Super-Schrödinger Symmetry and Holography

Jonas Probst

École Normale Supérieure, Paris

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Crete Centre for Theoretical Physics

Motivation

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Condensed Matter Systems:

- Often non-relativistic and strongly coupled
  $\rightarrow$ non-relativistic holography
- SUSY in the lab: e.g. fermions at unitarity

Non-relativistic limit as group embedding

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e.g. massless KG in d flat dimensions: $\partial^2 \tilde{\Phi} = 0$

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$\longrightarrow$ free Schrödinger equation in $(d-1)$ dimensions

- Schrödinger algebra Schr$(d-1) \subset$ Conformal(d) as the subalgebra that commutes with $-\mathrm{i}P_- =: -\mathrm{i}\sqrt{2}M$
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- the same embedding goes through for spinors: massless Dirac eqn. in $d \to$ Levy-Leblond eqn. in $(d-1)$
- procedure does in general not break SUSY (checked for free hypermultiplet)

$\to$ (Super-Schrödinger in $d-1$) $\subset$ (Superconformal in d)

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$$\mathrm{d}s^2 = \frac{-2\left(\mathrm{d}x^-\right)^2}{r^{2z}} + \frac{-2\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}x^i\mathrm{d}x^i + \mathrm{d}r^2}{r^2}$$

[Son 2008, Balasubramanian & McGreevy 2008]

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  - $z = 2 \to$ full Schrödinger symmetry
  - non-relativistic spacetime dimension $(d-1) > 2$
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- Task: Check that the boundary supercurrent is indeed in a Super-Schrödinger mulitplet:
  holographic calculation with gravitino in the Schrödinger bulk

Thank you for your attention.