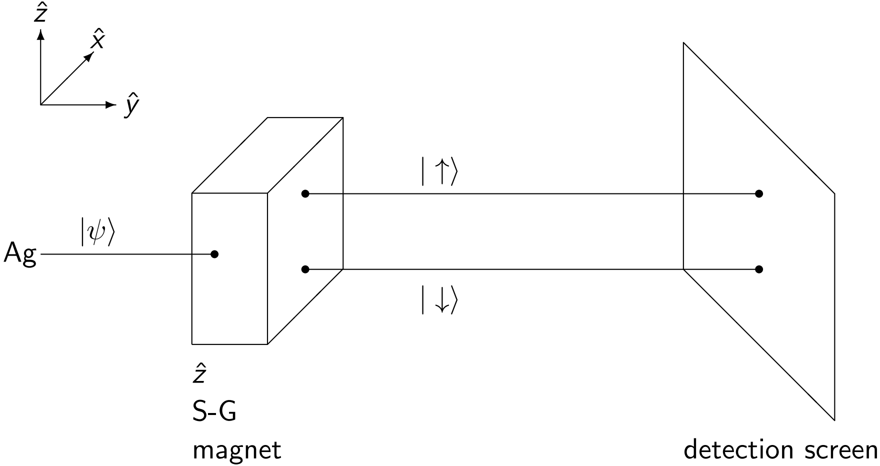
Brian Padden
Masters Student
Programme in Theoretical and Mathematical Physics
Ludwig-Maximilians-Universität München
Supervisor: Dr. Robert Helling

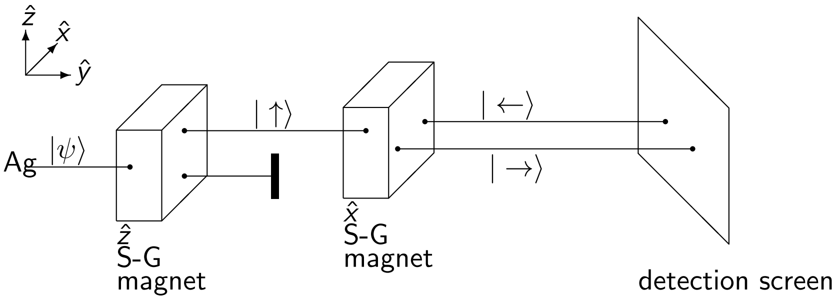
A Quantum Mechanics Problem

Stern-Gerlach Experiment: Level 1



Wow - not continuous but discrete. Quantum Mechanics!

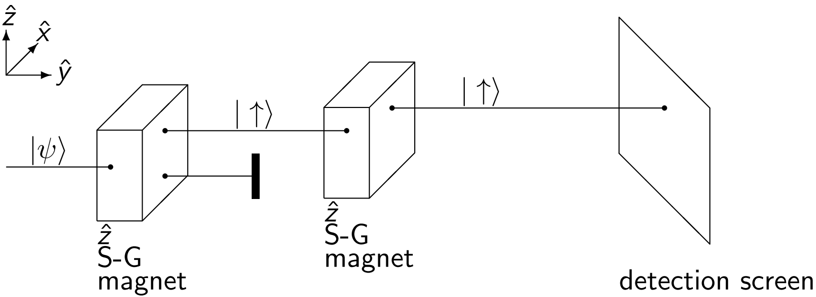
Stern-Gerlach Experiment: Level 2



Wow - a definite state for one observable

doesn't lead to definite outcomes for all observables!

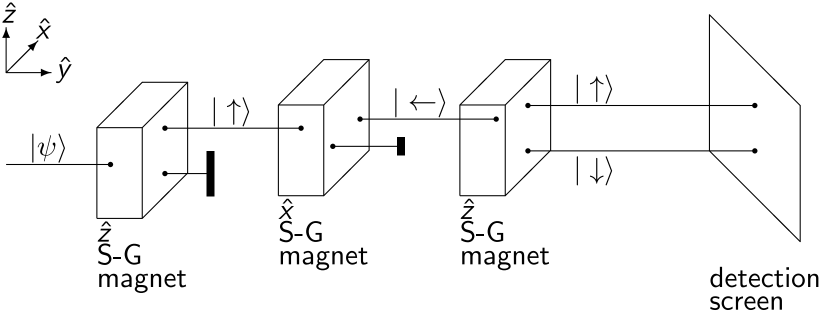
Stern-Gerlach Experiment: Level 2'



Wow - we don't just have random splitting!

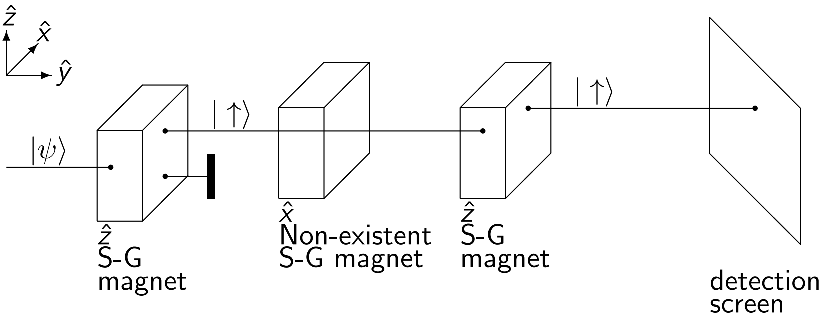
A second observation'similar' to the first is more probable.

