Niels Bohr Institutet

Three-point functions in SU(2) × SU(2) sector of ABJM

Presenter: Ara Martirosyan

arXiv:1211.1359

Cowriters: Agnese Bissi Charlotte Kristjansen and Marta Orselli

Strong/weak coupling

$\mathcal{N} = 4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit: λ – fixed, $N \to \infty$	Free strings: T – fixed, $g_s = 0$
Strong coupling: $N \to \infty$, $\lambda \gg 1$	Classical strings: $g_s = 0$, $T \gg 1$
Local operators	Strings states

Local gauge invariant operators $\longleftrightarrow$ string states
Conformal dimensions, $\Delta \longleftrightarrow$ energies of string states

$$\langle \mathcal{O}(x)\bar{\mathcal{O}}(y)\rangle \sim \frac{1}{|x-y|^{\Delta}}$$

The spectral problem of $\mathcal{N}=4$ SYM:
Determine $\Delta = \Delta(\lambda, N) \Leftrightarrow$ Diagonalize dilatation generator D

$$\langle \bar{\mathcal{O}}_I(x)\mathcal{O}_J(y)\mathcal{O}_K(z)\rangle = \frac{C_{IJK}}{|x-y|^{\Delta_I+\Delta_J-\Delta_K}|y-z|^{\Delta_J+\Delta_K-\Delta_I}|x-z|^{\Delta_I+\Delta_K-\Delta_J}}$$

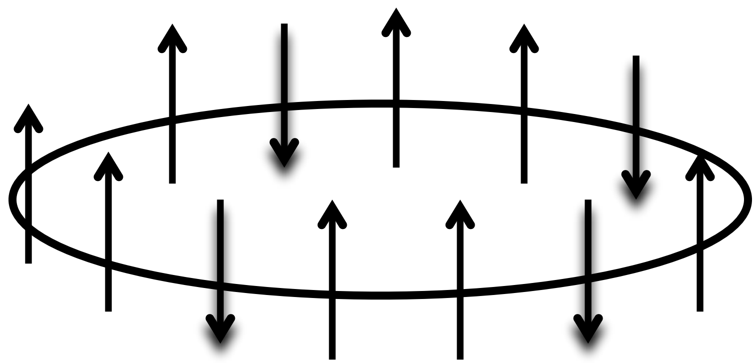
OPE $\longrightarrow$ $$\mathcal{O}_I(x)\mathcal{O}_J(y) = \sum_K \frac{C_{IJK}}{|x-y|^{\Delta_I+\Delta_J-\Delta_K}}\mathcal{O}_K(x)$$

C_{IJK} Δ, encode all the dynamics of CFT

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle = \frac{1}{|x-y|^{2L}}\left(1 - \frac{\lambda}{16\pi^2}\ln\Lambda^2|x-y|^2\sum_{\ell=1}^{L}(C - 2P_{\ell,\ell+1} + K_{\ell,\ell+1})\right)$$
$$\times\, \delta^{j_1}{}_{i_1}\cdots\delta^{j_L}{}_{i_L} + \text{cycles.}$$

1-loop Dilatation operator $\longrightarrow$ $$\Gamma_{SU(2)} = \frac{\lambda}{8\pi^2}\sum_{\ell=1}^{L}(1 - P_{\ell,\ell+1}).$$

$Tr(zzxzzxzxzzxz)$ $\longrightarrow$

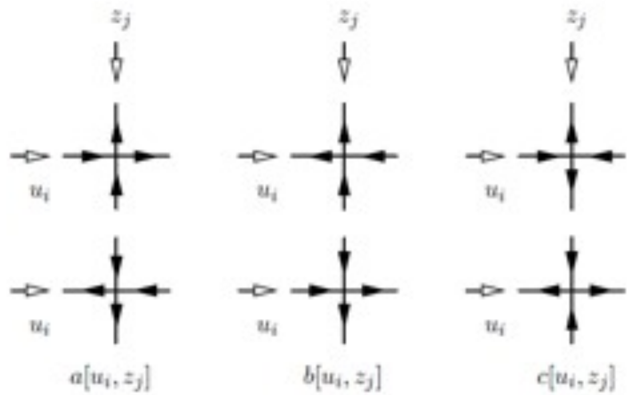


The Foda approach

Operators of the three-point function are treated as spin chain eigenstates produced by the algebraic Bethe ansatz.

