A Non-Renormalization Theorem in Gapped QFT

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Our goal

To show that in 3D massive QFT, the parity odd part of the 2-point functions of $U(1)$ currents $\langle j_\mu j_\nu \rangle$ and energy momentum tensors, $\langle T_{\mu\nu} T_{\rho\sigma} \rangle$, is one-loop exact.

We shall study j^μ, a (global) $U(1)$ current.
In massive theories, the parameterization of $\langle j_\mu(p) j_\nu(-p) \rangle$ is

$$a\delta_{\mu\nu} + \delta\kappa\, \epsilon_{\mu\nu\rho} p^\rho + O\left(p^2\right).$$

Note that only O (momentum) terms in the perturbative expansion can contribute to $\delta\kappa$ at zero momentum; we shall now see that no such terms exist.

Consider a generic QFT described by an action $\mathbf{S} = \int \mathcal{L}$, where

$$\mathcal{L} = \mathcal{L}_0 + \sum_i \lambda_i O_i$$

and O_i are scalar operators.

The perturbative expansion reads

$$\langle j_\mu(p) j_\nu(-p) \rangle = \langle j_\mu(p) j_\nu(-p) \rangle_0 - \sum_i \lambda_i \langle j_\mu(p) j_\nu(-p) O_i(0) \rangle_0 + \sum_{ij} \frac{1}{2} \lambda_i \lambda_j \langle j_\mu(p) j_\nu(-p) O_i(0) O_j(0) \rangle_0 + \ldots \,.$$

In gapped theories, there are no infrared singularities, and so $\langle j_\mu(p) j_\nu(-p) O_1(0) \dots O_n(0) \rangle$ is well defined as the limit

$$\lim_{k_i \to 0} \langle j_\mu(p) j_\nu(q) O_1(k_1) \dots O_n(k_n) \rangle .$$

We will take this limit in two steps, $k_{i \neq 1} \to 0$ followed by $k_1 \to 0$. Consider the most general tensor structure of

$$\langle j_\mu(p) j_\nu(q) O_1(k_1) O_2(0) \dots O_n(0) \rangle .$$

- The insertion of $O_1(k_1)$ allows p and q to be independent.
- The insertions at zero momentum do not impose or relax any constraints on the tensor structure.

Consequently, the parameterization of

$$\langle j_\mu(p) j_\nu(q) O_1(k_1) O_2(0) \ldots O_n(0) \rangle$$

does not depend on the number of insertions at zero momentum - so let's study the 3-point function

$$\langle j_\mu(p) j_\nu(q) O(k_1) \rangle .$$

We can now take the limit $k_1 \to 0$: if $O(\text{momentum})$ terms in $\langle j_\mu j_\nu O \rangle$ are for some reason forbidden, they must be absent from the rest of the perturbative corrections as well.

The Ward identity for the $U(1)$ symmetry is just

$$p^{\mu}\langle j_{\mu}(p) j_{\nu}(q) O(k_1)\rangle = 0.$$

The parameterization of $\langle j_{\mu}(p) j_{\nu}(q) O(k_1)\rangle$ is given by

$$a'\delta_{\mu\nu} + b\,\epsilon_{\mu\nu\rho}\,(p^{\rho} - q^{\rho}) + O\left(\text{momentum}^2\right).$$

Both a' and b must vanish to satisfy the Ward identity.

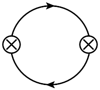
$$\Downarrow$$

there are no corrections to $\delta\kappa$

Remarks

1. What is so special about the one loop graph?

The only contribution to $\delta\kappa$, comes from $\langle j_\mu(p) j_\nu(q)\rangle_0$.
Since the current (in the free theory) is quadratic in the fields, $\langle j_\mu(p) j_\nu(q)\rangle_0$ corresponds to a one loop graph:



In the language of currents, the one-loop graph is a classical contribution.

Remarks

2. Why does the Ward identity forbid $O(\mathrm{momentum})$ terms in the tensor structure of $\langle j_\mu j_\nu O\rangle$?

Couple the global $U(1)$ current to a background gauge field a_μ, and the deformation O_i to a background source J_i.
We can then define

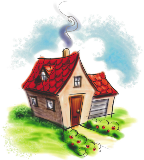
$$\langle j_\mu j_\nu O\rangle \equiv \frac{\delta}{\delta a^\mu}\frac{\delta}{\delta a^\nu}\frac{\delta}{\delta J} W\left[a, J_i\right]\Big|_{a=0, J_i=0}.$$

$O(\mathrm{momentum})$ terms in $\langle j_\mu j_\nu O\rangle$ correspond to terms in $W\left[a, J_i\right]$ with 2 $a's$, 1 J and only one derivative. There is one such term:

$$\int d^3x J \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho,$$

and it is ***NOT*** gauge invariant!

Take home message



Given two operators, A and B: if

$$\langle A(p) B(-p) \rangle$$

has a certian property, which is absent from the most general tensor structure of

$$\langle A(p) B(q) O(-p-q) \rangle$$

for an arbitrary scalar O -

that property is not renormalized!