Integrability beyond the $\mathcal{N} = 4$ paradigm

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DESY Theory

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Erasmus IP 2013 - Gong show

arXiv:0912.4918 and arXiv:1006.0015 with Abhijit Gadde and Leonardo Rastelli
arXiv:1105.3972 with Pedro Liendo and Leonardo Rastelli
arXiv:1105.3487 with Christoph Sieg
2 more to appear soon

Exact results in $\mathcal{N} = 4$ SYM

- The spectral problem is solved *at large* N_c. (**Integrability**)
- Wilson Loops
  - Circular WL (**Localization**) (*any* N_c)
  - Small Cusp WL (**Localization** plus **Integrability**) (*large* N_c)

In progress

- Scattering amplitudes (It seems $\exists$ deep connection with **Integrability**)
- n-point correlation functions (**Integrability** plays a crucial role)

The spin chain picture

We want to calculate the anomalous dimension of:

$$\mathcal{O} = \operatorname{tr}\left(Z^{L-M}X^{M}\right)$$

Map this problem to a spin chain (Minahan & Zarembo):

$$Z \quad \longleftrightarrow \quad |\uparrow\rangle \quad \text{and} \quad X \quad \longleftrightarrow \quad |\downarrow\rangle$$

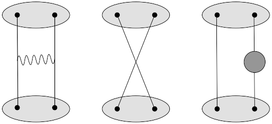
An operator with L constituent fields is mapped to a distribution of spins on a periodic one-dimensional lattice of length L:

$$\operatorname{tr}(ZZZXXZZZXZZZ\ldots) \qquad \longleftrightarrow \qquad |\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\ldots\rangle$$

- The map is one-to-one if the states are required to be translationally invariant.

The spin chain picture and Integrability

The **mixing matrix** acts linearly on the operators and thus can be interpreted as a Hamiltonian of a **spin chain**.



$$\Gamma = \frac{\lambda}{8\pi^2} \sum_{\ell=1}^{L} (\mathbb{I} - \mathbb{P}_{\ell,\ell+1}) \equiv \frac{\lambda}{8\pi^2} H_{XXX}$$

The XXX spin chain is integrable:

- from the 2-body problem you get the solution of the *n*-body.

The Complete $\mathcal{N} = 4$ spin chain contains all the members of the ultrasort multiplet $X, Y, Z, \bar{X}, \bar{Y}, \bar{Z}, \lambda^A_\alpha, \bar{\lambda}^{\dot{\alpha}}_A, \mathcal{F}_{\alpha\beta}, \bar{\mathcal{F}}_{\dot{\alpha}\dot{\beta}}$ plus derivatives at each lattice site.

Is there integrability beyond the $\mathcal{N}=4$ paradigm?

- $SU(2,1|2)$ subsector in any $\mathcal{N}=2$ SuperConformal gauge theory

$$\phi \qquad \lambda_{+}^{\mathcal{I}} \qquad \mathcal{F}_{++} \qquad \mathcal{D}_{+\dot{\alpha}}$$

$$\mathcal{H}_{\mathcal{N}=2}(g) = \mathcal{H}_{\mathcal{N}=4}(f(g)) \qquad g \longrightarrow f(g) = g + \zeta(3)g^3 + \dots$$

- $SU(2,1|1)$ subsector in any $\mathcal{N}=1$ SuperConformal gauge theory
- $SU(2,1)$ subsector in any $\mathcal{N}=0$ SuperConformal gauge theory

Outside these sectors (When Hypermultiplets collide)

- Twisted Yang Baxter equation