Oscillons

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Finite energy solution: Nielsen - Olesen vortex

- **Abelian Higgs model in (2+1) dimensions**

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(|\phi|)$$

$V(|\phi|) = \frac{\lambda}{4}\left(|\phi|^2 - \frac{v^2}{2}\right)^2, \quad \lambda, v > 0$

Broken phase $\rightarrow$ vev for Higgs field, $|\phi| = v/\sqrt{2}$

- **Important**: Look for localized solutions with non trivial v.e.v
  - Arbitrary phase: quantization of magnetic flux

$$\Phi = \left(\frac{2\pi}{e}\right) n$$

Finite energy solution in the form of a Vortex

Finite energy solution in (1+1) D: Oscillons

How to obtain localized solutions in 1+1 dimensions ?

- Broken phase: $\phi \rightarrow \phi = \frac{1}{\sqrt{2}}(\upsilon + H)$ and $m_A^2 = e^2\upsilon^2$, $m_H^2 = 2\lambda\upsilon^2$

$$
\begin{aligned}
(\Box + 1)A + 2AH + H^2 A &= 0 \\
(\Box + q^2)H + \frac{1}{2}q^2 H^3 + \frac{3}{2}q^2 H^2 + HA^2 + A^2 &= 0
\end{aligned}
$$

  Rescaled e.m, $q = m_H/m_A$ uniquely characterizes the dynamics!
- Multiple scale perturbation theory $\rightarrow$ use slow scales $\rightarrow$ expand fields and operators in powers of small parameter ϵ
- To the leading order

$$
\begin{aligned}
A &= fe^{-it} + c.c \\
H &= B\left(b|f|^2 + f^2 e^{-2it} + f^{*2} e^{+2it}\right)
\end{aligned}
$$

Nonlinear Schrödinger (NLS) equation: Oscillons

- Reduction from the initial non integrable system to an integrable

$$i\partial_T f + \frac{1}{2}\partial_X^2 f + s|f|^2 f = 0$$

NLS admits exact bright and dark soliton solutions.

Bright soliton (Bs) solutions $f = f_0 \mathrm{sech}\left(\sqrt{|s|} f_0 X\right) e^{-i\omega T}$

Dark soliton (Ds) solutions $f = f_0 \tanh\left(\sqrt{|s|} f_0 X\right) e^{-i\omega T}$

But how good are our approximate solutions for A and H in the full problem ??

Robust solutions

- We numerically solve the full system, using as initial condition the above analytical solutions.

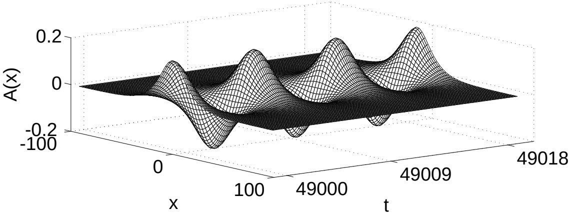

Figure: Evolution of the gauge field A as a function of (x, t). Parameter values used are $\epsilon = 0.1$, $q = 2.5$ and the oscillation period is $T_H = 2\pi$.

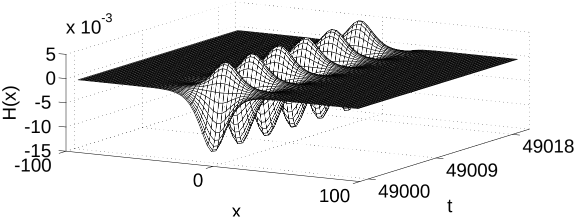

Figure: Evolution of the Higgs field H as a function of (x, t). Parameter values used are $\epsilon = 0.1$, $q = 2.5$ and the oscillation period is $T_H = \pi$.

- Oscillons are stable at least up to 10^4 oscillations.
- The numerical solution is exactly the same (up to the numerical error), to the analytical result for all times.
- **Using the same approach we wish to identify localized solutions to more complicated theories and also in higher dimensions**.