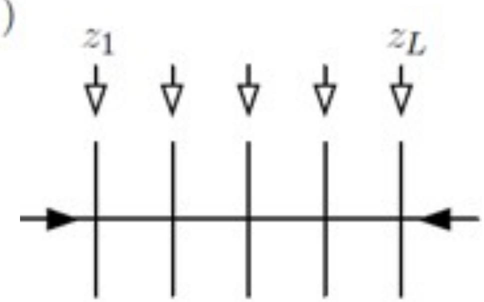
$$\mathcal{B}(u_1)\ldots\mathcal{B}(u_N)\,|\uparrow\ldots\uparrow\rangle$$

$$R_{ab}(u_a,u_b)=\begin{pmatrix} a[u_a,u_b] & 0 & 0 & 0 \\ 0 & b[u_a,u_b] & c[u_a,u_b] & 0 \\ 0 & c[u_a,u_b] & b[u_a,u_b] & 0 \\ 0 & 0 & 0 & a[u_a,u_b] \end{pmatrix}_{ab}$$



$$a[u_i,z_j]=\frac{(u_i-z_j+\eta)}{(u_i-z_j)},\quad b[u_i,z_j]=1\quad c[u_i,z_j]=\frac{\eta}{(u_i-z_j)}$$

$$M_a(x,\{z\}_L)=\begin{pmatrix} A(x) & B(x) \\ C(x) & D(x) \end{pmatrix}_a$$

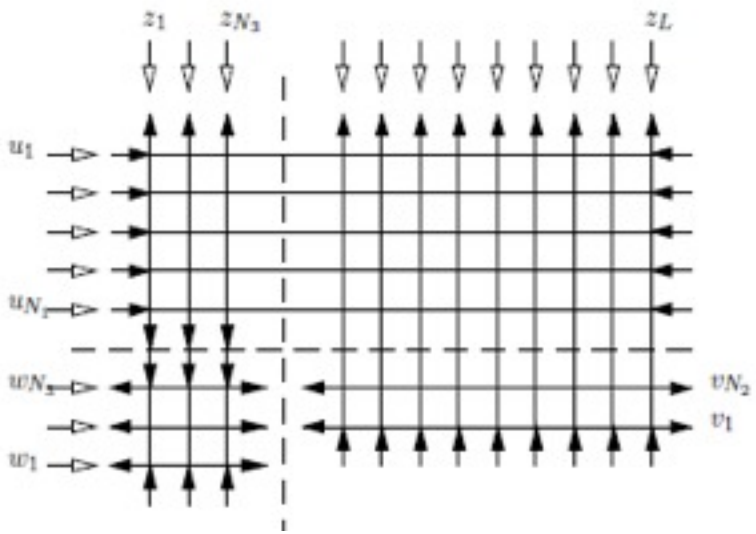


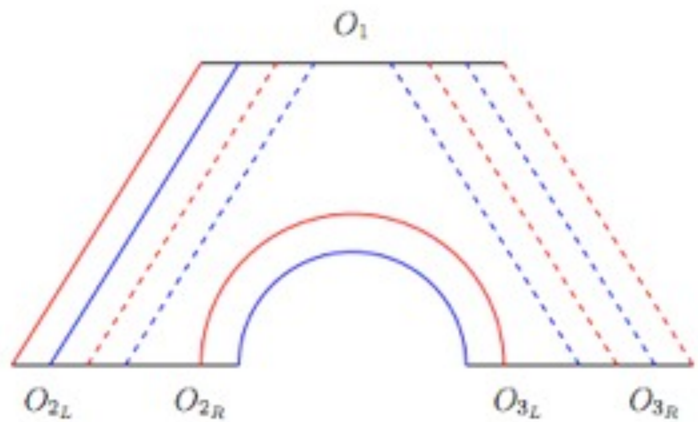
B-line

$$c_{123}^{(0)} = \mathcal{N}_{123} \sum_{a,b,c} {}_r\langle \mathcal{O}_{3_c} | \mathcal{O}_{1_a} \rangle_l \; {}_r\langle \mathcal{O}_{1_a} | \mathcal{O}_{2_b} \rangle_l \; {}_r\langle \mathcal{O}_{2_b} | \mathcal{O}_{3_c} \rangle_l$$

