Exact results in $\mathcal{N} = 4$ super Yang-Mills and AdS/CFT

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arXiv: 1106.5418 [hep-th], B. Fiol and BG
arXiv: 1202.5292 [hep-th], B. Fiol, BG and A. Lewkowycz
arXiv: 1302.6991 [hep-th], B. Fiol, BG and GT

Erasmus Intensive Programme "Non-perturbative quantum field theory"
Crete, April 19, 2013

- Non-perturbative computations are hard!

- Exact results in 4D QFT are extremely hard/impossible...

- The situation improves A LOT with additional symmetries:
  CFTs, SUSY, ...

$\mathcal{N} = 4$ SYM is both conformal and maximally supersymmetric!

- Various techniques can apply:
  Integrability, Localization, AdS/CFT, ...

- We will focus on observables related with external probes of $\mathcal{N} = 4$ SU(N) SYM.

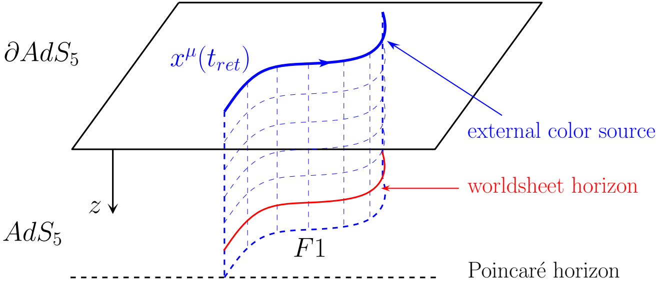
  ($\sim$ idealization of QCD with two parameters: λ and N)

- First, we will use AdS/CFT to obtain results valid to all orders in λ/N^2.

- Then we will use localization results to provide exact expressions, valid for all λ and N.

Fundamental representation $\longleftrightarrow$ **F1** $(AdS_2 \hookrightarrow AdS_5)$

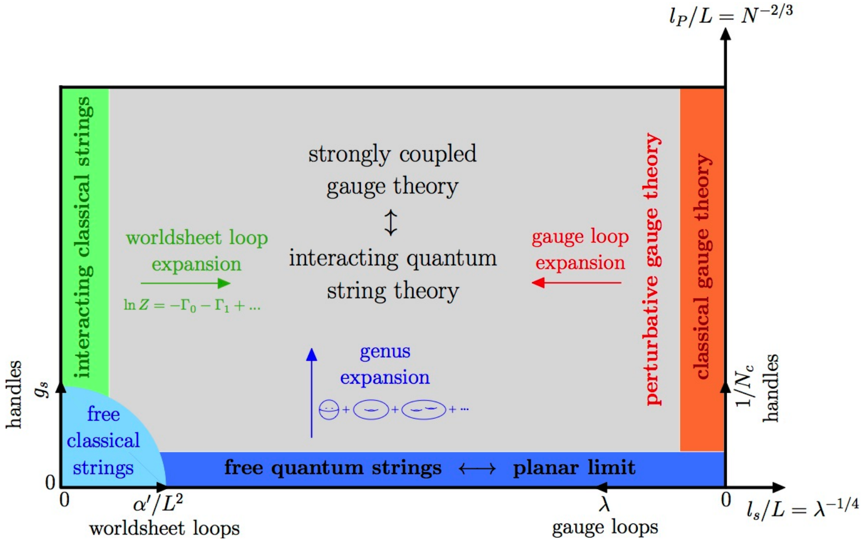
Consider a particle transforming in the fundamental representation of $SU(N)$. Its dual is a fundamental string, reaching the boundary of AdS at the particle word-line.



$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-|g|} = -\frac{\sqrt{\lambda}}{2\pi L^2}\int d^2\sigma\sqrt{-|g|}$$

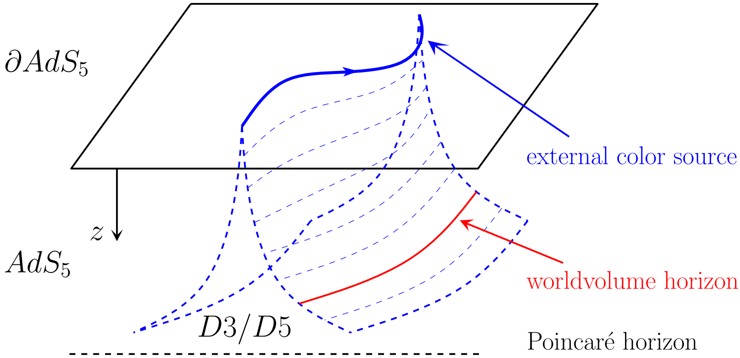
Fundamental representation $\longleftrightarrow$ **F1** $(AdS_2 \hookrightarrow AdS_5)$

