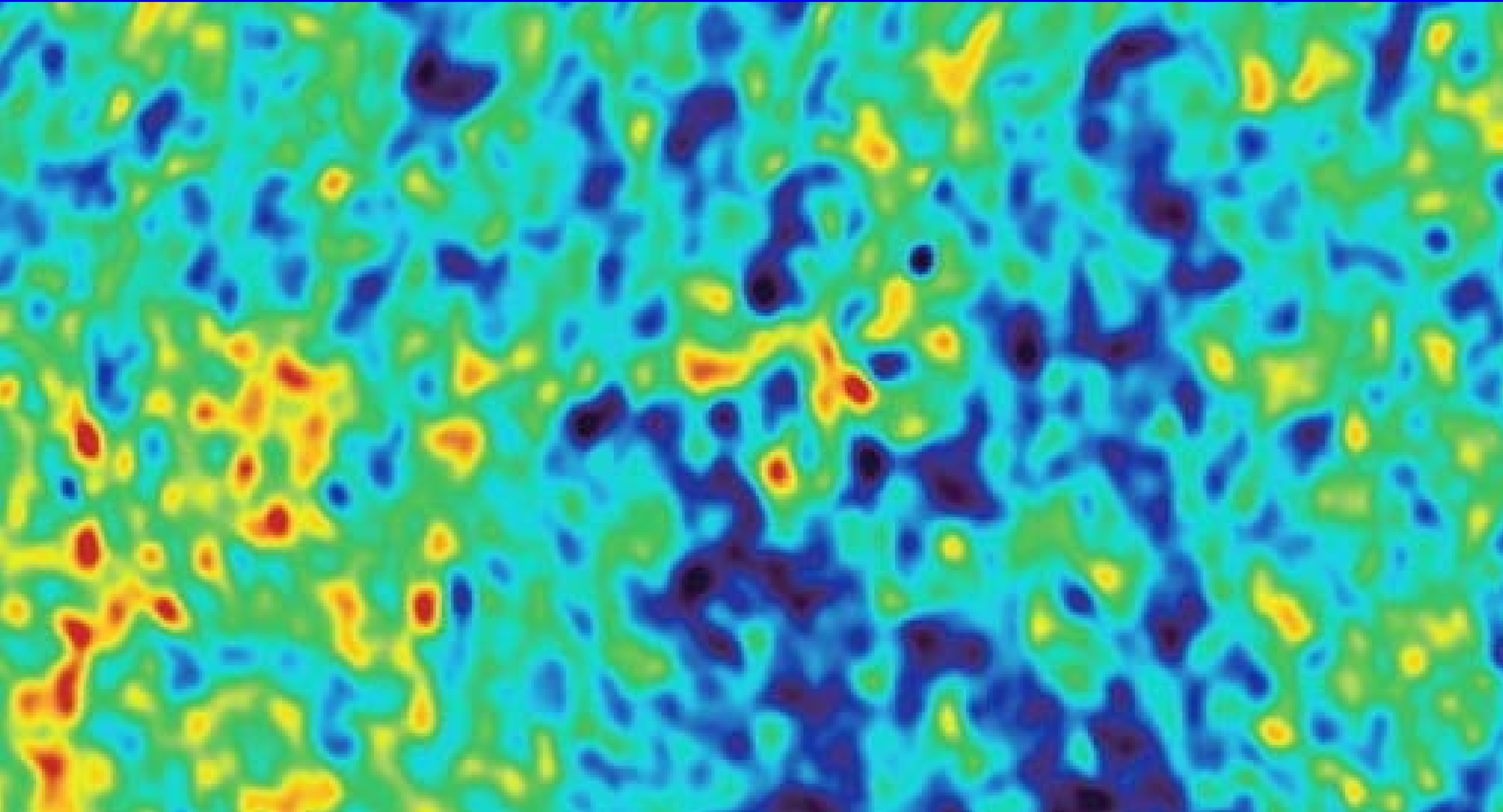
1. For early universe inflation $\lambda \leq 10^{-1}$
2. For dark energy $\lambda \leq 1$

For the dilaton $\lambda = \sqrt{2}$

For the total volume $\lambda = \sqrt{6}$

Both stringy moduli have very steep potentials incompatible with the data even for the current acceleration of the universe, particularly the total volume

A photographic image of quantum fluctuations blown up to the size of the universe



Inflationary slow roll parameters in units $M_P^2 \equiv 8\pi G_N = 1$

Primordial slope $n_s \equiv 1 - 6\epsilon + 2\eta$

$\eta[\varphi] = \frac{V''}{V} \sim \text{const}$ $\epsilon[\varphi] = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \sim \text{const}$

Derivatives w. r. to canonicaly normalized fields!

Observational data

$n_s = 0.98 \pm 0.02$

Flux compactification and moduli stabilization in IIB string theory (supergravity + local sources)

The potential with respect to dilaton and volume is very steep, they run down and V vanishes, the space tends to decompactify to 10d and string coupling tends to vanish, unless both are stabilized at some finite values.

Dilaton stabilization   Giddings, Kachru and Polchinski 2001

$$G_3 = F_3 - \tau H_3$$
$$\tau = C_0 + i e^{-\phi}$$
$$\tilde{F}_5 = F_5$$

Warping fixed by local sources (tadpole condition) and non-vanishing ISD fluxes

$$*G_3 = iG_3$$

This equation fixes the shape of CY and the dilaton-axion

4d description

* Susy at scale 1/R(CY) $\longrightarrow$ N=1 effective action
* Specify the Kahler potential, superpotential and gauge couplings
* At the leading order in α' and g_s

$$K = -3\ln[-i(\rho-\bar{\rho})] - \ln[-i(\tau-\bar{\tau})] - \ln[-i\int_M \Omega\wedge\bar{\Omega}]$$

* **Add the superpotential due to fluxes** $W[z_\alpha,\tau] = \int G_3\wedge\Omega$
* Solve equations

$$D_{z_\alpha}W = D_\tau W = 0$$

* Note that susy is broken if $W\neq 0$ $\quad D_\rho W = K_\rho W \neq 0$

The complex structure fields and the axion-dilaton are fixed.

The overall volume still has a runaway potential

Potential

No-scale

$$V = e^K\left(\sum_{\rho,z_\alpha,\tau} g^{k\bar{l}} D_k W \overline{D_l W} - 3|W|^2\right) \Rightarrow$$

$$V = e^K \sum_{z_\alpha,\tau} g^{a\bar{b}} D_a W \overline{D_b W} \geq 0$$

No potential for the volume moduli. Dilaton and shape moduli are generically fixed in Minkowski space!

*** Kahler moduli problem (in particular, overall volume)

***KKLT proposal

i) non-perturbative superpotential from Euclidean D3-branes wrapped on special 4-cycles

$$W \sim e^{-\mathrm{Vol}(D)} \sim e^{-\rho}$$

ii) non-perturbative superpotential from pure SYM on a stack of D7's on Σ_4

$$W \sim e^{-\mathrm{Vol}(\Sigma_4)/C_2(G)} \sim e^{-\rho/C_2}$$

Effective theory for the volume moduli

$$W = W_0(z_{cr}, \tau_{cr}) + Ae^{-ia\rho} + \dots \qquad \rho = \alpha + i\sigma$$

Solve $D_\rho W = 0$ $\Rightarrow$ $\sigma_{cr} \sim \frac{1}{a} \ln W_0$

$C_2(G) > 1$, $W_0 \ll 1$ **the volume is stabilized in AdS critical point in the regime of validity of calculations!**

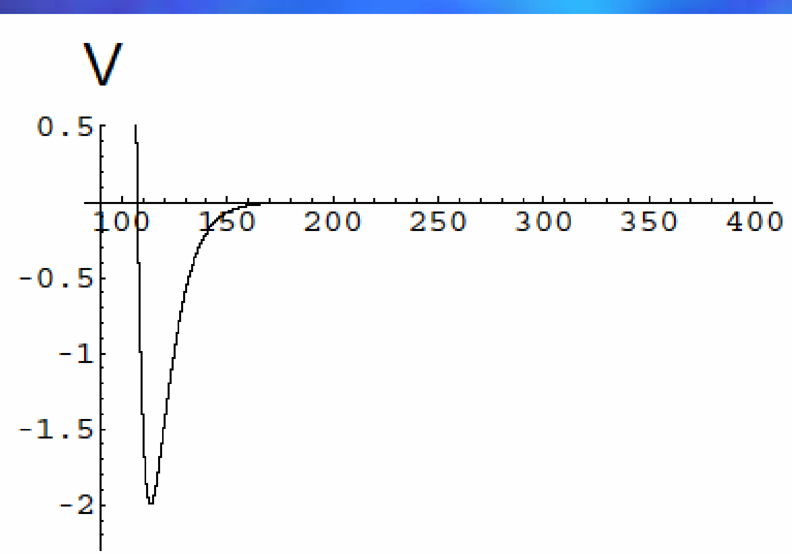
$$V_{cr} = -3e^{K} |W_{cr}|^2 , \qquad D_l W = 0$$

Volume stabilization

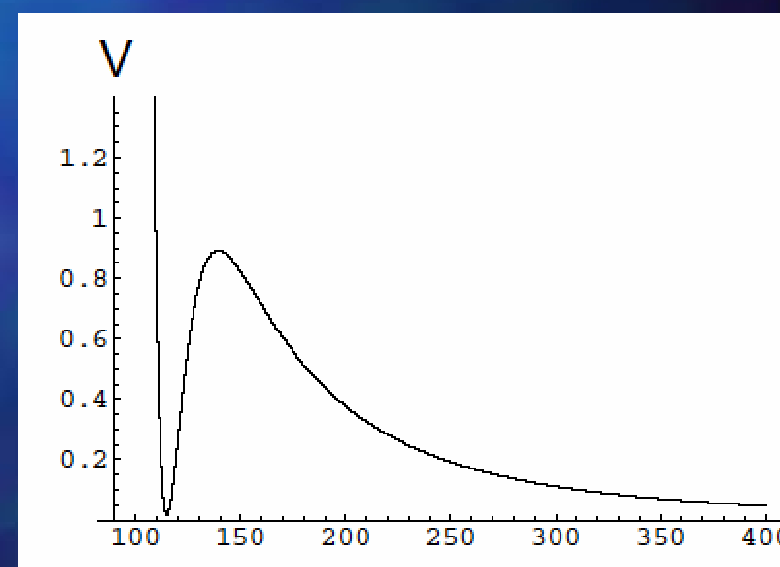
Basic steps:

- Warped geometry of the compactified space and nonperturbative effects ⟶ AdS space (negative vacuum energy) with unbroken SUSY and stabilized volume

- Uplifting AdS space to a metastable dS space (positive vacuum energy) by adding anti-D3 brane (or D7 brane with fluxes)