$${}_r\langle \mathcal{O}_2 | \mathcal{O}_3 \rangle_l = 1 \qquad\qquad {}_r\langle \mathcal{O}_2 | \mathcal{O}_1 \rangle_l = {}_l\langle \mathcal{O}_1 | \mathcal{O}_2 \rangle_r$$

$$c_{123}^{(0)} = \mathcal{N}_{123} \sum_{\alpha \cup \bar{\alpha} = \{u\}_{\beta N_1}} {}_r\langle \mathcal{O}_3 | \mathcal{O}_1 \rangle_l \; {}_l\langle \mathcal{O}_2 | \mathcal{O}_1 \rangle_r = \mathcal{N}_{123} \left({}_r\langle \mathcal{O}_3 |_l \bigotimes \langle \mathcal{O}_2 | \right) | \mathcal{O}_1 \rangle$$





2-loop dilatation operator in SU(2)xSU(2) sector

of N=6 superconformal Chern-Simons theory

$$\Gamma = \lambda^2 \sum_{l=1}^{2L} (1 - P_{l,l+2}) = \lambda^2 \sum_{l \in odd}^{2L} (1 - P_{l,l+1}) + \lambda^2 \sum_{l \in even}^{2L} (1 - P_{l,l+1})$$

two decoupled Heisenberg spin chains

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \Pi_{k \neq j}^{K_u} \frac{u_j - u_k + i}{u_j - u_k - i}, \qquad \left(\frac{v_j + i/2}{v_j - i/2}\right)^L = \Pi_{k \neq j}^{K_v} \frac{v_j - v_k + i}{v_j - v_k - i},$$

two chains being related only through the momentum constraint

$$\Pi_{j=1}^{K_u} \left(\frac{u_j + i/2}{u_j - i/2}\right) \Pi_{k=1}^{K_v} \left(\frac{v_k + i/2}{v_k - i/2}\right) = 1,$$

The scalar sector consists of two complex scalars Z_1 Z_2 $N\times\overline{N}$ $U(N)\times U(N)$
rep. of
and two complex scalars W_1 W_2 $\overline{N}\times N$

Scalars can be grouped into multiplets of of the R-symmetry group SU(4)

$$Z^a = (Z_1, Z_2, \overline{W}_1, \overline{W}_2) \qquad \overline{Z}_a = (\overline{Z}_1, \overline{Z}_2, W_1, W_2)$$

Z^a transforming in the fundamental representation and

$\overline{Z}_a$ in the anti-fundamental representation of SU(4)

A gauge invariant single trace operator containing only scalars

$$O = C^{b_1 b_2 \dots b_n}_{a_1 a_2 \dots a_n} Tr(Z^{a_1}\overline{Z}_{b_1} \dots Z^{a_n}\overline{Z}_{b_n})$$

