LIMITS ON MASSLESS PARTICLES

Steven WEINBERG and Edward WITTEN
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received 6 August 1980

We show that in all theories with a Lorentz-covariant energy–momentum tensor, such as all known renormalizable quantum field theories, composite as well as elementary massless particles with $j > 1$ are forbidden. Also, in all theories with a Lorentz-covariant conserved current, such as renormalizable theories with a symmetry that commutes with all local symmetries, there cannot exist composite or elementary particles with nonvanishing values of the corresponding charge and $j > 1/2$.

It has been known for many years that there are problems in the construction of lagrangian field theories for massless particles of higher spin. The difficulty is definitely not one of constructing free field theories. There are physically acceptable free-field lagrangians [1] for massless particles of arbitrary spin j, in which these particles are represented by tensor or tensor–spinor fields with j or $j - 1/2$ Lorentz indices, and the lagrangian satisfies a gauge invariance principle which eliminates unphysical degrees of freedom. Nor is there any difficulty in giving these particles interactions of some sort. For instance, we can construct interactions that trivially satisfy the gauge invariance conditions for higher spin, by simply requiring that a "curl" be taken on each Lorentz index. (Thus a spin-5/2 field $\Psi_{\mu\nu}$ would have to appear in the interaction in the form $\partial_\mu\partial_\nu\Psi_{\rho\sigma} - \partial_\mu\partial_\sigma\Psi_{\rho\nu} - \partial_\rho\partial_\nu\Psi_{\mu\sigma} + \partial_\rho\partial_\sigma\Psi_{\mu\nu}$.) Rather, the problem appears to do specifically with giving massless higher spin particles electromagnetic or gravitational interactions. The replacement of the derivatives in the lagrangians of ref. [1] with gauge-covariant or generally covariant derivatives yields a lagrangian that for high spin does not satisfy the appropriate higher-spin gauge invariance conditions, and leads to field equations that are not even algebraically consistent [2].

We find it difficult to believe that this problem is solely a limitation on the properties of *elementary* massless particles whose fields appear in the lagrangian, and that this limitation need to apply to composite particles. If there did exist massless higher spin composite particles with electromagnetic or gravitational interactions, then could we not describe their interactions by an effective lagrangian? There may well be a distinction between elementary and composite particles, but this distinction must have to do with whether or not they appear in the fundamental lagrangian, and not with whether they can appear in any lagrangian at all.

It seems likely to us instead that whenever it proves impossible to construct a lagrangian field theory for certain kinds of massless particles, then such particles simply cannot exist, whether elementary or composite [‡1]. In support of this view, we offer a pair of very simple theorems [‡2], which rule out higher-spin massless particles in certain contexts.

Theorem 1. A theory that allows the construction of a Lorentz-covariant conserved four-vector current J^μ cannot contain massless particles of spin $j > 1/2$ with nonvanishing values of the conserved charge $\int J^0\, d^3x$.

[‡1] There is an S-matrix-theoretical argument against the possibility of giving gravitational interactions to massless particles with $j = 5/2$, by Grisaru et al. [3].

[‡2] A result similar to our theorem 1 has been derived by Coleman [4].

Theorem 2. A theory that allows the construction of a conserved Lorentz covariant energy–momentum tensor $\theta^{\mu\nu}$ for which $\int \theta^{0\nu}\, d^3x$ is the energy–momentum four-vector cannot contain massless particles of spin $j > 1$.

These theorems are proved by studying the matrix elements $\langle p', \pm j \mid J^\mu \mid p, \pm j\rangle$ and $\langle p', \pm j \mid \theta^{\mu\nu} \mid p, \pm j\rangle$ of J^μ and $\theta^{\mu\nu}$ between one-massless-particle states of helicity $\pm j$ and four-momenta p' and p. We first note that under the assumptions of these theorems, the matrix elements cannot vanish in the limit $p' \to p$, and we then show that for all p' and p with $(p - p')^2 \neq 0$, the matrix element of J^μ must vanish for $j > 1/2$, and the matrix element of $\theta^{\mu\nu}$ must vanish for $j > 1$.