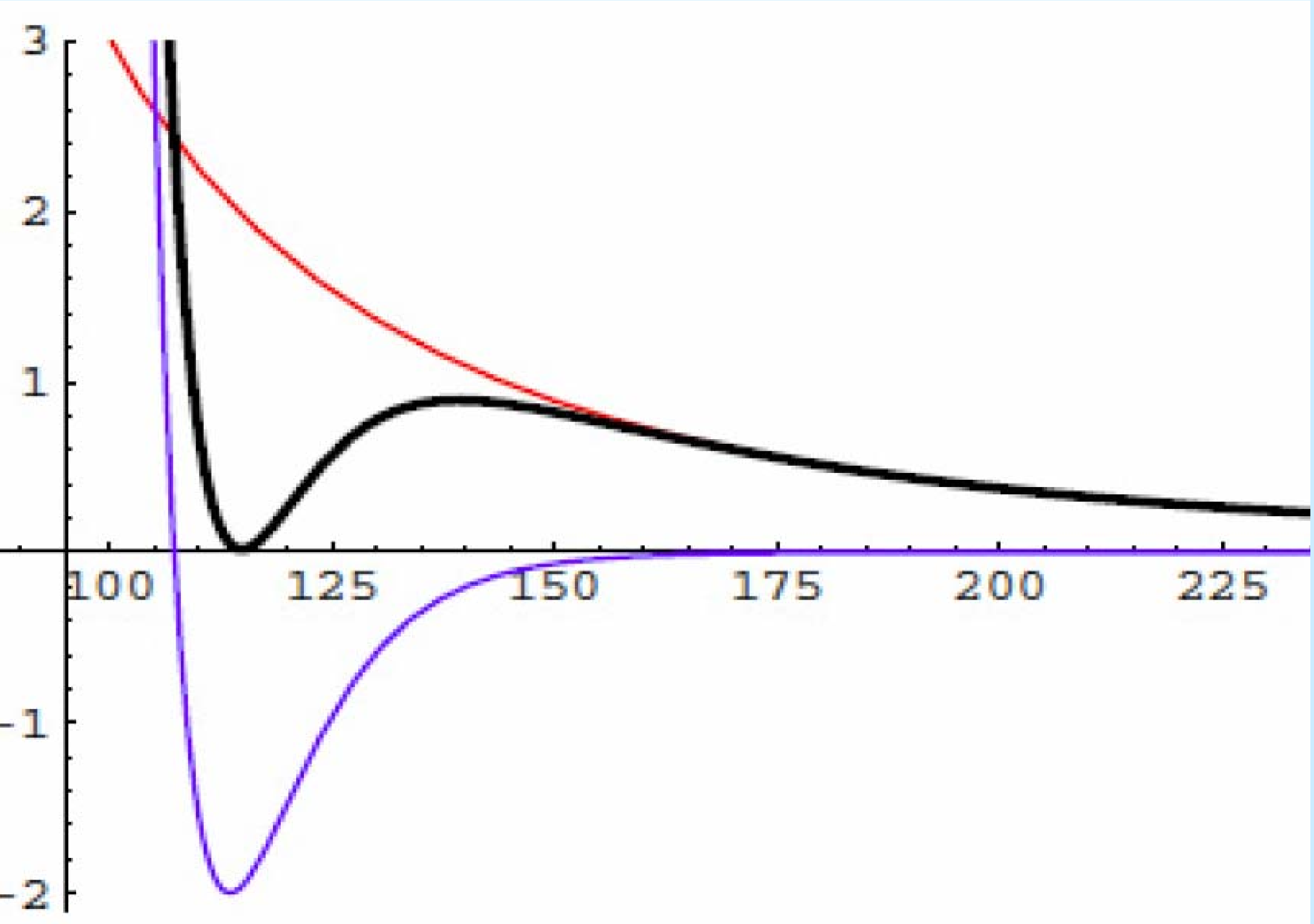
AdS minimum



Metastable dS minimum

$V(\overline{D3}) = \sqrt{g}_4 \sim 1/\sigma^2$ In KS warped geometry

V(AdS) + V(anti-D3)= V(dS)



Metastable dS vacua

- There is no claim that the cosmological constant is computable;

only that the expected corrections in models with small $g_s, \quad W_0$

are smaller than barierr height stabilizing vacuum.

KKLT based new ideas

Landscape Susskind	Statistics of **Flux Vacua** Douglas

Kachru, R. K., Maldacena, McAllister, Linde and Trivedi 2003

Model of Inflation

Cosmic Strings Produced by the end of Inflation

Dvali, R. K. , Van Proeyen; Copeland, Myers, Polchinski

Two types of string inflation models:

- **Moduli Inflation.** The simplest class of models. It uses only the fields that are already present in the KKLT model.
- **Brane inflation.** The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.

Racetrack Inflation

the first working model of the moduli inflation

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo
hep-th/0406230

Superpotential:

$$W = W_0 + A\, e^{-aT} + B\, e^{-bT}$$

Kahler potential:

$$K = -3\log(T + T^*)$$

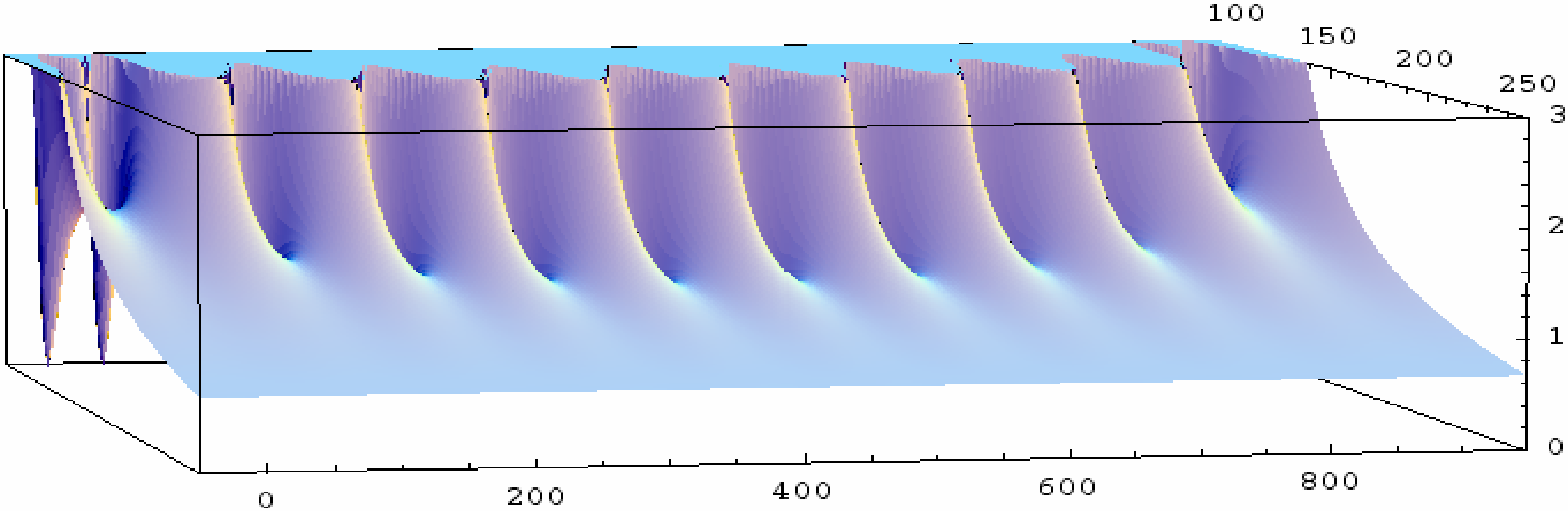
Effective potential for the field T = X + i Y

$$\begin{aligned} V \;=\; & \frac{E}{X^\alpha} + \frac{e^{-aX}}{6X^2}\left[aA^2\,(aX+3)\;e^{-aX} \;+3W_0 aA\cos(aY)\right] + \\ & + \frac{e^{-bX}}{6X^2}\left[bB^2\,(bX+3)\;e^{-bX} \;+3W_0 bB\cos(bY)\right] + \\ & + \frac{e^{-(a+b)X}}{6X^2}\left[AB\,(2abX+3a+3b)\cos((a-b)Y)\right] \end{aligned}$$

This model is similar to the "natural inflation" of Freese, Frieman and Olinto, 1990. The difference is that in the original version of the natural inflation scenario the real part of the field f was supposed to be stabilized at $f_a >> 1$ for all values of the axion field, as in the Mexican hat potential, and inflation would occur along the axion direction with the potential

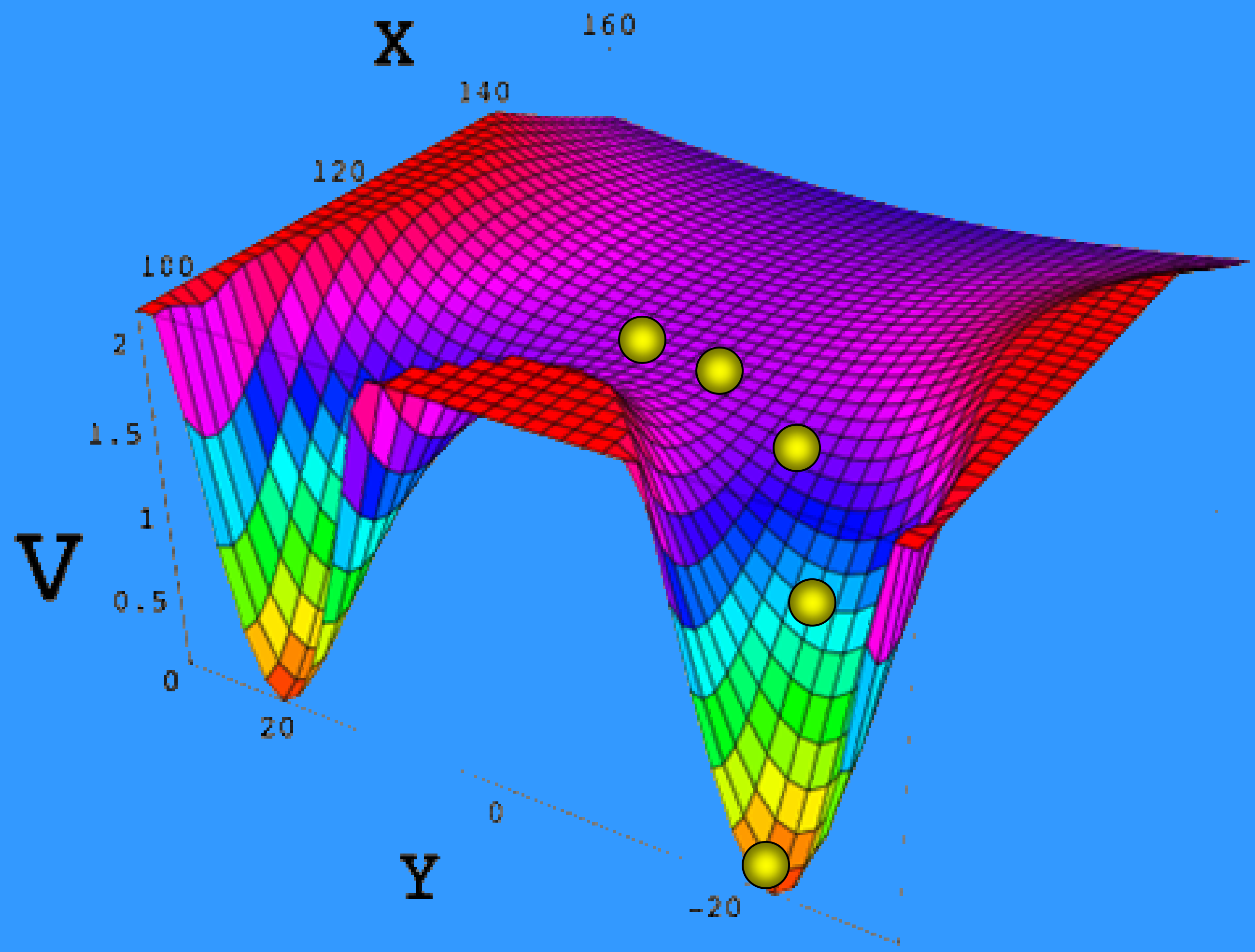
$$V(\phi) = \Lambda^4 [1 \pm \cos(N\phi/f)]$$

