The conformal bootstrap and higher spin symmetry

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Outline

CFT: Motivation and Definition

- Understanding QFT: fixed points of RG flows- special points of enhanced symmetry in the space of field thoeries. Define QFT in the EFT picture as a relevant deformation of a CFT.
- Phase transitions: describe universal features of systems near criticality.
- Holography: can describe quantum theories of gravity (at least in AdS) via AdS/CFT.

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Conformal Symmetry and CFT

- Conformal Group: $SO(D,2)$ in D spacetime dimensions ($D \geq 3$). All fields transform in representations of $SO(D,2)$. Representation labelled by Cartans of the compact subgroup $SO(D) \times SO(2)$: $\mathcal{R}$ and dimension Δ.
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- O_Δ: local Primary ($K_\mu O_\Delta = 0$) operator with scaling dimension Δ; $\mathcal{R}$: representation of $SO(D)$ in which O_Δ transforms.
- Operator Product Expansion: $O_i(x)O_j(0) = \sum_k c_{ijk} F(x, \partial_y) O_k(y) \mid_{y=0}$
- Unitarity $\rightarrow \Delta \geq \Delta_{min}(\mathcal{R})$

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The Conformal Bootstrap

- Basic idea of bootstrap: Use general principles like Unitarity, Analyticity, Symmetry to determine physical observables of interest: S matrices (or correlation functions).
- Conformal symmetry is constraining: 2 and 3 point functions of scalar conformal primary operators fixed by conformal invariance:
- $\langle \phi_\Delta(x_1)\phi_\Delta(x_2)\rangle = \frac{k}{x_{12}^{2\Delta}}$; normalise to set $k = 1$
- $\langle \phi_{\Delta_1}(x_1)\phi_{\Delta_2}(x_2)\phi_{\Delta_3}(x_3)\rangle = \frac{c_{123}}{x_{12}^{2\alpha_{123}} x_{23}^{2\alpha_{231}} x_{31}^{2\alpha_{312}}}$

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4-point functions: not fixed but functional form quite constrained.

- $\langle \phi_\Delta(x_1)\phi_\Delta(x_2)\phi_\Delta(x_3)\phi_\Delta(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}x_{34}^{2\Delta}} f(u,v)$
- $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \quad conformal\ cross - ratios$
- Crossing symmetry (OPE associativity): can do OPE contraction of different operators within the correlation function- different ways should give same results. Leads to further constraints on f:
- $v^\Delta f(u,v) = u^\Delta f(v,u)$

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CFTs with higher spin operators

- 2-point function again completely fixed by conformal symmetry
- $\langle O_{s,\Delta}(1) O_{s,\Delta}(2) \rangle = \frac{\textit{unique tensor structure}}{x_{12}^{2\Delta}}$
- 3-point function is determined as a sum of finite number of tensor structures with undetermined constant coefficients
- $\langle O_{s_1,\Delta_1}(1) O_{s_2,\Delta_2}(2) O_{s_3,\Delta_3}(3) \rangle = \frac{\textit{finitely many tensor structures}}{x_{12}^{2\alpha_{123}} x_{23}^{2\alpha_{231}} x_{31}^{2\alpha_{312}}}$

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Higher Spin Symmetry

- Conformal symmetry is constraining but not enough. Maybe together with some other symmetry it can help us bootstrap .
- Idea: use higher spin symmetry- infinite tower of exactly conserved higher spin currents $\partial_{\mu_1} J^{\mu_1\mu_2\cdots\mu_s} = 0$
- However, unfortunately, we encounter the Maldacena-Zhiboedov theorem:
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- Possible way out: use broken (in a special way) higher spin symmetry. This is still quite constraining. H.S. Symmetry broken by finite N effects in a large N CFT
- Example: 3d CS gauge theory coupled to fundamental matter (boson or fermion)
- We get anomalous "conservation"eqs.
- $\partial.J_s = \frac{1}{N} J_{s_1} J_{s_2} + \frac{1}{N^2} J_{s_1} J_{s_2} J_{s_3}$

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Large N solution for N-point functions in such a theory (schematic)

- $\partial.\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \frac{1}{N} \langle J_s J_{s'} J_{s_2} J_{s_3} \rangle + O(1/N^2)$
- Use large N factorisation
- $\frac{1}{N} \langle J_s J_{s'} \rangle \langle J_{s_2} J_{s_3} \rangle + perm. + O(1/N^2)$
- So leading order (to $1/N$) expression for the 3-point function is the solution of the above simple diff. eq. Schematically
- $\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{N=\infty} + \frac{1}{N} \int \langle J_s J_{s'} \rangle \langle J_{s_2} J_{s_3} \rangle + perm.$

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