To see that the matrix elements do not vanish for $p' \to p$, note that Lorentz invariance dictates their form in this limit to be [‡3]

$$\langle p' \mid J^\mu \mid p\rangle \to g p^\mu / E(2\pi)^3 , \tag{1}$$

$$\langle p' \mid \theta^{\mu\nu} \mid p\rangle \to f p^\mu p^\nu / E(2\pi)^3 . \tag{2}$$

The coefficient g is the one-particle value of the charge $\int J^0\, d^3x$, and does not vanish by hypothesis. The quantity $f p^\mu$ is the one-particle value of the energy–momentum four-vector $\int \theta^{0\nu}\, d^3x$, and so $f = 1$.

To show that the matrix elements *do* vanish for high spin, we adopt a Lorentz frame in which

$$p = (\boldsymbol{p}, |\boldsymbol{p}|) , \quad p' = (-\boldsymbol{p}, |\boldsymbol{p}|) . \tag{3}$$

(This is always possible for $(p' - p)^2 \neq 0$, because then $p' + p$ is timelike, and we need simply choose a frame in which $p' + p$ has no space component.) Consider the effect of a rotation $R(\theta)$ by an angle θ around the $\boldsymbol{p}$ direction. The one-particle states undergo the transformations

$$|p, \pm j\rangle \to \exp(\pm i\theta j)\, |p, \pm j\rangle , \tag{4}$$

$$|p', \pm j\rangle \to \exp(\mp i\theta j)\, |p', \pm j\rangle . \tag{5}$$

(The difference of the signs in the exponents arises because $R(\theta)$ is a rotation of $+\theta$ around $\boldsymbol{p}$ but of $-\theta$ around $\boldsymbol{p}' = -\boldsymbol{p}$.) Rotational invariance tells us then that

$$\exp(\pm 2i\theta j) \langle p', \pm j \mid J^\mu \mid p, \pm j\rangle = R(\theta)^\mu{}_\rho \langle p', \pm j \mid J^\rho \mid p, \pm j\rangle , \tag{6}$$

$$\exp(\pm 2i\theta j) \langle p', \pm j \mid \theta^{\mu\nu} \mid p, \pm j\rangle = R(\theta)^\mu{}_\rho R(\theta)^\nu{}_\sigma \langle p', \pm j \mid \theta^{\rho\sigma} \mid p, \pm j\rangle . \tag{7}$$

But the rotation matrix $R(\theta)$ has Fourier components $e^{i\theta}$, 1, and $e^{-i\theta}$ only, so these equations require the matrix element of J^μ to vanish unless $2j = 0$ or 1, and the matrix element of $\theta^{\mu\nu}$ to vanish unless $2j = 0,1$ or 2. Since these matrix elements thus vanish for $j > 1/2$ or $j > 1$ in the special Lorentz frame defined by eq. (3), and the helicities of massless particles are Lorentz invariant while J^μ and $\theta^{\mu\nu}$ are assumed to be Lorentz covariant, the matrix elements would have to vanish in all frames, and hence for all p' and p with $(p' - p)^2 \neq 0$. This concludes the proof.

Of course, there are acceptable theories that have massless charged particles with spin $j > 1/2$ (such as the massless version of the original Yang–Mills theory), and also theories that have massless particles with spin $j > 1$ (such as supersymmetry theories or general relativity). Our theorem does not apply to these theories because they do not have Lorentz-covariant conserved currents or energy–momentum tensors, respectively. For instance, interpreting the Yang–Mills theory as a theory of charged massless spin-one particles and photons, the electric current is

$$J^\mu = e\, \mathrm{Im}[B^{\mu\dagger}(\partial_\mu B_\nu - \partial_\nu B_\mu) - \partial^\mu (B_\mu^\dagger B_\nu)] ,$$

where B^μ is the complex field of the charged bosons. This is not a four-vector, because under a Lorentz transformation $x^\mu \to \Lambda^\mu{}_\nu x^\nu$ the boson field B^μ transforms into $\Lambda^\mu{}_\nu B^\nu$ plus a term proportional to a derivative $\partial^\mu \Phi$. Similarly, the energy–momentum pseudotensor in general relativity is not a Lorentz tensor,