So far, we were unable to find models of exactly this type, but we have found the regions where the potential has <u>isolated inflationary saddle points</u> in the axion direction:



Parameters and Potential

$$A = \frac{1}{50}, \qquad B = -\frac{35}{1000}, \qquad a = \frac{2\pi}{100}, \qquad b = \frac{2\pi}{90}, \qquad W_0 = -\frac{1}{25000}$$



Inflationary Predictions:

COBE-normalized spectrum of perturbations of metric

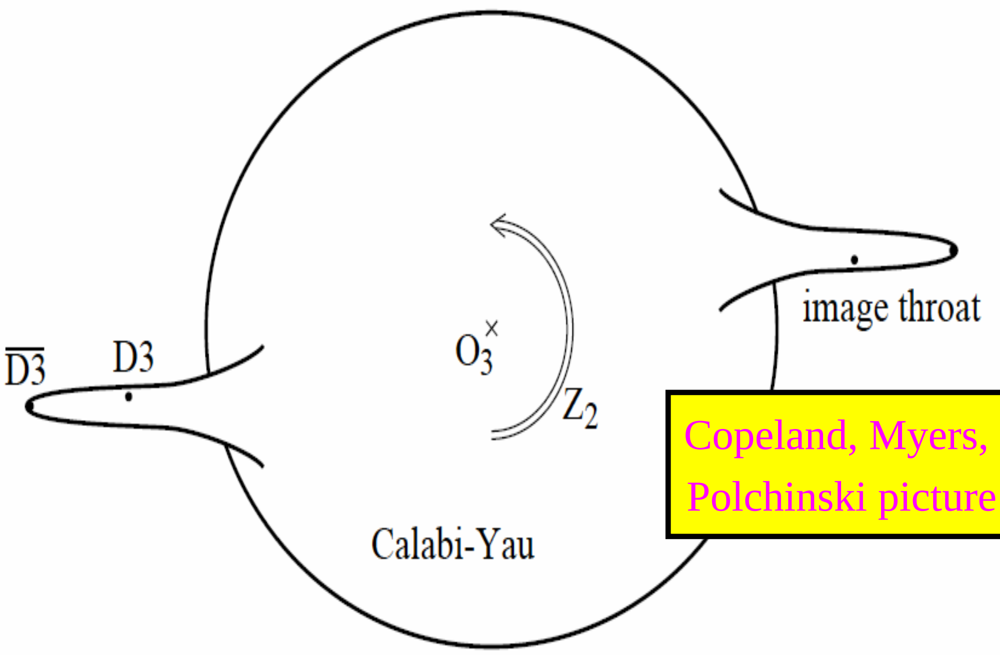
Flat spectrum of metric perturbations with $n_s = 0.95$

No tensor perturbations

Inflation in this scenario is eternal

Parameters require fine-tuning with accuracy O(0.1%), which may not be a problem if one takes into account string theory landscape

The KLMT model



$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = z$$

Deformed Conifold

$$e^{A_{min}} \sim z^{1/3} \sim e^{-\frac{2\pi K}{3Mg_s}}$$

The throat geometry has a highly warped region

$$ds^2 = e^{2A(y)} ds_4^2 + ds_y^2 \qquad e^{2A} \ll 1$$

The role of warping factor in uplifting AdS vacuum to dS

Small z (resolution of conifold singularity)

$$V = V_F + V(\overline{D3}) \qquad V(\overline{D3}) = \frac{C}{\sigma^2}$$

$$C \sim z^{2/3} \sim e^{4A_{min}} \ll 1$$

Small C is necessary for dialing the anti-D3 energy to AdS scale to preserve and uplift the minimum

The redshift in the throat plays the key role in KLMT

Advantage: source of small parameters

Disadvantage: highly warped region of KS geometry corresponds to conformal coupling of the inflaton field (position of D3-brane in the throat region)

$$M^2_{\text{infl}} \sim H^2$$

Flatness of the Inflaton Potential and of the Perturbation Spectrum Require $M^2_{\text{infl}} \sim 10^{-2} H^2$

- Volume Stabilization and Brane Inflation ?

Dvali and Tye

Volume Stabilization, Kähler Potential, Non-Perturbative Superpotential

- KKLMMT (warped flux compactification models)

$$K = -3 \ln\left(-i(\rho - \bar{\rho}) - \bar{\phi}\phi\right)$$

$$W = W_0 + Ae^{-ia\rho}$$

D3-anti-D3 brane inflation with volume stabilization

In the warped deformed conifold KS geometry leads to:

$$M^2_{\mathrm{infl}} \sim H^2$$

Without fine-tuning

KKLMMT-type models with fine-tuning

One can fine-tune the parameters in some models to provide a flat potential with volume stabilization.

More recent work on models with warping →

Burgess, Cline, Stoica, Quevedo
DeWolfe, Kachru, Verlinde
Iizuka, Trivedi
Berg,Haack, Kors
Buchel, Ghodsi

Can we do better in string theory? Use symmetries?

Supersymmetry and Inflation

- Hybrid Inflation

Linde, 91

- F-term, D-term Inflation

Copeland, Liddle, Lyth, Stewart, Wands;
Dvali, Shafi, Shafer, 94
Binetruy, Dvali; Halyo, 96

- D3/D7 Brane Inflation as D-term Inflation

Dasgupta, Herdeiro, Hirano, R.K., 2002

- **Include Volume Stabilization:**

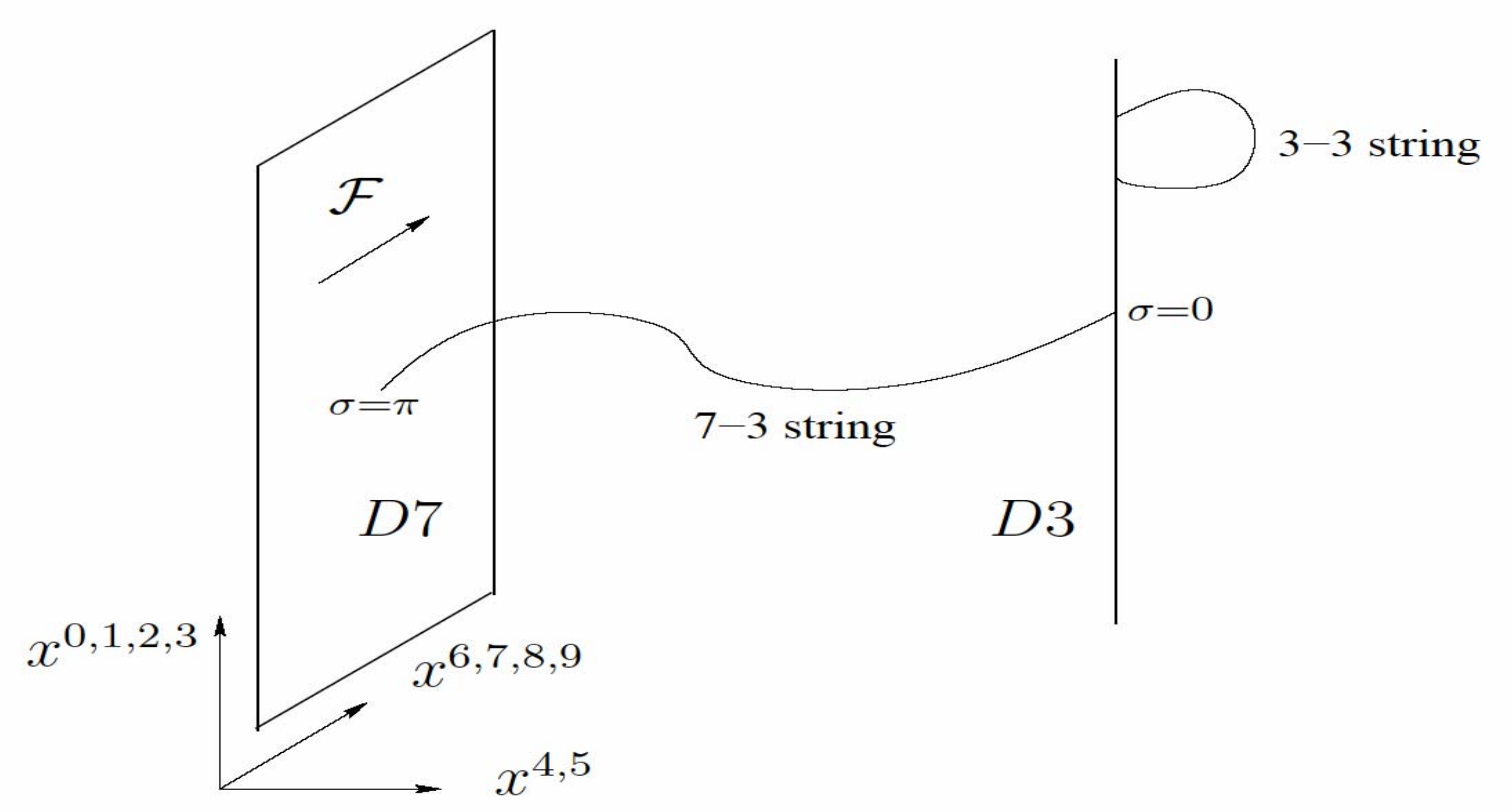
F-term for KKLT+ Shift Symmetry slightly broken by quantum corrections