Stern-Gerlach Experiment: Level 3



Wow - measurements change the physical state!

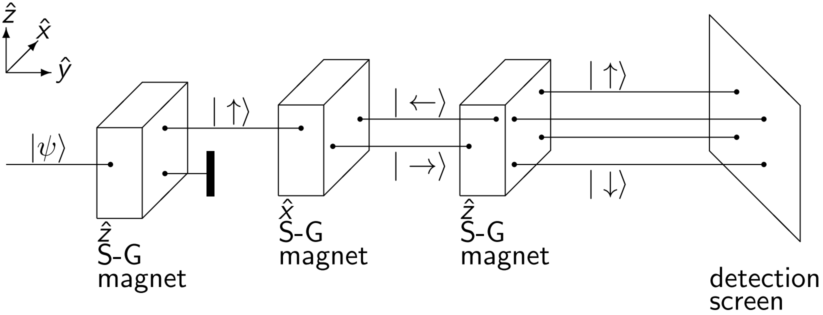
Stern-Gerlach Experiment: Level 3' (= Level 2')



Wow - turning off the magnet means we have not measured!

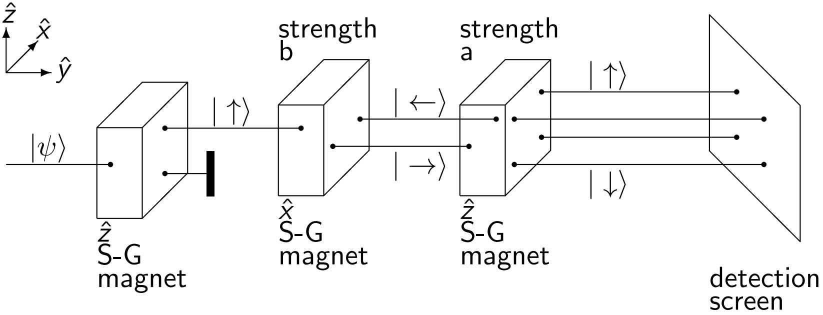
But - what if $\hat{x}$ is measured with ε- strength?

Stern-Gerlach Experiment: Level 4



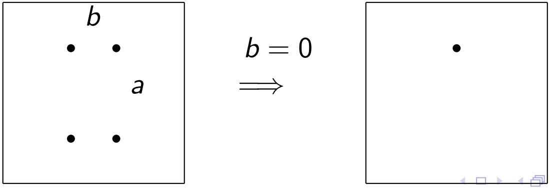
Let's smoothly adjust the strength of the $\hat{x}$ magnet down to zero

Stern-Gerlach Experiment: Level 4



Let's smoothly adjust the strength of the $\hat{x}$ magnet down to zero

Atom's-eye view of detection screen



Consideration of principles

Why the phase transition?
Where does the measurement take place?
How does the transition take place? At some b_{crit}?
Poll
Does the measurement take place at the site of the magnet?

IF: the magnet performs a measurement

Then at the screen, the wavepacket is contained in one of the four dots.
Then we need to find some scale in the magnet which can set b_{crit}.

IF: the magnet performs unitary time evolution

Then we can hope for interference later on.

The scale setting b_{crit} should be the width of the wave packet, σ.

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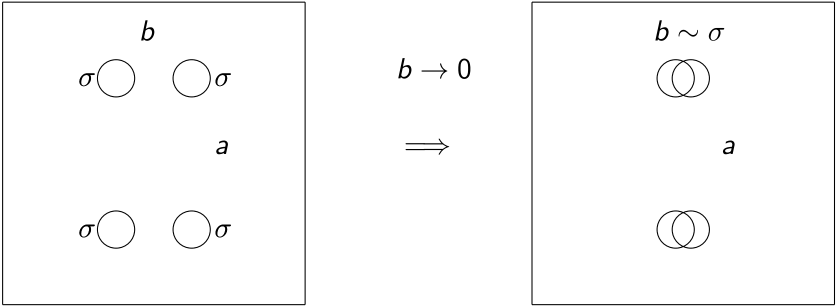
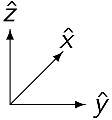
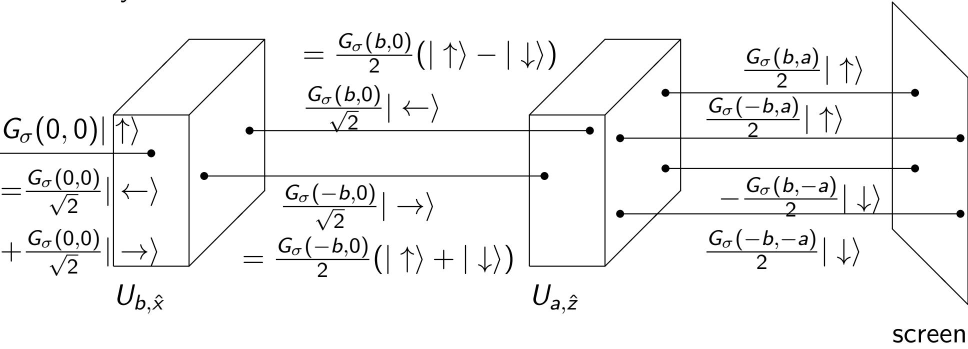