Static particle	$\langle \mathcal{L}(\vec{x}) \rangle = \frac{1}{16\pi^2} \frac{\sqrt{\lambda}}{|\vec{x}|^4}$	[Danielsson *et al.*, Callan-Güijosa '98]
Accelerated particle	$P_{_F} = \frac{\sqrt{\lambda}}{2\pi} a^\mu a_\mu$	[Mikhailov '03]
Circular WL	$\ln \langle W(C) \rangle = \sqrt{\lambda}$	[Berenstein *et al.* '98]
Potential $q\bar{q}$	$V_{q\bar{q}} = -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})} \frac{\sqrt{\lambda}}{L}$	[Rey-Yee, Maldacena '98]
Momentum diff. coefficient	$\kappa = 4\pi\sqrt{\lambda}\, T^3$	[Xiao, '08]



$l_P/L = N^{-2/3}$
interacting classical strings
strongly coupled
gauge theory
↕
interacting quantum
string theory
worldsheet loop
expansion
$\ln Z = -\Gamma_0 - \Gamma_1 + ...$
gauge loop
expansion
perturbative gauge theory
classical gauge theory
genus
expansion
handles
g_s
free
classical
strings
free quantum strings ⟷ planar limit
$1/N_c$
handles
0
0
α'/L^2
worldsheet loops
λ
gauge loops
0
$l_s/L = \lambda^{-1/4}$

So far we discussed Strings $\Longleftrightarrow$ Fundamental WL
String theory also contains BRANES $\Longleftrightarrow$ Other probes?



$$S = S_{DBI} + S_{WZ}$$

D3, k units of electric flux $\longleftrightarrow$ k-symmetric rep.

D5, k units of electric flux $\longleftrightarrow$ k-antisymmetric rep.

[Drukker-Fiol, Hartnoll-Kumar, Yamaguchi, Gomis-Passerini'06]

Range of validity for D3:

$$\underbrace{\frac{N^2}{\lambda^2} >> \overbrace{k}^{\text{SUGRA approx.}}}_{\text{probe approx.}} >> \frac{N}{\lambda^{3/4}}$$

$k = 1$ excluded

Observable:	F1 Result:	D3 Result:
Static particle	$\langle \mathcal{L}(\vec{x}) \rangle_{S_k} = \frac{\sqrt{\lambda}}{16\pi^2 \, \lvert\vec{x}\rvert^4}$	$\times k\sqrt{1+\frac{k^2\lambda}{16N^2}}$
Accel. particle	$P_{S_k} = \frac{\sqrt{\lambda}}{2\pi} \, a^\mu a_\mu$	$\times k\sqrt{1+\frac{k^2\lambda}{16N^2}}$
Circular WL	$\ln\langle W(C) \rangle_{S_k} = \sqrt{\lambda}$	$\times \frac{k\sqrt{1+\frac{k^2\lambda}{16N^2}}}{2} + 2N \sinh^{-1}\frac{k\sqrt{\lambda}}{4N}$ [Drukker-Fiol'05]
Diff. coefficient	$\kappa = 4\pi\sqrt{\lambda}\,T^3$	$\times k\sqrt{1+\frac{k^2\lambda}{16N^2}}$ [Fiol-BG-GT '13]

All those observables can be computed exactly (to all orders in λ and N) using localization techniques (QFT side).

Surprising result:

String corrections at all $\frac{\lambda}{N^2}$ orders $\Longleftrightarrow$ D3 results for $k = 1$

OUT OF THE RANGE OF VALIDITY!

- Results of D3 branes carrying $k = 1$ units of electric flux match exact results obtained with other techniques (without AdS/CFT)

- This suggests an alternative (and simple!) calculational method.

- Exact results comparable to those of other formalisms (localization, etc.) $\Rightarrow$ Nontrivial AdS/CFT check.

- BUT: We still don't know why it works!

- Blackfold? Quantum Corrections?

- (Any insight?)