Hsu, R. K., Prokushkin
Hsu, R. K., 2003-2004

- D-term Inflation with type IIB string theory parameters

D3/D7 BRANE INFLATION MODEL

In type IIB string theory compactified on $K3 \times \frac{T^2}{Z^2}$

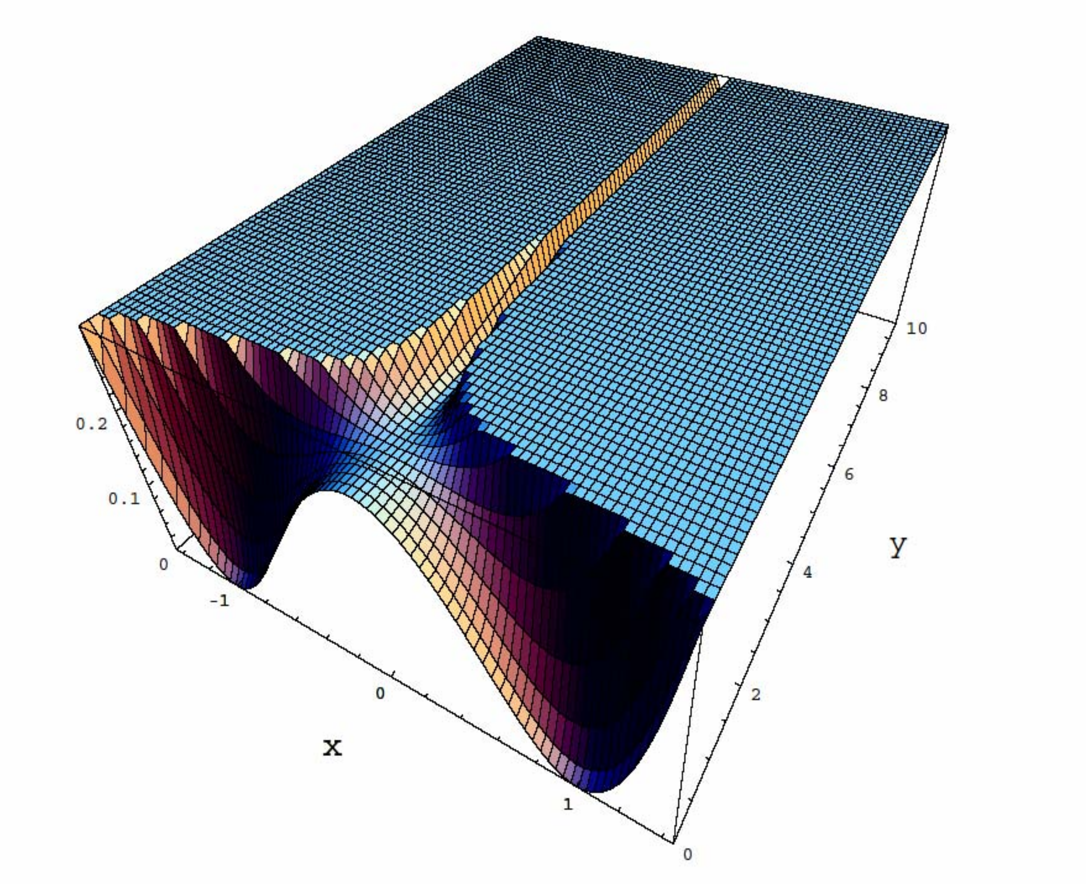


Dasgupta, Herdeiro, Hirano, R.K., 2002

D-term Inflation

Binetruy, Dvali; Halyo,1996

The Potential of the Hybrid D3/D7 Inflation Model



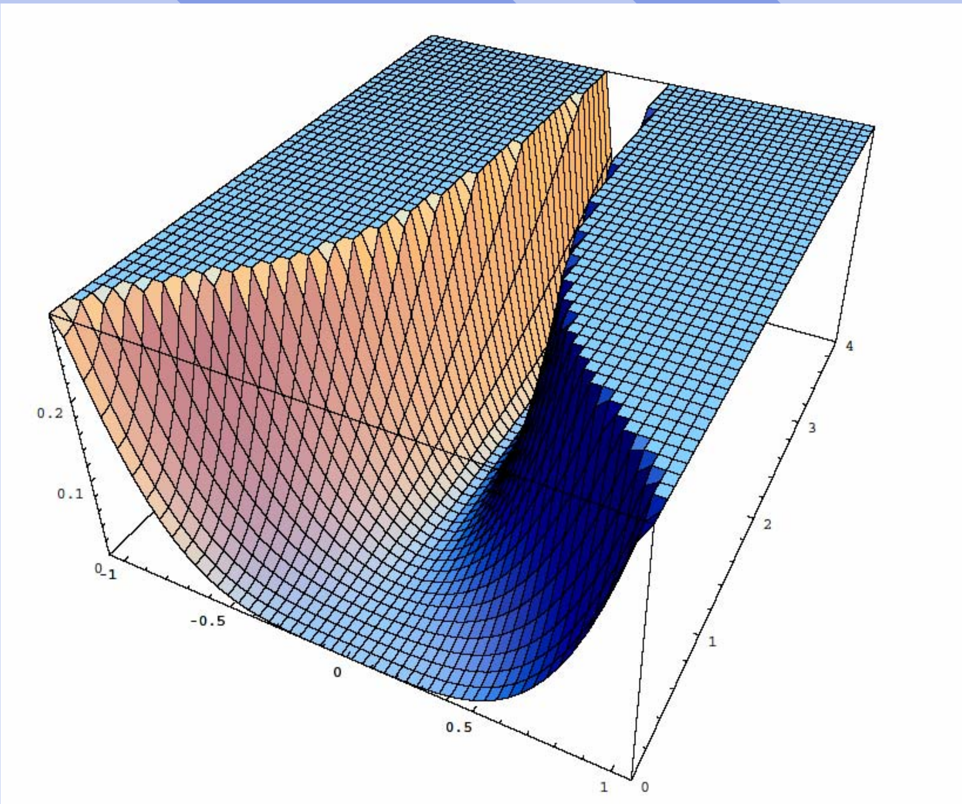
$$V = S^2 \Phi^\dagger \Phi + \frac{g^2}{2} D^2$$

$$\vec{D} = \Phi^\dagger \vec{\sigma} \Phi - \vec{\xi}$$

Φ is a hypermultiplet

$\vec{\xi}$ is an FI triplet: resolution of the singularity

Same Potential without Fayet-Iliopoulos term



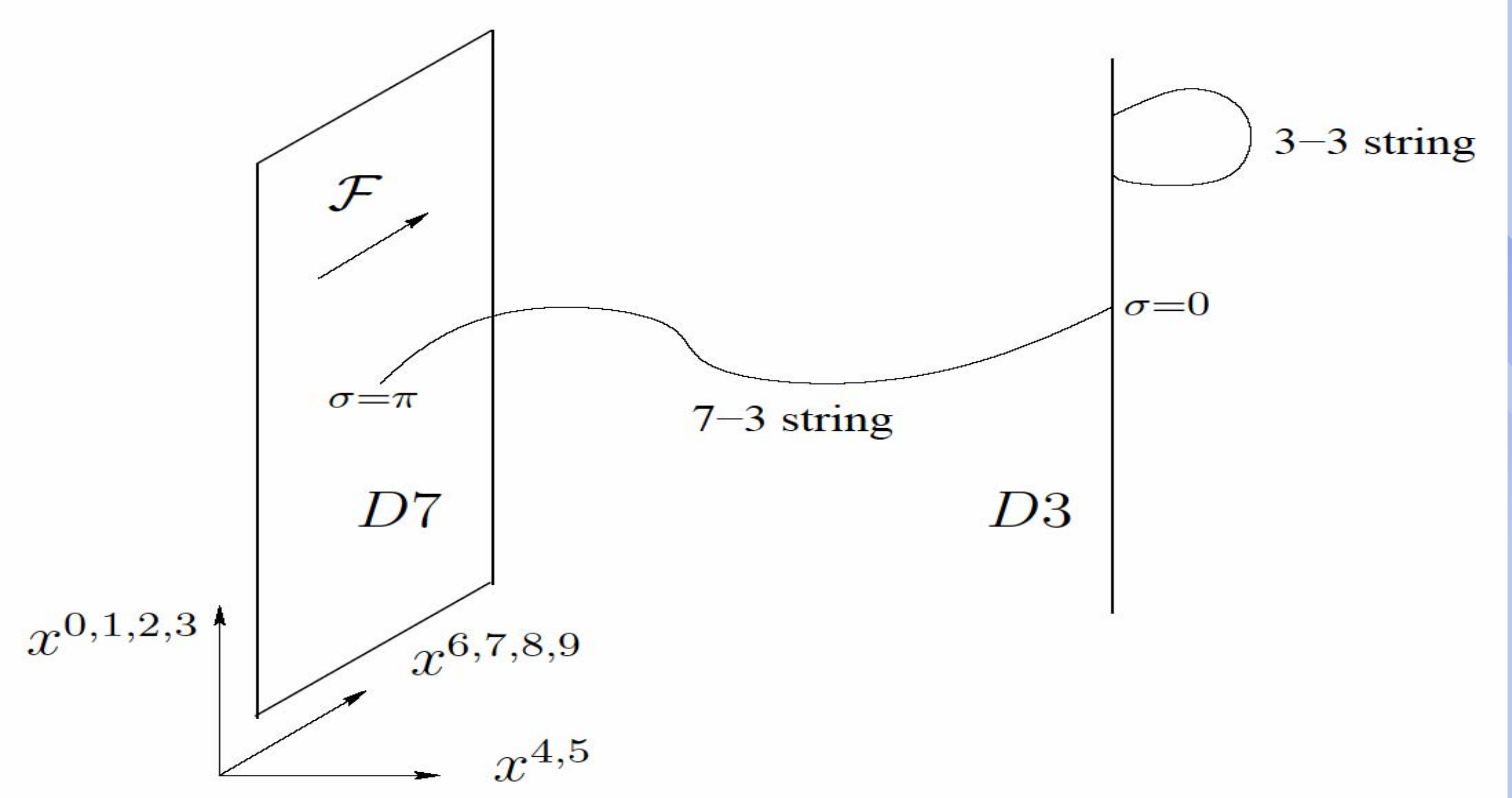
$$V = S^2 \Phi^\dagger \Phi + \frac{g^2}{2} D^2$$

$$\vec{D} = \Phi^\dagger \vec{\sigma} \Phi$$

$$\vec{\xi} = 0$$

Flat direction corresponding to the singularity in the moduli space of instantons in D3/D7

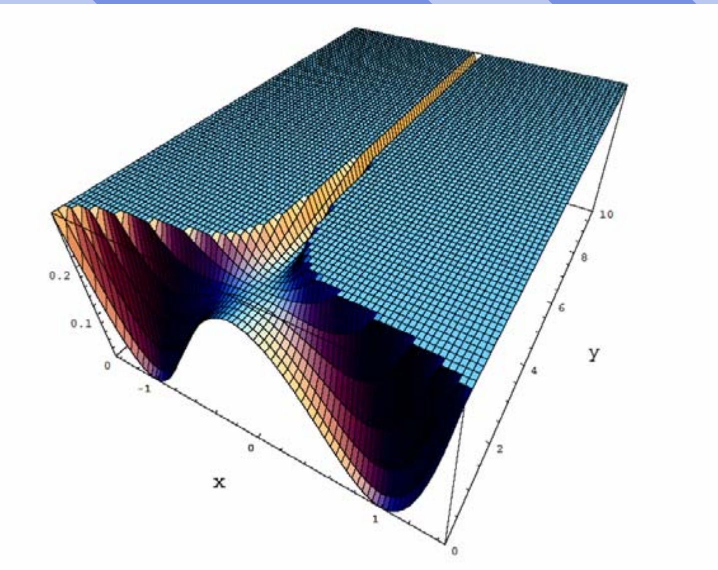
D3/D7 BRANE INFLATION MODEL



The mass of D3-D7 strings (hypers) is split due to the presence of the anti-self-dual flux on D7

$$V_{1-\text{loop}} = \frac{g^2\xi^2}{2}\left[1 + \frac{g^2}{8\pi^2}\ln\frac{S^2}{S_{cr}^2}\right]$$

De Sitter stage $\longrightarrow$ Waterfall $\longrightarrow$ Ground State



$$\Phi = 0 \qquad S \gg S_{cr} \qquad \vec{D} = \vec{\xi} \qquad V = \frac{g^2 \xi^2}{2}$$