Calculation

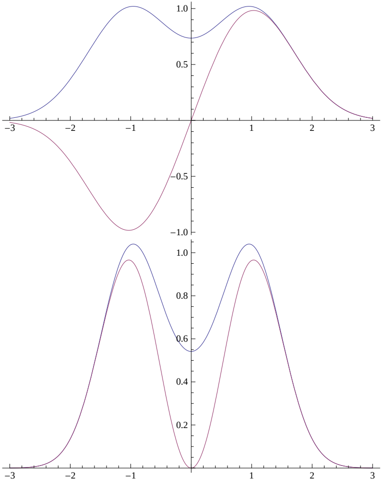
$$U_{a,\hat{z}}\psi(\vec{x})|\uparrow\rangle = \psi(\vec{x}+a\hat{z})|\uparrow\rangle$$
$$U_{a,\hat{z}}\psi(\vec{x})|\downarrow\rangle = \psi(\vec{x}-a\hat{z})|\downarrow\rangle$$



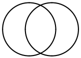
$$G_\sigma(x_0, z_0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2+(z-z_0)^2}{2\sigma^2}}$$



Interference: Probability Migration

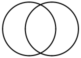


$$\frac{1}{2}(G_\sigma(a,b) + G_\sigma(a,-b))|\uparrow\rangle$$



$b \sim \sigma$

a



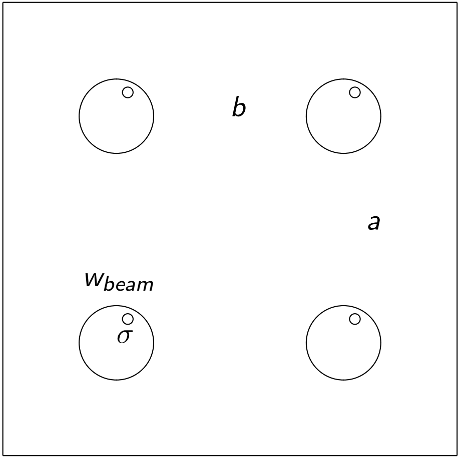
$$\frac{1}{2}(G_\sigma(-a,-b) - G_\sigma(-a,b))|\downarrow\rangle$$

The Answer

A magnet can measure spin if it is strong enough to split the wavepacket into spatially distinct regions.

Stern-Gerlach Experiment: Level 5

What if the wavepackets are not as wide as the beam?



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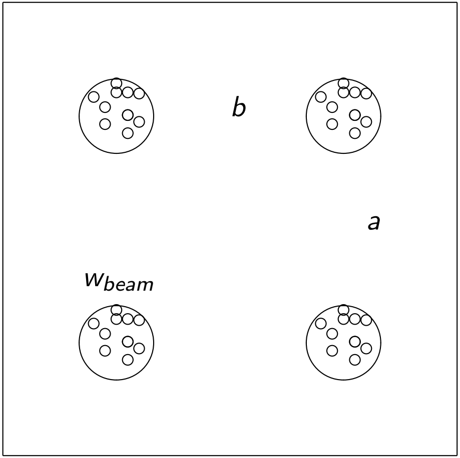

Stern-Gerlach Experiment: Level 5

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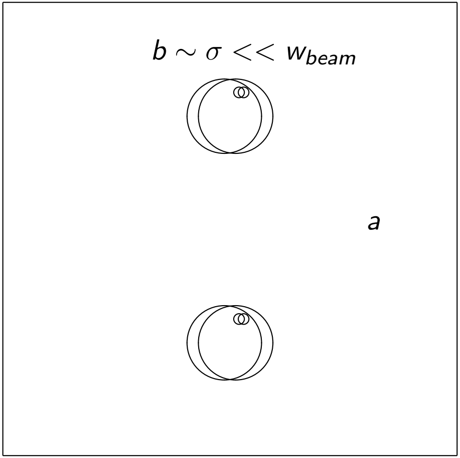

Then: probability migration

does not occur until $b \sim \sigma$

Pedagogical Value

- Trains the concept of interference
- If posed without hints: a good exercise in creative problem solving
- Understanding state vectors: spatially dependent spinors
- Forces students to think about unitary evolution and measurement
- More detailed look at how measurements are actually performed.
- Trains students to find the relevant scale to determine the answer.

Works Cited:
J.J. Sakurai: Modern Quantum Mechanics