‡3 It is important that we define g and f in terms of limits of matrix elements as the momentum transfer $p' - p$ approaches zero, and not in terms of the values of matrix elements at $p' - p = 0$. Our definition corresponds to the method by which charges, energies, and momenta are actually determined: by measuring the nearly forward scattering caused by exchange of spacelike but nearly lightlike massless vector bosons or gravitons, or else by evaluating $\int_V J^0 \times d^3x$ or $\int_V \theta^{0\mu}\, d^3x$ for a large but finite volume V. Of we had defined g and f in terms of matrix elements with $p' - p = 0$, we could not have used the vanishing of the higher-spin matrix elements for $(p - p')^2 \neq 0$ to conclude that $g = 0$ for $j > 1/2$ and $f = 0$ for $j > 1$, without invoking an assumption of continuity between spacelike and lightlike momentum transfers. Such a continuity assumption seems entirely plausible, but with our definition of f and g, it is not necessary.

because it involves the gravitational field $h_{\mu\nu}$, which has a non-tensor behavior under Lorentz transformations. (Of course, we can make J^μ or $\theta^{\mu\nu}$ into tensors by introducing unphysical helicity states, such as longitudinal and timelike charged bosons in the Yang–Mills theory, but then the proof of our theorem would be invalid because the helicities of physical states would not be Lorentz invariant.)

Our theorem does apply to all known renormalizable quantum field theories such as quantum chromodynamics, with the qualification that theorem 1 only applies to conserved currents associated with symmetries that commute with any local symmetries. It can be shown by direct construction that these theories have a Lorentz-covariant energy–momentum tensor $\theta^{\mu\nu}$, and a Lorentz-covariant Noether current J^μ for all symmetries that commute with local symmetries. Thus we conclude that all these theories have no massless bound states with $j > 1$, and quantum chromodynamics has no flavor nonsinglet massless bound states with $j > 1/2$.

It is perhaps not surprising that ordinary field theories like quantum chromodynamics do not have massless bound states of high spin. What is somewhat surprising is that this result can be proved so easily, and with such generality.

We close by noting some applications of this result to current research.

Because of the difficulties with general relativity as a quantum theory, it has occasionally been suggested that the graviton is not an elementary particle, but a massless spin-two bound state that arises in an ordinary renormalizable field theory [5]. (The couplings of a massless spin-two particle, composite or not, must at low energies mimic general relativity.) However, our theorems rule out this possibility [‡4].

Our theorems also remove one objection to the idea that the familiar quarks and leptons are bound states of more nearly fundamental particles. It might be thought that in this case, quarks and leptons would have to form closely lying states of differing angular momenta (like atoms and nuclei) in disagreement with the observation that quarks and leptons all seem to have spin 1/2. However, the absence of any detectable quark or lepton structure indicates that quark and lepton masses are very much less than the characteristic energy scale of the binding forces, so it seems reasonable to suppose that in the absence of electroweak or other perturbations, the quark and lepton masses would vanish. Assuming that the constituent particles and binding forces are described by an ordinary renormalizable theory, our theorem then rules out any other particles with $j > 1$ that would also be massless in the absence of electroweak or other perturbations to the binding forces. There may well exist excited quarks and leptons with high spin, but they are likely to have masses of the order of the characteristic scale of the binding forces, and hence to be much heavier than the ordinary quarks and leptons.