De Sitter: Inflation or current acceleration

$$\Phi^\dagger \vec{\sigma} \Phi = \vec{\xi} \qquad S = 0 \qquad \vec{D} = 0 \qquad V = 0$$

$$V = S^2 \Phi^\dagger \Phi + \frac{g^2}{2} D^2$$

$$\vec{D} = \Phi^\dagger \vec{\sigma} \Phi - \vec{\xi}$$

Ground state: D3/D7 bound state

Higgs branch: non-commutative instantons

NS non-commutative instantons:

Higgs branch, bound state of D0/D4

D3 can move away from D7 when the deformation parameter vanishes, the moduli space is singular:

there is no de Sitter space

Resolution of singularity of the moduli space of instantons in D3/D7 Higgs branch

$$\vec{\xi} \neq 0 \qquad \vec{D} = \Phi^\dagger \vec{\sigma} \Phi - \vec{\xi} = 0$$

requires that the Coulomb branch has a non-vanishing D-term potential

$$V_D = \frac{g^2 \xi^2}{2}$$

Deformation-non-commutativity-resolution of singularity

de Sitter space

DBI kappa-symmetric action and non-linear deformed instantons

Seiberg,Witten, 99; Marino, Minassian, Moore, Strominger, 99

$$\mathcal{F} = F - B \qquad F = dA$$

D3/D7 bound state and unbroken supersymmetry

Bergshoeff, R. K., Ortin, Papadopoulos, 97

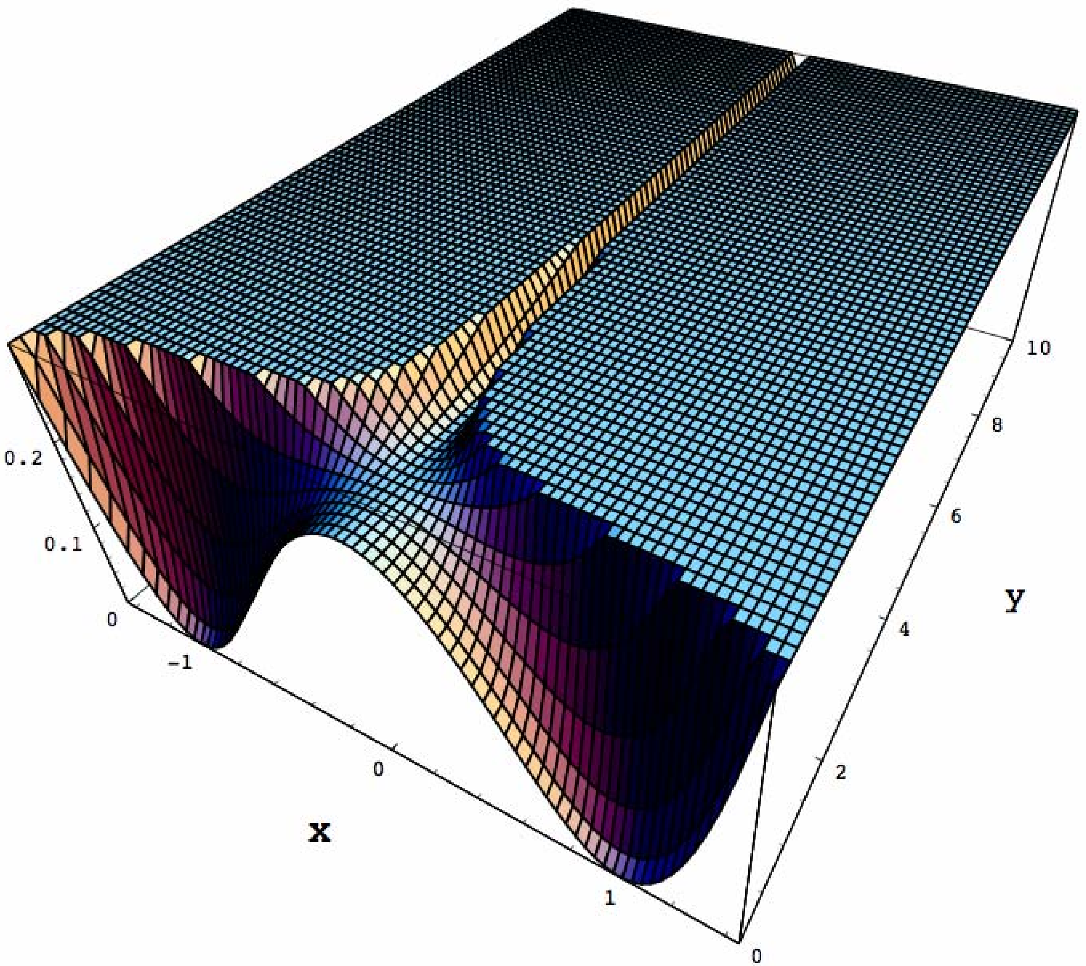
$$(1-\Gamma)\epsilon = 0 \qquad \Gamma = e^{-a/2}\Gamma_0 e^{a/2}$$

$$a = \frac{1}{2} Y_{ik} \Gamma^{ik} \sigma_3 \qquad \mathcal{F} = \text{"}tan\text{"}\, Y$$

$$\frac{\mathcal{F}^-}{1+Pf\mathcal{F}} = -\frac{B^-}{1+PfB}$$

Deformed flux on the world-volume → Non-linear deformed instanton

Hybrid D3/D7 Inflation Model



How to make this model valid in string theory with the **volume stabilization**

String theory does not have constant FI terms!

Shift Symmetry?

D-term volume stabilization

Instead of anti-D3 add D7 with flux. The D-term potential depends on the ASD deformed flux and volume modulus

$$V_D \sim \frac{(\theta_1 - \theta_2)^2}{\sigma^3} \approx \frac{(\mathcal{F}^-)^2}{\sigma^3}$$

$$\mathcal{F}_{67} = \tan\theta_1 \qquad \mathcal{F}_{89} = \tan\theta_2$$

2 possibilities to make this mechanism working

1) Place D7 in highly warped region of space Burgess, R. K., Quevedo

2) Use deformation: irrational B cannot be gauged away into $F = dA$

$$\int F = 2\pi n$$

Deformation parameter (non-commutativity) is not quantized, it can be small!

Cosmology, Supersymmetry and Special Geometry

In familiar case of **Near Extremal Black Holes**

DUALITY SYMMETRY protects exact entropy formula from large quantum corrections

DUALITY SYMMETRY (shift symmetry)

may protect the **flatness of the potential**

in D3/D7 inflation model from large quantum corrections

Shift Symmetry of $\mathcal{G}$

- Flatness of the effective supergravity inflaton potential follows from the shift symmetry of $\mathcal{G} \equiv K + \ln |W|^2$

$$V = e^{\mathcal{G}}[|\mathcal{G}_{,z}|^2 - 3]$$

We need models where the position of the D3 brane after stabilization of the volume is still a modulus

SHIFT SYMMETRY
and volume stabilization

Distance between branes $\phi = x^4 + ix^5$

Volume-axion field $\rho = \alpha + i\sigma$

$$\phi \to \phi + \text{Re}f \qquad \phi - \bar{\phi} \to \phi - \bar{\phi}$$

$$K(\rho - \bar{\rho}, \phi - \bar{\phi}) \to K(\rho - \bar{\rho}, \phi - \bar{\phi})$$

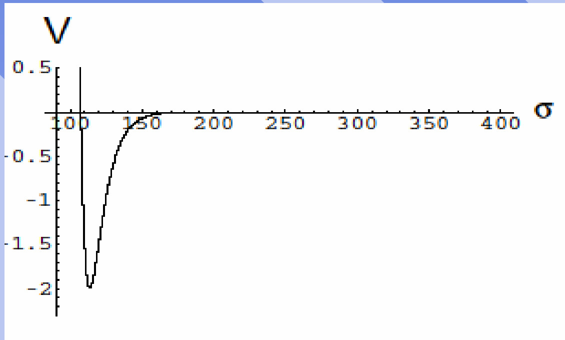
$$W(\rho) \to W(\rho)$$

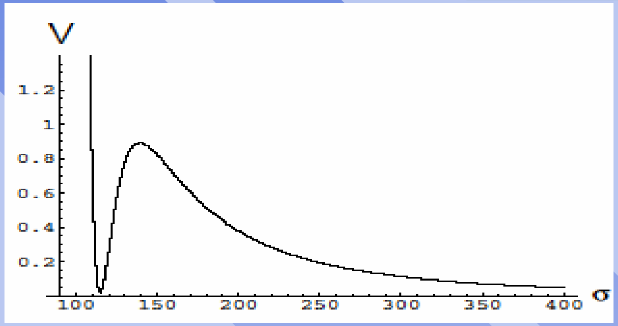
To avoid ambiguities with holomorphic change of variables

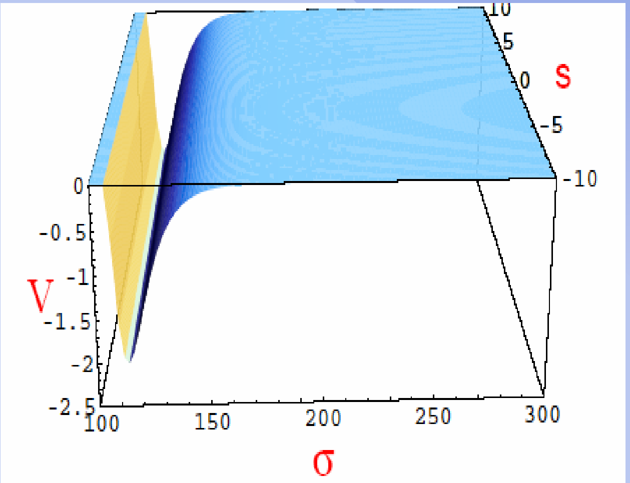
$$\mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi}) \to \mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi})$$

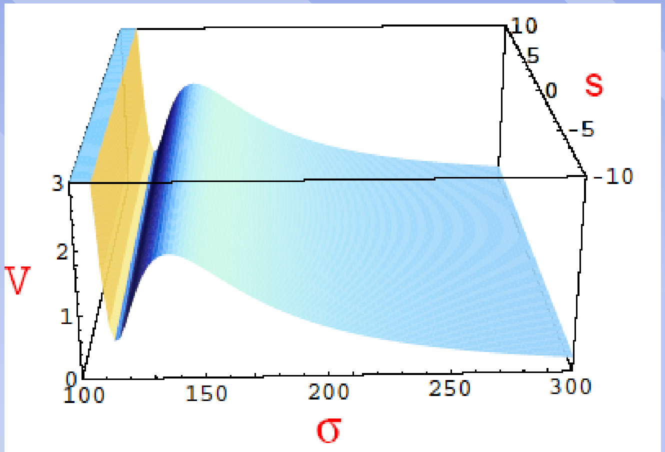
Inflaton Trench

Hsu, R.K., Prokushkin









- Supersymmetric Ground State of Branes in Stabilized Volume

SHIFT SYMMETRY

The motion of branes does not destabilize the volume

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 branes

The prepotential does not exist!