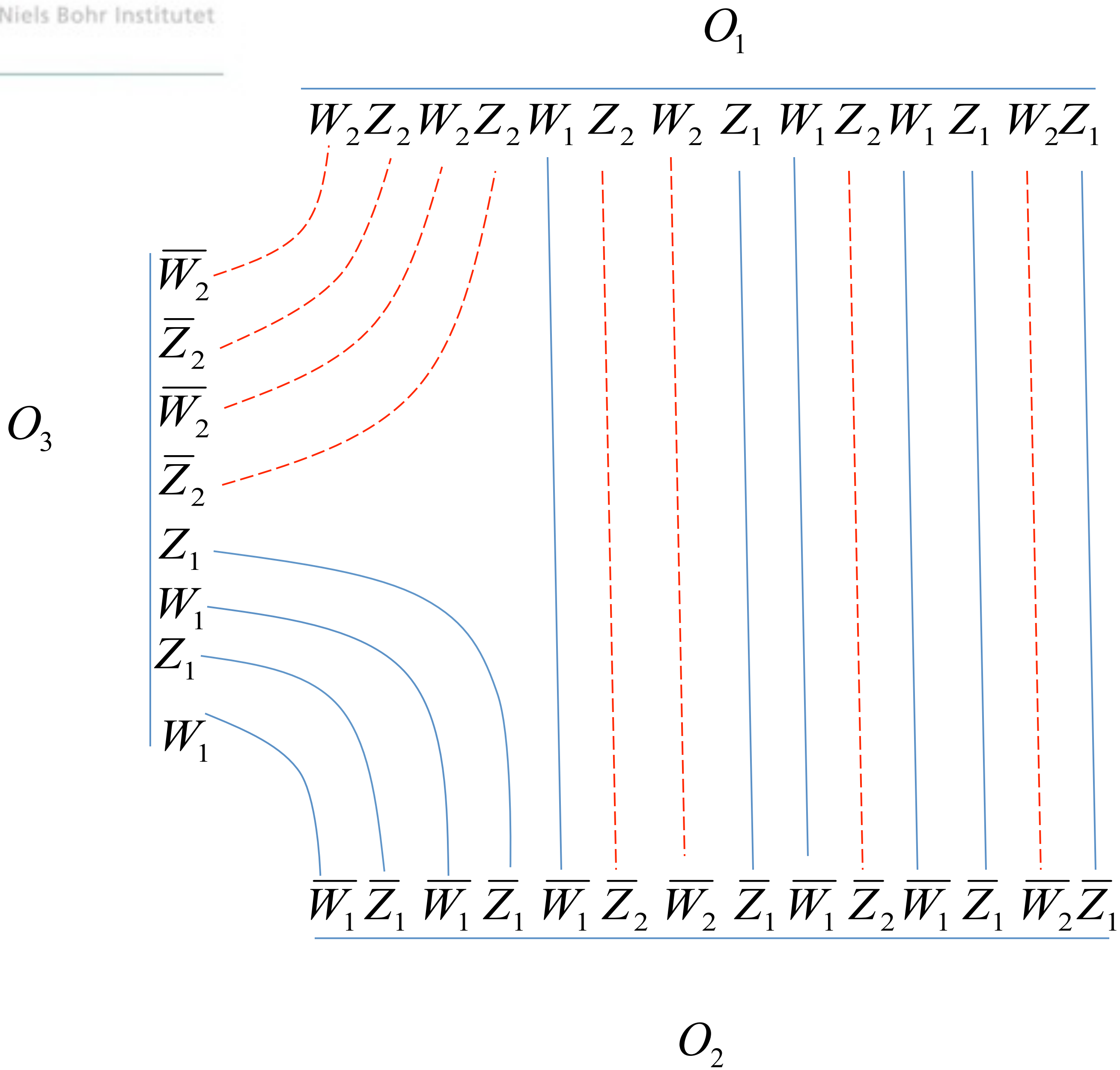
Operator O has an interpretation of a spin chain state of length 2n with the spins in the odd sites transforming in the fundamental representation and spins on the even sites transforming in the anti-fundamental representations of SU(4)

Operator	Vacuum odd	Excitation	Vacuum even	Excitation
O_1	$(J-J_1)$ Z_1	J_1 Z_2	$(J-J_2)$ W_1	J_2 W_2
O_2	$(J-J_2+j_1)$ $\overline{W}_1$	(J_2-j_2) $\overline{W}_2$	$(J-J_1+j_1)$ $\overline{Z}_1$	(J_1-j_2) $\overline{Z}_2$
O_3	j_1 Z_1	j_2 $\overline{W}_2$	j_1 W_1	j_2 $\overline{Z}_2$

