# Lipatov's Effective Action Approach to High Energy QCD

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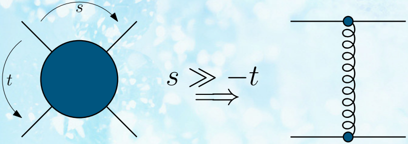
† *Based on work in collaboration with G. Chachamis, M. Hentschinski & A. Sabio Vera: 1202.0649, 1211.2050, 1212.4992 and work to appear soon*

# Why Studying the High Energy Limit of QCD?




- Purely Phenomenological Interest
- Emergence of the Pomeron and Reggeization

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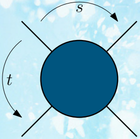
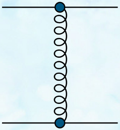



$s \gg -t$
$\Longrightarrow$



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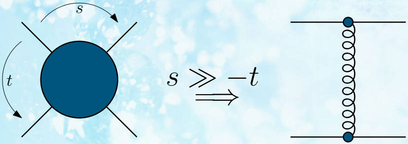


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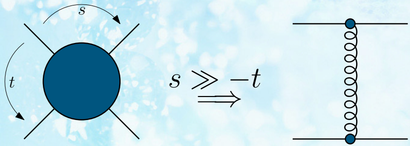


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[Mueller'90]

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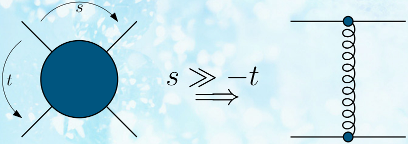


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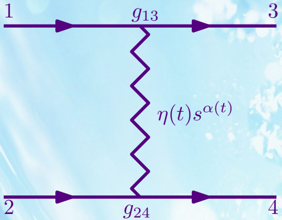
# Reggeons in QCD

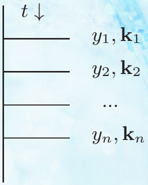


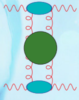

**Amplitudes in Multi-Regge Kinematics: Reggeization**

$\mathcal{M}^{\mathrm{LLA}}_{2\to 2+n} = \mathcal{M}^{\mathrm{tree}}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}$

$\omega(t)$ = Regge Trajectory

- Reggeon Emerges in QCD from Resumming an Infinite Tower of Virtual Corrections Enhanced by $\ln(s/|t|)$ Factors



$Y \sim \ln s$

$\mathbf{k}_i \sim \mathbf{k}_{i+1}$

$y_i << y_{i+1}$

$$\alpha_s^n \int_0^Y dy_1 \int_0^{y_1} dy_2 ... \int_0^{y_{n-1}} dy_n \sim \frac{(\alpha_s Y)^n}{n!}$$

**BFKL Equation**

$$\omega f_\omega(\mathbf{k}, \mathbf{k}') = \delta^2(\mathbf{k} - \mathbf{k}') + \int d^2\boldsymbol{\kappa}\, \mathcal{K}(\mathbf{k}, \boldsymbol{\kappa}) f_\omega(\mathbf{k}, \mathbf{k}')$$

# Lipatov's High-Energy Effective Action

## Effective Field Theory Approach

- Powerful computational tool
- Unitarity directly restored
- Ultimately should lead to 2d reggeon field theory enjoying conformal invariance, probably integrable

$$S_{\text{eff}} = S_{\text{QCD}/\mathcal{N}=4\,\text{SYM}} + S_{\text{ind}};$$

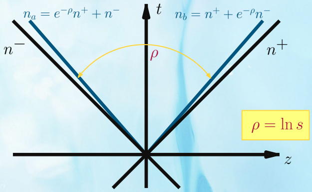
$$S_{\text{ind}} = \int d^4x \,\text{Tr}\left[(W_+[v(x)] - \mathscr{A}_+(x))\,\partial_\perp^2 \mathscr{A}_-(x)\right] + \int d^4x \,\text{Tr}\left[(W_-[v(x)] - \mathscr{A}_-(x))\,\partial_\perp^2 \mathscr{A}_+(x)\right];$$

$$W_\pm[v] = v_\pm \frac{1}{D_\pm}\partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

$\mathscr{A}_\pm$: reggeons, $v_\mu$: gluons $\partial_\pm \mathscr{A}_\mp(x) = 0$

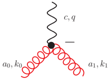
- Not a Wilsonian effective action: new degrees of freedom added
- Need of consistent gauge invariant subtraction procedure...

- ... and regularization of spurious divergences

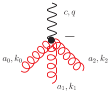


# Putting the Effective Action to Work


$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c}(n^-)^{\nu_0},$$



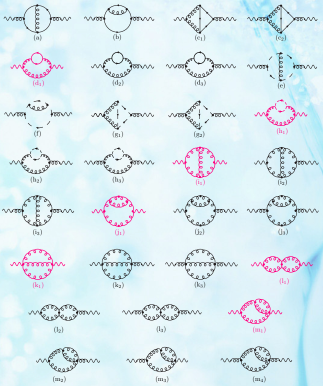

$$= g\boldsymbol{q}^2 f^{a_0 a_1 c}\frac{1}{k_0^-}(n^-)^{\nu_0}(n^-)^{\nu_1},$$




$$= \Delta^{\nu_0\nu_1\nu_2 -}_{a_0 a_1 a_2 c} = ig^2\boldsymbol{q}^2\left(\frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-}\right)(n^-)^{\nu_0}(n^-)^{\nu_1}(n^-)^{\nu_2},$$


$$= \frac{i}{2\boldsymbol{q}^2}.$$

...and higher-order vertices




Exact result for 2-loop Regge trajectory (& 1-loop reggeon vertices)

# Points to Take Home (if interested;-)

- The High-Energy Limit of Gauge Theories Hides Deep Structures and Enhanced Symmetry
- Effective Theory Description Expected to Be Very Useful
- We Have Consistently Extended Lipatov's Approach Beyond Tree Level and Checked It to 2-Loops

## Still to Be Done...

- Connection to Other Formalisms and 2d Reggeon Field Theory
- Applications to Phenomenology, etc.

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