Coset Space

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2+n_3)}{SO(2)\times SO(2+n_3)}$$

Duality

Special Geometry of D3/D7 system

$$s = C_{(4)} - \mathrm{i}\,\mathrm{Vol}(K_3), \qquad t = \frac{g_{12}}{g_{22}} + \mathrm{i}\,\frac{\sqrt{\det g}}{g_{22}}, \qquad u = C_{(0)} + \mathrm{i}\,e^{\phi}$$

x^k and y^r are the positions of the D7 and D3–branes along T^2 respectively

- **Symplectic Section**

$$\begin{pmatrix} X^{\Lambda} \\ F_{\Lambda} \end{pmatrix}$$

$$\begin{aligned}
X^0 &= \tfrac{1}{\sqrt{2}}\left(1 - t\,u + \tfrac{(x^k)^2}{2}\right), \\
X^1 &= -\tfrac{t+u}{\sqrt{2}}, \\
X^2 &= -\tfrac{1}{\sqrt{2}}\left(1 + t\,u - \tfrac{(x^k)^2}{2}\right), \\
X^3 &= \tfrac{t-u}{\sqrt{2}}, \\
X^k &= x^k, \\
X^r &= y^r,
\end{aligned}$$

$$\begin{aligned}
F_0 &= \frac{s\left(2-2\,t\,u+(x^k)^2\right)+u\,(y^r)^2}{2\,\sqrt{2}}, \\
F_1 &= \frac{-2\,s\,(t+u)+(y^r)^2}{2\,\sqrt{2}}, \\
F_2 &= \frac{s\left(2+2\,t\,u-(x^k)^2\right)-u\,(y^r)^2}{2\,\sqrt{2}}, \\
F_3 &= \frac{2\,s\,(-t+u)+(y^r)^2}{2\,\sqrt{2}}, \\
F_k &= -s\,x^k, \\
F_r &= -u\,y^r.
\end{aligned}$$

$$\mathcal{F}(s,t,u,x^k,y^r) = stu - \frac{1}{2}\,s\,x^k x^k - \frac{1}{2}\,u\,y^r y^r$$

x^k and y^r are the positions of the D7 and D3–branes along T^2

Kähler Potential

$$K = -\log[-8\,(\mathrm{Im}(s)\,\mathrm{Im}(t)\mathrm{Im}(u) - \frac{1}{2}\,\mathrm{Im}(s)\,(\mathrm{Im}(x)^k\,)^2 - \frac{1}{2}\,\mathrm{Im}(u)\,(\mathrm{Im}(y)^r\,)^2)]\,,$$

How to find out if there is a shift symmetry?

- Using special geometry represent the shift symmetry of a D3 position as a duality symmetry. Find out the transformation properties of the gauge coupling and axion on D7. If they are invariant, there is a shift symmetry!

Hsu, R.K.,

- Direct method: calculate the period matrix

D' Auria, Ferrara, Trigiante

and check the D7 part of it:

It works!

Special Kähler geometry

N=2 supergravity with vector multiplets

- SymplecticVectors

de Wit, Van Proeyen,1984

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}' = \begin{pmatrix} A & -B \\ C & D \end{pmatrix} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}$$

$$Sp(2(n+1), R)$$

$$K = -\log\left[\mathrm{i}(\overline{X}^\Lambda F_\Lambda - \overline{F}_\Lambda X^\Lambda) \right]$$

- Kähler potential is a symplectic invariant

- **Supersymmetric Black Hole Entropy**

Symplectic Invariant

Ferrara, R. K., Strominger, 1996

Duality and symplectic transformations

$$\mathcal{L}_1 = \frac{1}{4}(\mathrm{Im}\ \mathcal{N}_{\Lambda\Sigma})\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\mu\nu\Sigma} - \frac{i}{8}(\mathrm{Re}\ \mathcal{N}_{\Lambda\Sigma})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma}_{\rho\sigma}$$

coupling constants or functions of scalars

$$\begin{aligned} \partial^\mu \mathrm{Im}\ \mathcal{F}^{+\Lambda}_{\mu\nu} &= 0 \quad \text{Bianchi identities} \\ \partial_\mu \mathrm{Im}\ G^{\mu\nu}_{+\Lambda} &= 0 \quad \text{Equations of motion} \end{aligned}$$

$$\mathcal{F}^{\pm}_{\mu\nu} = \frac{1}{2}\left(\mathcal{F}_{\mu\nu} \pm \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma}\right)$$

$$G^{\mu\nu}_{+\Lambda} \equiv 2i\frac{\partial\mathcal{L}}{\partial\mathcal{F}^{+\Lambda}_{\mu\nu}} = \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{+\Sigma\,\mu\nu}$$

DUALITY

$$\begin{pmatrix} \tilde{\mathcal{F}}^+ \\ \tilde{G}_+ \end{pmatrix} = \mathcal{S}\begin{pmatrix} \mathcal{F}^+ \\ G_+ \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \mathcal{F}^+ \\ G_+ \end{pmatrix}$$

For consistency:

$$\tilde{G}^+ = (C + D\mathcal{N})\mathcal{F}^+ = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}\tilde{\mathcal{F}}^+$$

$$\rightarrow \boxed{\tilde{\mathcal{N}} = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}}$$

L should be symmetric

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2(n+1), \mathbb{R})$$

- B=0, perturbative duality

Inflaton Shift is a Duality

$$(y^r)' = y^r + \beta^r \qquad s' = s \qquad t' = t \qquad u' = u$$

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

- We found

$$B = 0$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 0 & 0 & -\beta^2/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta^2/2 & 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & \beta & 0 \end{pmatrix}.$$

Hsu, R.K.

$$A^T C - C^T A = 0 \ , \quad B^T D - D^T B = 0 \ , \quad A^T D - C^T B = I$$

Duality of the period matrix

$$(\mathcal{N})' = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}$$

- Using duality matrix for the shift of D3 we found that the D7 part of $\mathcal{N}_{\Lambda\Sigma}(X)$
  - **IS DUALITY (SHIFT) INVARIANT**

Therefore the non-perturbative superpotential and potential are invariant under the inflaton shift symmetry $\phi \to \phi + \Delta x^4$

Conclusion: $\mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi})$

Bringing KKLT and D3/D7 together:

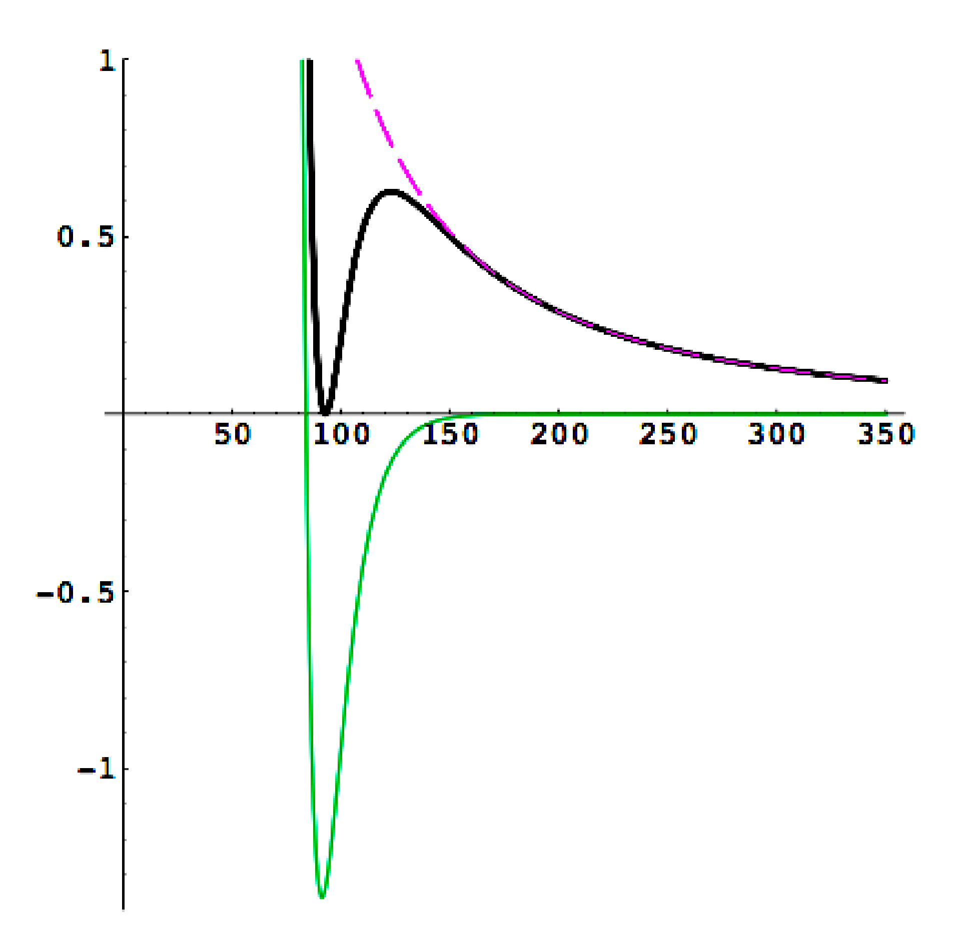
Double Uplifting

$$V_{\rm tot} = V_{\rm KKLT}(\sigma) + V_2(\sigma, s, \phi_\pm)$$

KKL, in progress

First uplifting: *KKLT*

$$V_{\rm KKLT} = V_{AdS} + \frac{D}{\sigma^2}$$

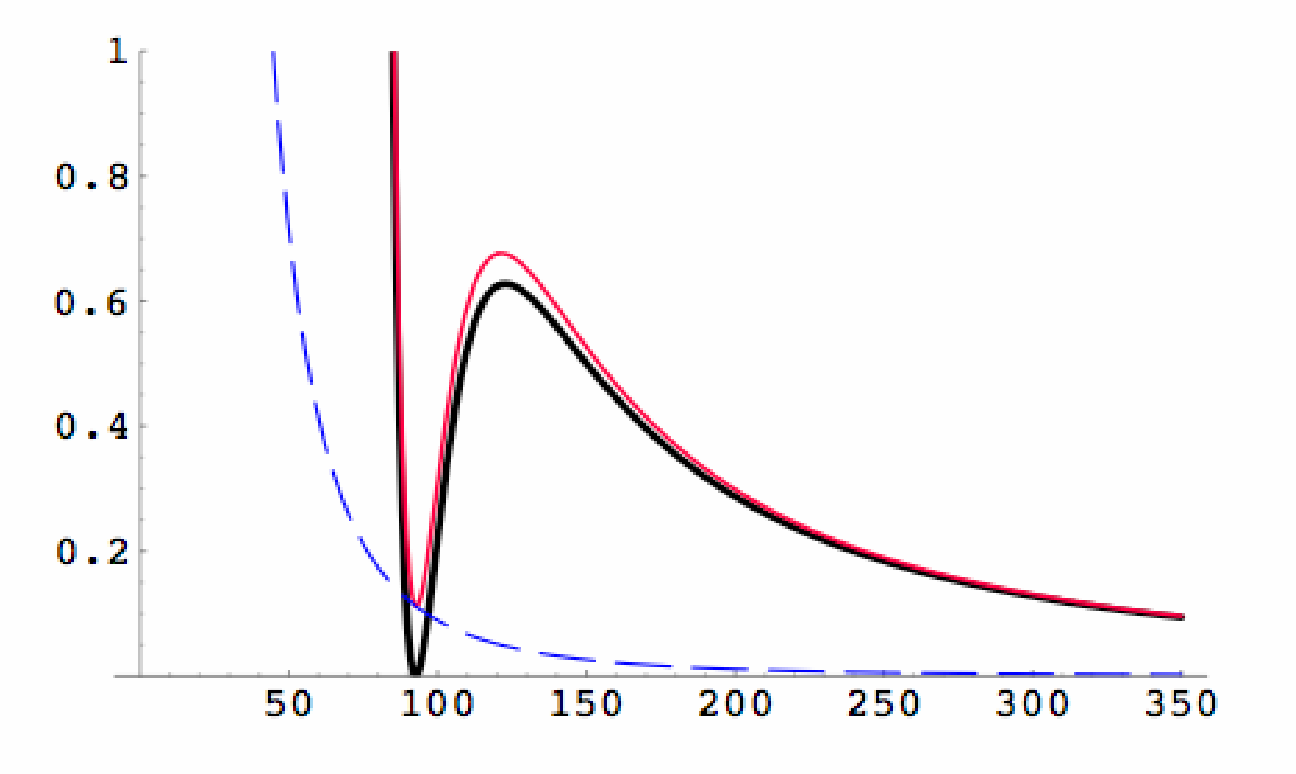


Second uplifting in D3/D7 model

$$V(s,\sigma,\phi_\pm = 0) = V_{AdS} + \frac{D}{\sigma^2} + \frac{C}{\sigma^3}$$

$\overline{\text{D3}}$ contribution

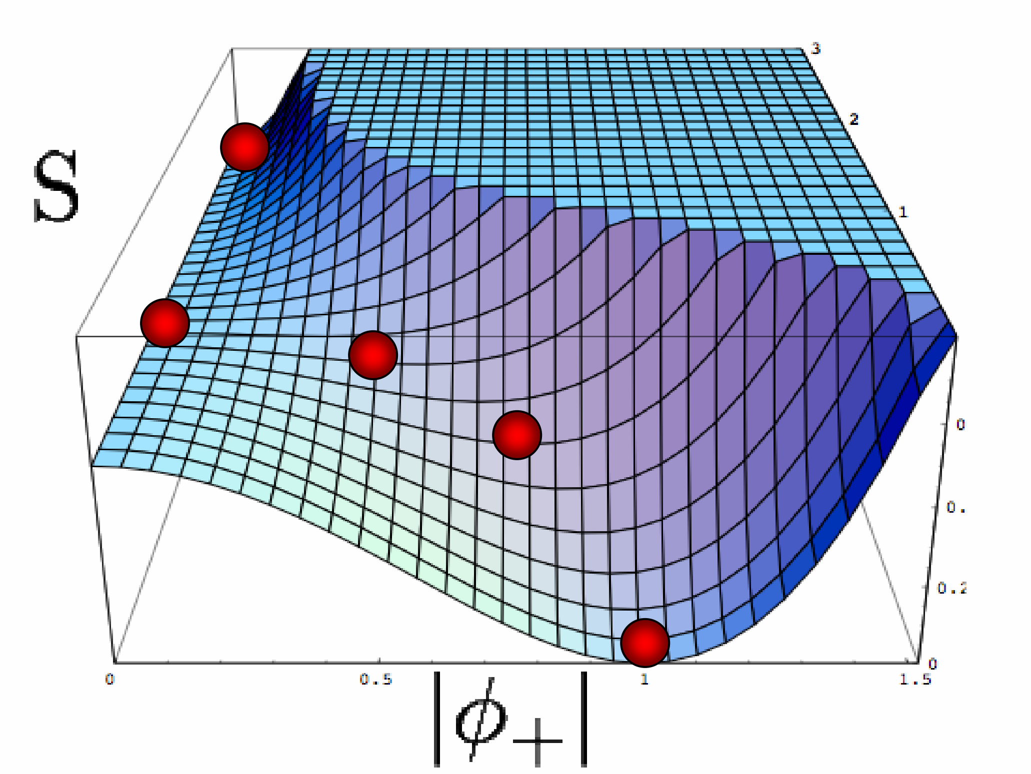
effective FI



Potential of hybrid inflation with a stabilized volume modulus

$$V_2(\phi_+,\phi_-,s,\sigma_{\rm cr}) = \frac{g^2}{2}\,|\phi_+\phi_-|^2 + \frac{g^2}{2}s^2\left[|\phi_+|^2+|\phi_-|^2\right] + \frac{g^2}{8}\left[|\phi_+|^2-|\phi_-|^2-2\xi\right]^2$$

$$V_{1-\rm loop} = \frac{g^2\xi^2}{2}\left(1+\frac{g^2}{8\pi^2}\ln\frac{|S^2|}{|S_c^2|}\right)$$



Unlike in the brane-antibrane scenario, inflation in D3/D7 model does not require fine-tuning

D3/D7 Phenomenology with Stabilized Volume and Inflation

In uplifted dS valley at $\phi_+ = \phi_- = 0$

The last N e-foldings of inflation take place starting from the value of the field s^2 related to the distance between D3 and D7

$$s^2 \sim s_N^2 \qquad s_N^2 = \xi + \frac{g^2 N}{\pi^2} \gg \xi$$

The conditions for successful inflation require that

$$\xi \sim 1.5 \times 10^{-5}$$

To find other parameters one should use the dictionary between brane construction and D-term model

Dasgupta, Hsu, R.K., A. L., Zagermann

$$V_{\rm eff} \approx (2\pi g_s) 10^{-10} M_P^4$$

The effective value of the gauge coupling and the FI term can be deduced from the brane construction and D-term model relation

$$g^2 \sim g_3^2 = 2\pi g_s$$

g_s is the string coupling

$$\frac{g^2 \xi^2}{2} \sim \frac{1}{\sigma^3} \frac{|\mathcal{F}^-|^2}{8 g_7^2} \sim \frac{C}{\sigma^3}$$

$|\mathcal{F}^-|^2 \equiv \int_{K3} (\mathcal{F} - *\mathcal{F})^2$ and $g_7^2 = (2\pi)^5 g_s (\alpha')^2$.

$$|\mathcal{F}^-|^2 \sim 10^{-10} 8 (2\pi)^6 g_s^2 \sigma^3 (M_P \sqrt{\alpha'})^4$$

$$|\mathcal{F}^-| \sim (2\pi)^{-5/2} \frac{\sigma^3}{g_s} 10^{-5}$$

The estimate for the factor $\frac{\sigma^3}{g_s}$ is $\sim 10^8$ for $\sigma \sim 10^2$ and $g_s \sim 10^{-2}$ $(g^2 \sim 2\pi g_s \sim 6 * 10^{-2})$

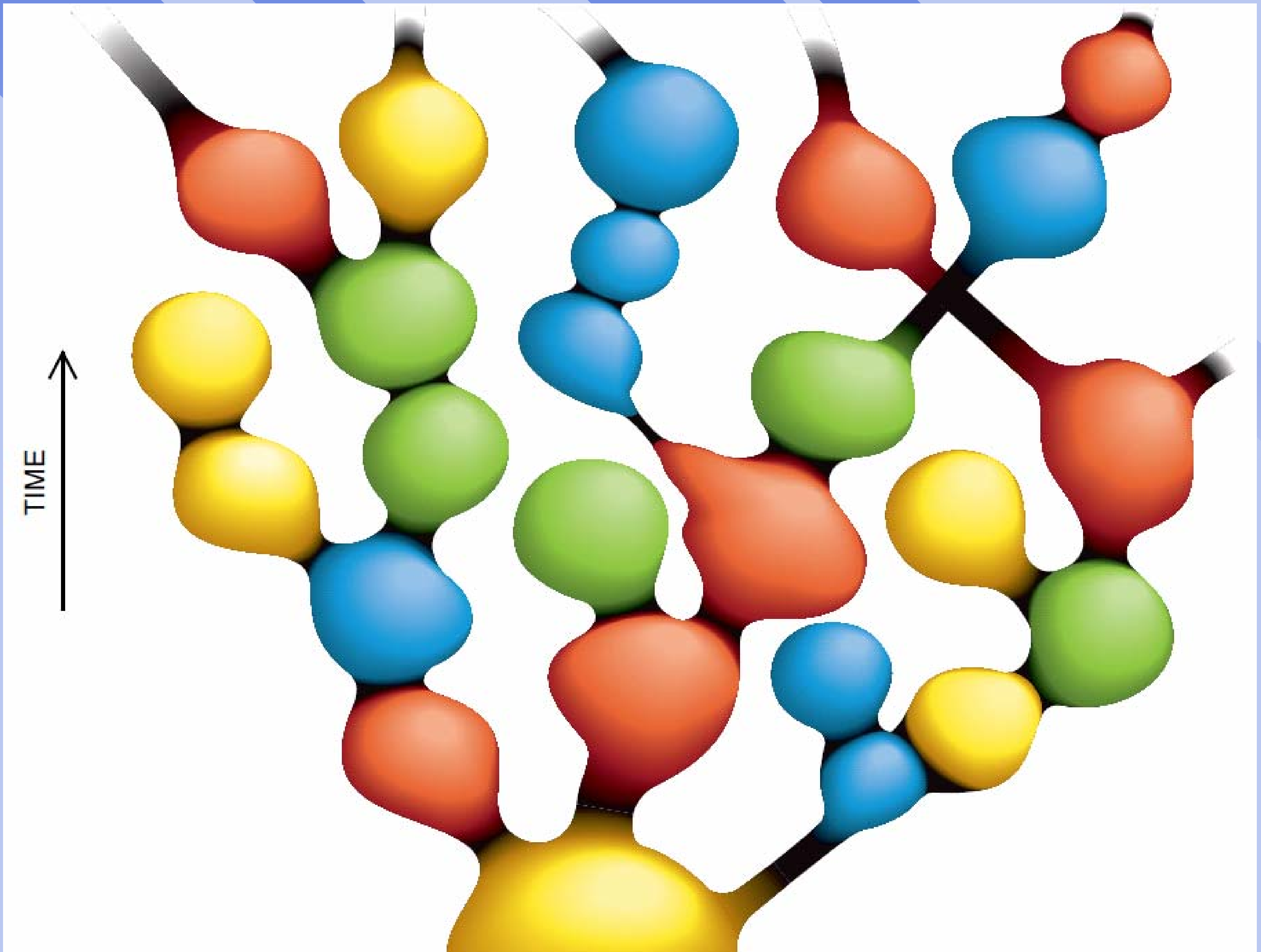
$$|\mathcal{F}^-| \sim 2\pi$$

We believe that such restrictions on the anti-self-dual flux on D7 can be met, which will lead to a successful realization of inflation in string theory.