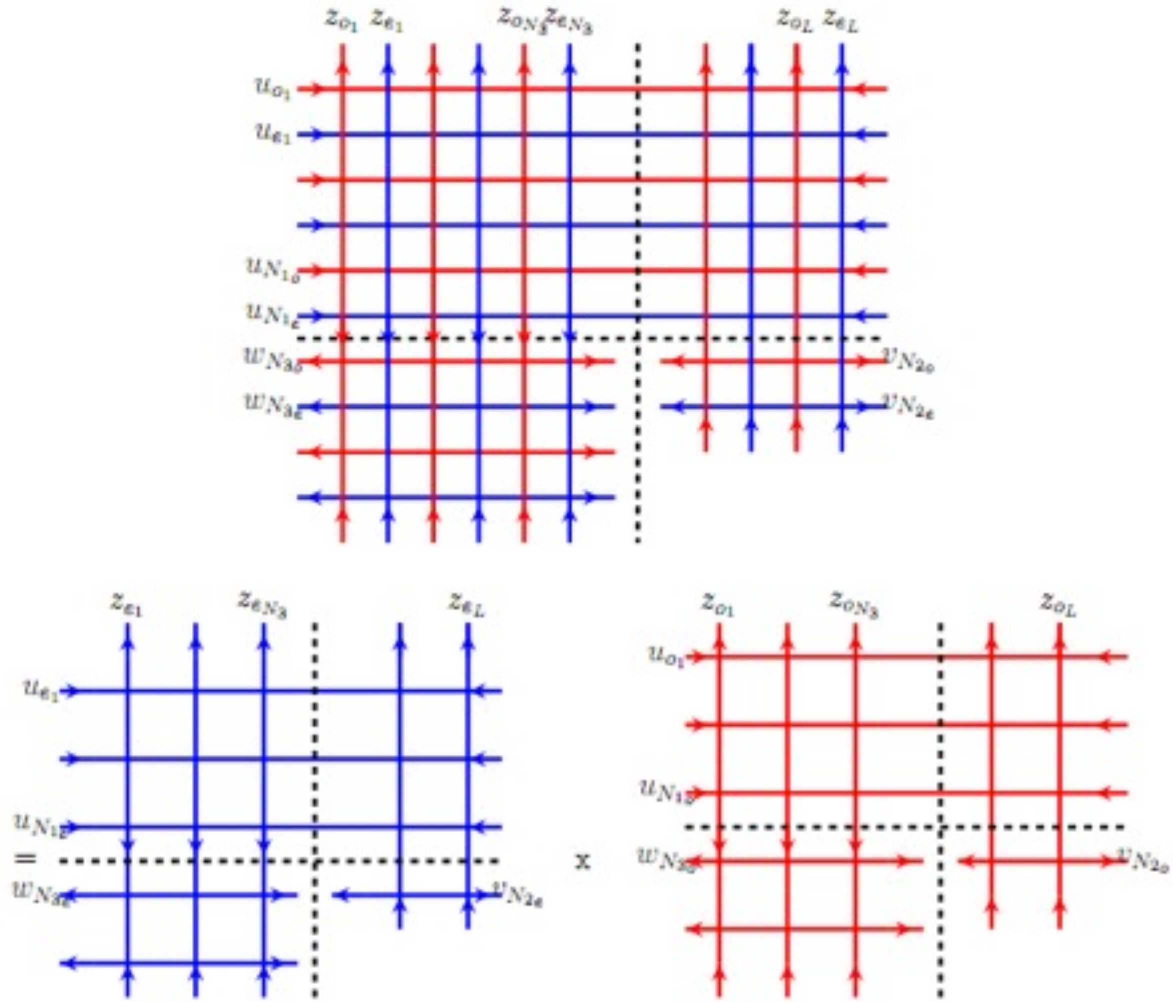
Operator	odd	even
O_1	$(Z_1, Z_2, 0, 0)$	$(0\ ,0, W_1, W_2)$
O_2	$(0\ ,0, \overline{W}_1, \overline{W}_2)$	$(\overline{Z}_1, \overline{Z}_2, 0, 0)$
O_3	$(Z_1, 0, 0, \overline{W}_2)$	$(0, \overline{Z}_2, W_1, 0)$

$$O_1 = C^{j_1 j_2 \ldots j_J}_{i_1 i_2 \ldots i_J} Tr(Z_{i_1} W_{j_1} \ldots Z_{i_J} W_{j_J})$$

Niels Bohr Institutet



Niels Bohr


z_{o_1} z_{e_1} $z_{o_{N_3}}$ $z_{e_{N_3}}$ z_{o_L} z_{e_L}
u_{o_1}
u_{e_1}
$u_{N_{1o}}$
$u_{N_{1e}}$
$w_{N_{3o}}$
$w_{N_{3e}}$
$v_{N_{2o}}$
$v_{N_{2e}}$
=
x

THANK YOU!