In particular, our theorems also close a loophole in recent discussions [6] of the implications of Adler–Bell–Jackiw triangle anomalies. It has been argued that theories with triangle anomalies in the amplitudes of conserved currents must involve massless untrapped physical particles. Our theorems confirm the conjecture by 't Hooft [6] that these massless particles would have to have spin zero or one-half. Our theorems do leave open the possibility that there are massless spin-one particles that are neutral under all symmetries that commute with gauge symmetries, but there is an argument by Frishman et al. [6] that such particles could not produce the anomaly, so that there would still have to be massless particles of spin zero or one-half.

[‡4] However, the theorem clearly does not apply to theories [7] in which the gravitational field is a basic degree of freedom but the Einstein action is induced by quantum effects.

We are grateful for valuable conversations with S. Coleman, S. Deser, M. Grisaru, Y. Ne'eman and H. Schnitzer. This research is supported in part by the National Science Foundation under Grant No. PHY77-22864.

References

[1] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) 173 (1939) 211;
W. Rarita and J. Schwinger, Phys. Rev. 60 (1941) 61;
J. Schwinger, Particles, sources, and fields (Addison-Wesley, Reading, MA, 1970);
L.P.S. Singh and C.R. Hagen, Phys. Rev. D9 (1974) 898, 910;
A. Kawakami and S. Kamefuchi, Nuovo Cimento 48A (1976) 239;
C. Fronsdal, Phys. Rev. D18 (1978) 3624;
J. Fang and C. Fronsdal, Phys. Rev. D18 (1978) 3630;

F.A. Berends, J.W. van Holten, P. van Nieuwenhuizen and B. de Wit, Phys. Lett. 83B (1979) 188; Nucl. Phys. B54 (1979) 261;
T. Curtright, Phys. Lett. 85B (1979) 219;
C. Aragone and S. Deser, Phys. Rev. D21 (1980) 352; and to be published in Nucl. Phys. B (1980);
B. de Wit and D. Freedman, Phys. Rev. D21 (1980) 358.

[2] C. Aragone and S. Deser, Phys. Lett. 85B (1979) 161;
F.A. Berends, J.W. van Holten, B. de Wit and P. van Nieuwenhuizen, J. Phys. A Math. Gen. 13 (1980) 1643;
B. de Wit and D. Freedman, ref. [1];
for earlier discussions of related difficulties, see K. Johnson and E.C.G. Sudarshan, Ann. Phys. (NY) 13 (1961) 126;
G. Velo and D. Zwanziger, Phys. Rev. 186 (1967) 1337.

[3] M.T. Grisaru, H.N. Pendleton and P. van Nieuwenhuizen, Phys. Rev. D15 (1977) 996.

[4] S. Coleman, unpublished.

[5] G. Papini, Nuovo Cimento 39 (1965) 716;
H. Terazawa, Y. Chikashigi, K. Ikama, and T. Matsuki, Phys. Rev. D15 (1977) 1181;
F. Cooper, G. Guralnik and N. Snyderman, Phys. Rev. Lett. 40 (1978) 1620.

[6] G. 't Hooft, Cargese lectures (1979);
Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, Weizmann Institute preprint WIS-80/27 (1980);
S. Coleman and E. Witten, Phys. Rev. Lett. 45 (1980) 100.

[7] A.D. Sakharov, Dokl. Akad. Nauk SSSR 177 (1967) 70 [Sov. Phys. Dokl. 12 (1968) 1040];
O. Klein, Phys. Scr. 9 (1974) 69;
P. Minkowski, Phys. Lett. 71B (1977) 419;
A. Zee, Phys. Rev. Lett. 42 (1979) 417;
L. Smolin, Nucl. Phys. B160 (1979) 253;
K. Ikama, Y. Chikashigi, T. Matsuki and H. Terazawa, Prog. Theor. Phys. 60 (1978) 868;
S. Adler, Phys. Rev. Lett. 44 (1980) 1567; Phys. Lett. 95B (1980) 241;
B. Hasslacher and E. Mottola, Phys. Lett. 95B (1980) 237.