Discussion

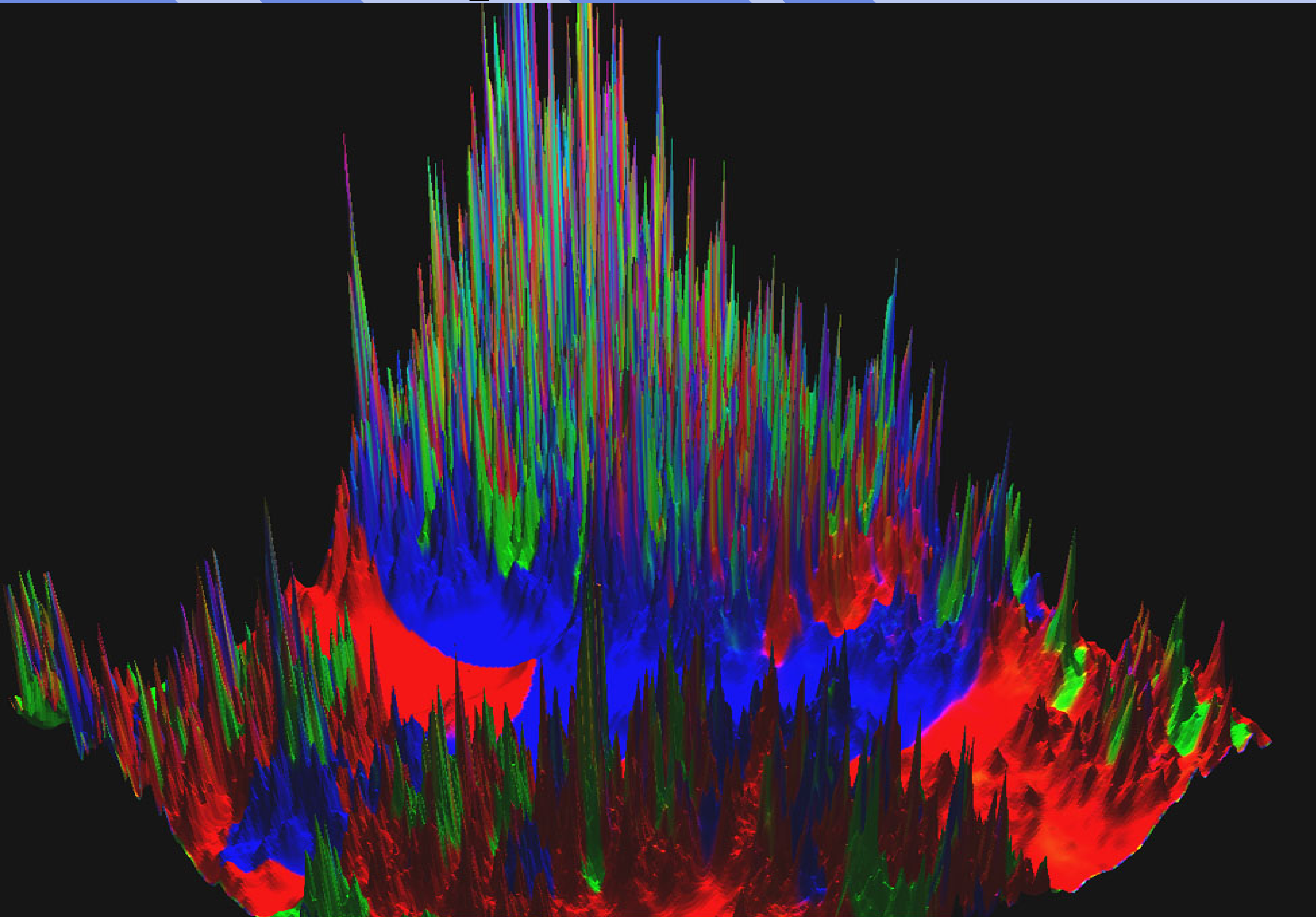
Cosmological Observations	⟷	**Fundamental Theory**
Inflation, Acceleration of the Universe	⟷	String Theory Supergravity
WMAP, Supernova, Large Scale Structure, SNAP, Planck LHC	⟷	

Self-reproducing Inflationary Universe

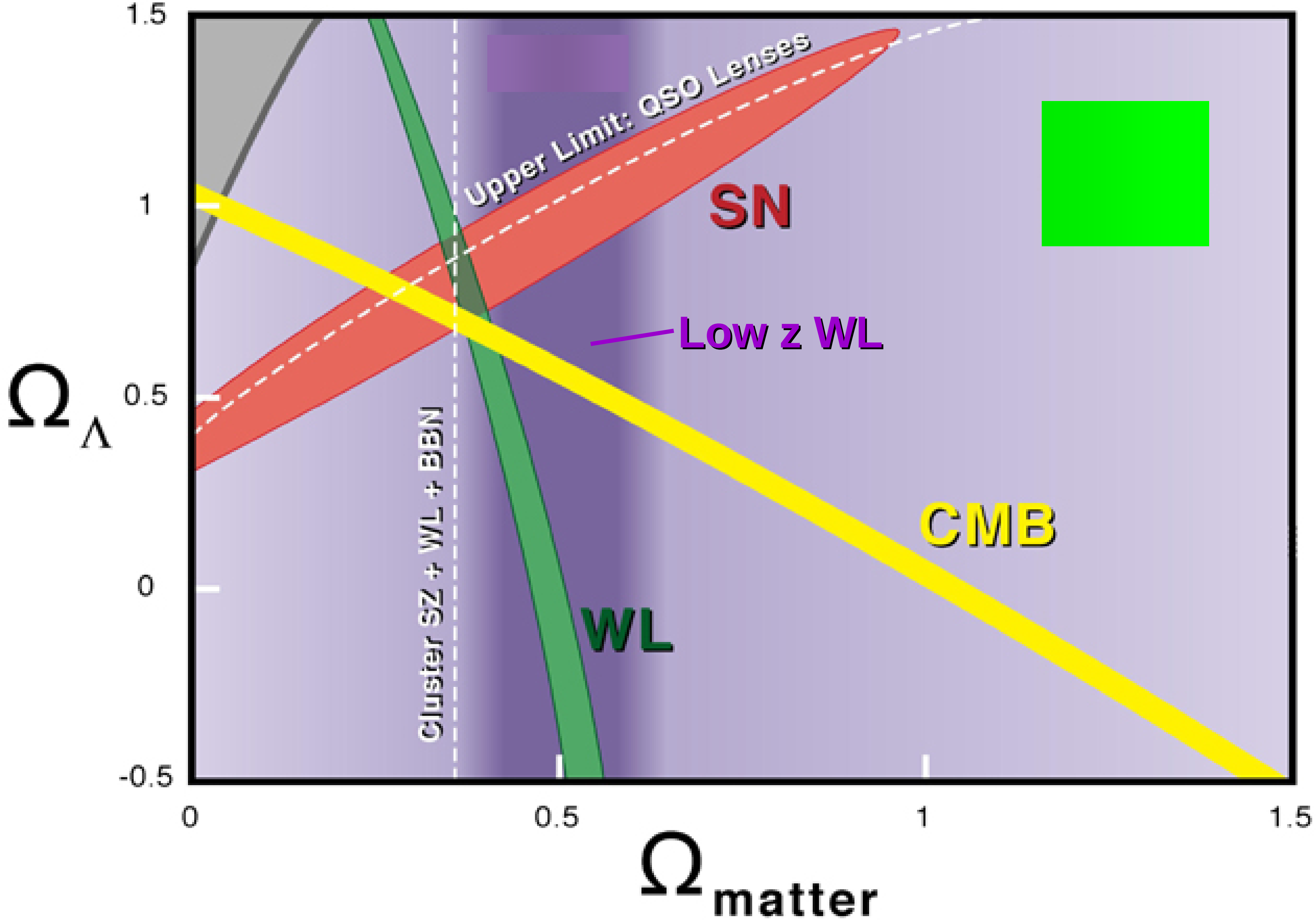


String Theory Landscape

Landscape of eternal inflation

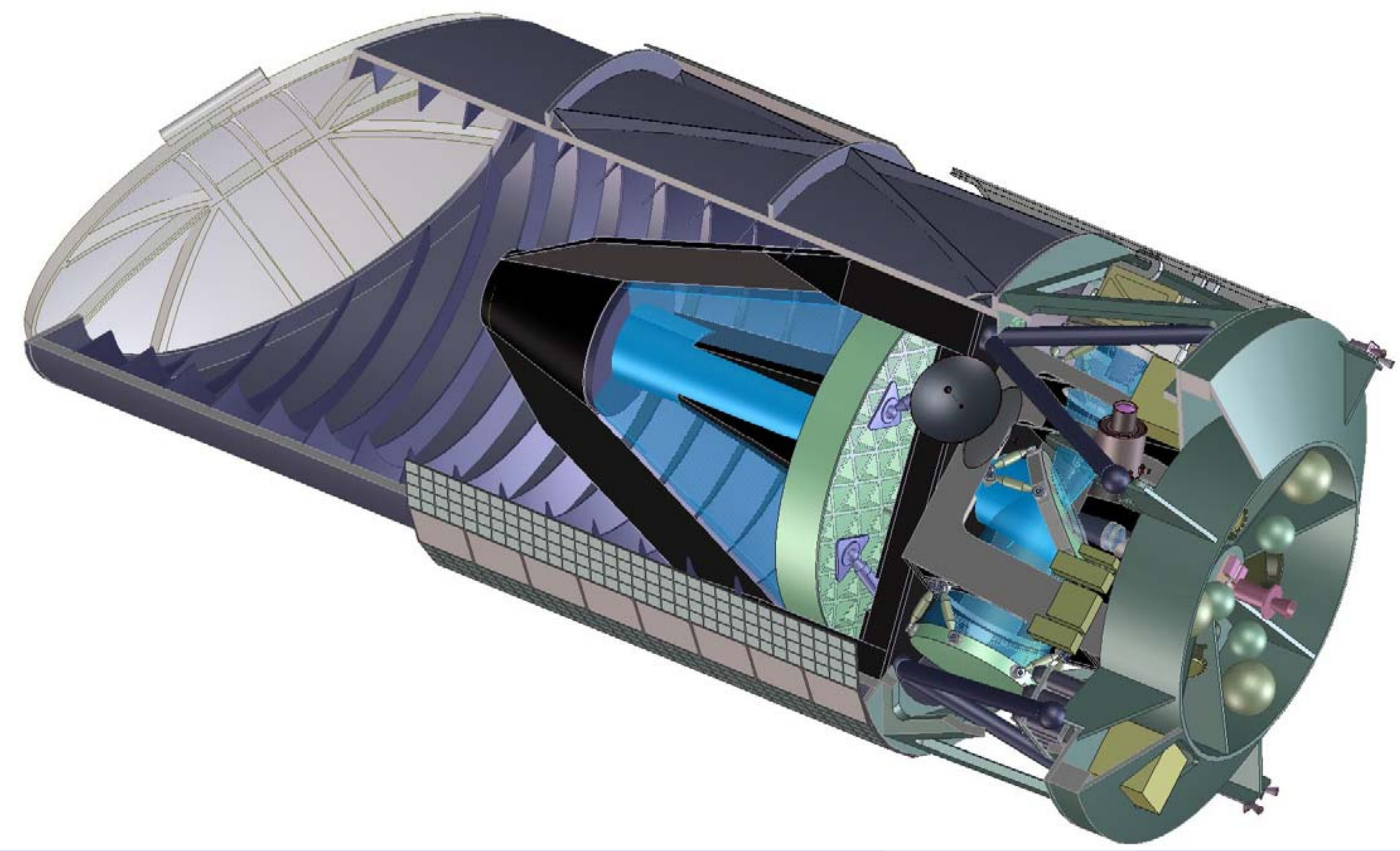


Situation in next decade



Dark Energy – The Next Generation

Dedicated dark energy probe



SNAP: Supernova/Acceleration Probe

The Large Survey Synoptic Telescope (LSST)

The LSST will be a large, wide-field ground-based telescope designed to deeply image the entire visible sky every few nights.